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A DYNAMIC SUPPLY SIDE GAME APPLIED TO THE EUROPEAN GAS MARKET

by

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Abstract

This paper discusses optimal investment plans for large gas exporters to Western Europe. We discuss market power on the supply side, while assuming price taker behaviour on the demand side.

A static game approach is compared to a dynamic, and we argue that the use of static game models does not capture important market forces. A dynamic Nash game, where the players can observe previous actions and react according to them, is introduced.

Contents

Part I
INTRODUCTION

1 The background

This paper discusses optimal investment plans for a large gas exporter to Western Europe. There are 4 major suppliers to this market: The Soviet Union, the Netherlands, Algeria and Norway. The UK is also a large producer, but still a net importer (Frigg-gas from Norway), and so far no link to the continental Europe is planned.

Since the European gas market is a regional market with a limited number of buyers and sellers, game theory is appropriate as an analytical approach. In this paper we shall analyze the consequences of market power on the supply side, assuming price taker behaviour on the demand side, in spite of the fact that the buyers are represented by national monopsonies (the distribution companies Ruhrgas, Gas de France, Gasunie, etc. are dominant firms). We make this choice so as to be able to study the relationship and interdependence between sellers in an oligopoly. By giving up some realism in this respect, we are able to construct a numerical model.

In the short run, production of natural gas is constrained by the existing capacity and exports are settled through contracts between buyers and sellers. In the longer term, however, the capacity limits can be extended, contracts can be renegotiated, and the problem is to decide on optimal capacity and investments within a planning horizon. In the case of price taking suppliers this is a fairly simple problem. For a given expected price path, the best project is put onstream when the price p exceeds some value p1 at some point of time, then the next best project will follow when the price exceeds the value $p2$, $p2 > p1$, etc. Calculations of this type were done by M. Hoel for the government committee "Tempoutvalget" (1983). Hoel recommended a rapid development of Norwegian off-shore oil fields. The price taker assumption may still be defended for oil, since the Norwegian share of world oil production is small.

Similar calculations implying price taking behaviour for gas suppliers to Continental Europe, however, are not plausible. To our knowledge, little has been done to shed light on the problem of how to extract non-renewable resources in an oligopolistic market of this type. An exception is a spatial trade model for the European natural gas market presented by Mathiesen, Roland and Thonstad (1986), which also discusses degrees of market power on the selling side. This model has a short term Nash Cournot solution (1983) when production capacities are given, and a long term solution (2000) when the players are allowed to invest in new capacity. Like other trade models the model focuses on other aspects than those of primary interest here, and was designed to describe trade patterns under simplistic behavioral assumptions like competitive, Nash Cournot and collusive.

We begin with a discussion of the use of static versus dynamic game models. We will argue that the use of a static game model will not capture important market forces. A dynamic Nash game is therefore introduced. In a static game, the players are not allowed to react to the moves of the other players. The players have only one move each, and they are moving simultaneously. In a dynamic game, the players can observe previous actions by the other participants and react according to them. We show that if one player makes an investment, the potential payoffs on all other possible investments are reduced. If the players are allowed to consider the consequences in later periods of their actual investments, they are acting strategically. In a dynamic model the players maximize discounted sum of payoffs over the planning horizon. Compared to a static game, we find that the solution leads to increased investments and production in all periods. The investors are willing to sacrifice large profits in

Supplies	USSR	Norway	Algeria	Total Imports	Total Consumption	Marketshare USSR
Buyers						
Austria	3.8			3.8	5.2	73.8
Belgium		1.7	2.3	4.0	9.4	
France	6.8	2.8	7.7	17.3	26.2	25.9
Italy	5.9		7.8	13.7	30.7	19.2
Netherlands		1.9		1.9	37.1	
West Germany	12.1	6.5		18.6	46.5	26.0
Others		0.7	1.6	2.3	7.4	
Total :	28.6	13.6	19.4	61.6	162.5	17.6

Source: BP review of word gas 1986.

Table 1: Imports of gas to the European Continent 1985 (bcm)

early periods to gain market shares and later profits. This means that an investment project may appear profitable, even though the market does not seem mature enough for it.

2 A brief description of the market

2.1 The Supply side

The indigenous production of gas on the Continent was about 109 bcm in 1985 (Source: BP 1986), most of it being produced by the Netherlands. Estimates of gas reserves in the Netherlands have been growing recently, mainly due to the upgrading of known fields. Thus the Netherlands will be a dominant producer for many years to come. We have, however, excluded the Netherlands from the supply side game to reduce the number of players. The reason is that we are focusing on the investment game, not on the static supply game for given investments. The Dutch problem is how to extract developed fields rather than how to invest optimally in new fields.

The remaining share of the gas demand is mainly covered by imports from the Soviet Union, Norway and Algeria. The UK is assumed to stay a net importer, and is thus not included in this market.

2.1.1 The Soviet-Union

The Soviet Union with its huge recoverable reserves has the greatest potential to supply gas to Western Europe. However, there is a general political goal that imports from the Soviet Union should be limited. EC has recommended that imports from the Soviet Union should not be higher than 30 % of total consumption of gas in each country. At present only Austria exceeds this limit (see table 1). The total market share is about 17 % of total consumption. These political limitations are dependent on the present political situation. Attitudes could change over the time horizon used in this model.

The trade balance in USSR is strongly dependent on supply of western currency through sales of petroleum. Crude oil reserves are small compared to the natural gas reserves. The gas trade will thus play a more important role in the economy of the Soviet Union in the future.

Exports of natural gas from USSR to the European Continent in 1985 was about 29 bcm. Exports can be doubled by utilizing the excess capacity of the existing pipeline. The transportation capacity can be further increased by adding compressors or new pipelines.

2.1.2 Norway

Norway has the largest reserves of natural gas in Western Europe. All the fields are off-shore and expensive to develop. Norwegian exports to the European Continent in 1985 was 13.6 bcm, mainly produced from the fields Ekofisk and Valhall.

Increased Norwegian gas exports will require exploitation of the large gas reserves of the Troll and Sleipner fields. Sleipner can increase the export by 8.8 bcm/year. Troll can be divided in two steps, eastern and western Troll. According to the Troll Contract (Stortings Prop. Nr.1 (1987)) the first Troll platform will increase the export capacity of 24 bcm/year. We assume that a similar volume can be extracted later on Western Troll. The development costs of these fields are large due to deep waters.

2.1.3 Algeria

The economic situation of Algeria makes the country strongly dependent on the incomes it receives from exports of natural gas. The government has insisted on a gas price based on parity with the price of crude oil. This gas price has exceeded the market price, making Algeria unable to sell all the gas they can produce. Algeria has recently been forced to abandon this policy.

Gas from Algeria is distributed partly trough a pipeline to Italy and partly as LNG (Liquified Natural Gas) shipments. Algeria exported 19.4 bcm to Western Europe in 1985. The exports can be increased by utilizing idle LNG capacity, and increasing the transport through pipelines by installing compressors or building a new pipeline to Europe.

2.2 The Demand side

Total consumption on the Continent was 162.5 bcm in 1985: the main consumers are West Germany (29 % of total consumption), France(16 %), Italy (19%) and the BeNeLux countries (29 %). The import situation in 1985 is listed in table 1. 90 % of the total imports are consumed by West Germany, France and Italy.

The Continental natural gas market developed rapidly in the 1960-ies and early 1970-ies. The gas grid is developed and covers 83 % of the population in Western Germany, 84% in Belgium, 69% in France, 59% in Italy and 97% in The Netherlands (Colloque internationtal de marketing gazier (1986)).

3 Alternative gas market models

Competitive models assuming consumers and producers as price-takers have been used in a number of energy market models. An analysis of the European gas market is done in Dahl and Gjelsvik (1987). They have a two-good (oil and gas) model with 5 demand regions: France, Vest Germany, Belgium, Netherlands and Italy (all buyers of Troll-gas). Netherlands, USSR, Algeria and Norway are gas suppliers and there are a number of oil suppliers. The model determines the prices and quantities which maximize social surplus (consumer surplus + producer surplus - transportation costs) for given supply and demand curves in each region.

An attractive property is that the model gives a simultaneous equilibrium in oil and gas, and the model is easily expanded to more goods and regions. A competitive model may be attractive, but hard to adapt to real markets. The model simulates better with constraints on transport arcs, making trade flows more diversified. A more doubtful assumption is the modeling of the supply side as static price takers. Neither market power nor dynamics are captured.

Static game models has been introduced as an alternative. The Nash Cournot model has an intuitive appeal in markets with a few suppliers and many buyers. Some may find it difficult to accept a passive demand side in the European gas market. But given the demand curves, capacities and short term marginal cost curves, the Cournot solution is identic to the Nash solution. This is the short term solution in Mathiesen & al.(1986.)

The problems arise when the static model is used to solve a dynamic optimization problem. Suppose we want to make a forecast for the year 2000. We know the demand functions in all future years, and the suppliers are assumed to be non-cooperative Nash players. (For a further description of the concept, see section 4.) Now, both capacities and supplies are endogenous. The problem can be thought of as finding optimal supply for each possible vector of capacities. This game can be formalised in the following way:

Suppose optimal capacities and production is to be decided simultaneously. $Q = (q_1, \ldots, q_i)$ denotes the production vector and Q_i the production vector Q except the production of supplier i (q_i). Similar, let $K = (k_1, \ldots, k_i)$ be the vector of production and K_i be the vector of capacities except k_i

The payoff is given by :

$$\forall\, i \;:\; \pi_i = \pi_i(q_i, Q_i, k_i, K_i) \tag{1}$$

The Nash solution Q_i^*, K_i^* (if it exists) is defined by

$$\forall\, i \;:\; \forall\, q_i, k_i \;:\; \pi_i(q_i^*, Q_i^*, k_i^*, K_i^*) \geq \pi_i(q_i, Q_i^*, k_i, K_i^*) \tag{2}$$

To make this problem easy to solve, Mathiesen & al assumes that the sample room of capacities is discrete, i.e. there are a few, actual investment projects to be considered, like new pipelines and some field developments. These projects may be arranged in a natural order, i.e. by increasing unit costs. Furthermore, variable unit costs are assumed constant until production approaches capacity limits, when costs are increasing sharply.[1] The long term marginal cost curves are then continuous, step wise increasing functions. The solution of the model is defined by those investments and quantities which maximise profits in the year 2000.

The game is illustrated in figure 1. The initial capacities (1983) are given (K^0). In period 3 (2000) the demand function D^3 is known and the players find optimal capacities K^3. This is a simultaneous one-move-game, and the market development before and after year 2000 are not (explicitly) considered. The players are not even allowed to observe investments completed in the period 1983-2000.

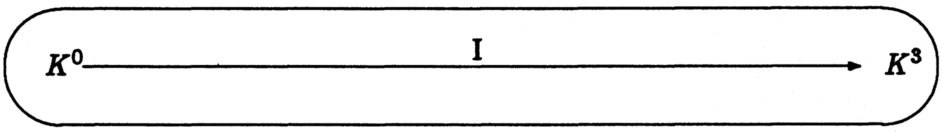
In the same figure another model is illustrated. The 1983-2000 period is divided in three periods, (1,2,3). In each period the players decide optimal capacity in the beginning of next period. The players are acting as if each period was the last. I.e the responses to their actions in succeeding periods are not taken into consideration.

This gives three games of the same type as in Mathiesen & al. (1986), with a closer horizon. In period 2, the players can observe the investments done in the first period, and optimize given this information. Otherwise, there is no difference between this sequence of games and the one step game above. Still the games are static, in period 1 they do not consider the consequences in later periods of their actions.

Obviously the solution in 2000 of these games will be different from the one step game. The solution will also depend on length of time periods (or number of games in period 1983-2000). This shows that both models discussed so far are not satisfying and lead to suboptimal solutions.

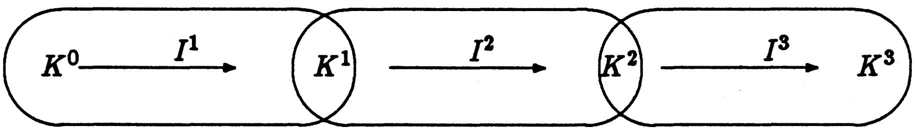
[1] Mathiesen & al.(1986) p.17,fig.2

MRT-MODEL-ONE STEP STATIC MODEL



Max π *s.t* K^0 and D^3

A SEQUENCE OF STATIC GAMES

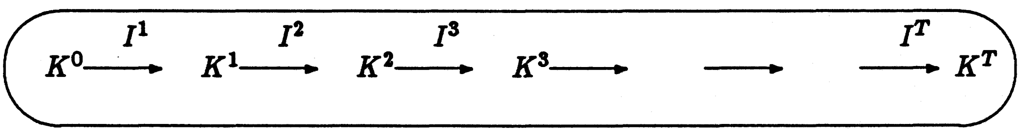


Max π^1 *s.t* K^0, D^1

Max π^2 *s.t* K^1, D^2

Max π^3 *s.t* K^2, D^3

DYNAMIC GAME



$$\text{Max} \sum \frac{\pi^t}{(1+r)^t} \; s.t. \; K^0, D^t, \quad t = 0 \ldots, T$$

K^t	:	Capacity vector
I^t	:	Investment vector
D^t	:	Demand
π^t	:	Profit
boxes	:	Indicate game structure.

Figure 1: Illustration of different games

The lower part of figure 1 shows a dynamic game. In period 0 the players make a total plan of actions for all subsequent periods (T). In the beginning of each period they observe the actions (here investments) done in the last period and make their investment decisions according to their investment plans. This means they must have a plan for how to act in every possible situation in every period. This game may be difficult to solve, but if a solution is found, it will be an optimal solution to the dynamic problem. A further description of this game is given in part II.

Part II
NASH EQUILIBRIA AND DYNAMIC GAMES

4 Nash equilibrium

Consider a game with N players. Each player has a finite number of alternative pure strategies[2] Denote a pure strategy for player i as $s_i \in S_i$. Let $s = (s_1, \ldots, s_N)$ be a vector of strategies. The payoff of player i is a function of the strategy vector $\nu_i = \nu_i(s)$. A strategy vector s^* is a Nash equilibrium if:

$$\forall i : \forall s_i \in S_i : \nu_i(s^*) \geq \nu_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_N^*) \tag{3}$$

A Nash equilibrium may not exist, or there may be many of them. It is possible to prove that a Nash equilibrium always exists in mixed strategies, but the problem of which to choose if there are many would remain. We only consider pure strategies in this paper.

A possible defence of the Nash equilibrium concept is given by Leif Johansen (1984). He gives four postulates of rationality in a game theoretical context, and argues that the only solution satisfying all these postulates is the Nash solution. The postulates are [3]:

Postulate 1. *A player makes his decision on the basis of, and only on the basis of, information concerning the action possibility set of all players, and the preference functions of all players.*

Postulate 2. *When choosing his own decision, a player assumes that the other players are rational in the same way as he himself is rational.*

Postulate 3. *If a decision is the rational decision for an individual player, then this decision can be correctly predicted by other players.*

Postulate 4. *Being able to predict the actions to be taken by other players, a player's own decision maximizes his preference function corresponding to the predicted action of other players.*

The ideas behind the postulates is roughly: If s^* is the set of rational decisions for all players, then by postulate 3, player i will know the rational decision of the others. By postulate 4 he thus has to choose an action according to (3), which gives a Nash equilibrium.

The postulates ensure that the solution will be unique. Postulate 3 implicitly assumes that the solution is unique. The problem is now what do the players do when there are two or more Nash equilibria. We have to dispense from one of the postulates if we want to find a "rational" decision for the players, i.e. postulate 3 cannot hold if there are two equilibria. It is beyond the scope of this paper to give a definition of rationality that applies to this situation.

[2] We assume that the reader is familiar with the distinction between pure and mixed strategy. If not, a definition will be found in most textbooks in game theory, f.ex. Basar and Olsder (1982).

[3] The postulates have been slightly reformulated to avoid introducing Johansen's notation.

5 Dynamic versus static games

We will follow Basar and Olsder (1982) and call agame dynamic if at least one player is allowed to use a strategy that depends on other players' previous actions.

Some examples will make this clear. Consider a usual Stackelberg game with two players. Player one moves first. We denote his set of possible actions A_1. Since he has not observed any previous action, his set of possible strategies is equal to the set of possible actions $S_1 = A_1$. Assume his choice is the strategy $s_1 = a_1 \in A_1$. Player 2 will observe this action before he makes his move. The action of player 2 depends on a_1. A strategy for player two is a plan for what to do in every possible situation. That is a mapping $s_2 : A_1 \rightarrow A_2$ where A_2 is the set of possible actions for player 2. Thus player 2 is allowed to use strategies that prescribes different actions to different s_i. *This game is dynamic.*

It is important to note the distinction between the set of possible actions A_i and the set of strategies S_i, which is the set of mappings from the set of possible previous actions to the set of possible actions. Another important observation is that the Stackelberg solution is in fact a Nash equilibrium. Let $s_2^*(a_1)$ be chosen to satisfy:

$$\forall a_1 \in A_1 : \nu_2(a_1, s_2^*(a_1)) = \max_{a_2 \in A_2} \nu_2(a_1, a_2) \tag{4}$$

and let $s_1^* = a_1^*$ satisfy:

$$\nu_1(a_1^*, s_2^*(a_1^*)) = \max_{a_1 \in A_1} \nu_1(a_1, s_2^*(a_1)) \tag{5}$$

Then (s_1^*, s_2^*) is a Nash equilibrium, and $(a_1^*, s_2^*(a_1^*))$ is a Stackelberg solution.

An important example of a game that is *not* dynamic is the following: There are T periods and 2 players. Let the set of possible actions for player i in period t be A_{it}, and assume that it does not depend on previous actions. If we restrict the set of possible strategies for player i to the set of vectors $s_i = (a_{i1}, \ldots, a_{iT})$ where $a_{it} \in A_{it}$ *then this game is not dynamic.* The reason is that no player is allowed to use a strategy that depends on previous observed actions. But, to cite Basar and Olsder: ".., by an abuse of language, such games is often also called dynamic."

This last game would, however become dynamic if we allowed at least one of the players to use a strategy that is a (non-constant) function of the other players' previous actions. If all players are allowed to choose their strategy from the set of all such mappings, then we call the solution to the game the *closed loop solution.* The solution of the static game described above is called the *open loop solution.*

This last point is very important. Let us look at an example with 2 players and two periods, and let $A_{it} = \{L, R\}$ for all it. The game is static if the set of strategies for i is restricted to the set of vectors :$\{(L, L), (L, R), (R, L), (R, R)\}$. The interpretation of the strategy (L, R), is that i plans to play L the first period, and R in the second period (no matter what happened in the first). The interpretation of the other alternatives are similar.

On the other hand the game is dynamic if the set of strategies for i is the set of (s_{i1}, s_{i2}) where $s_{i1} \in \{L, R\}$ but where s_{i2} is in the set of all mappings $\{L, R\} \rightarrow \{L, R\}$. To take an example let $s_{11} = L$ and $s_{12}(L) = R$ and $s_{12}(R) = L$ be a strategy for player 1 (P1). This strategy says that P1 will play L the first period, while in the second period he will play R if P2 has played L the first period, and L if P2 has played R.

We have to introduce some additional notation. Let x_{it} be typical variables indexed by player and time (like a_{it} and s_{it}). We can form two types of vectors of these variables. We use the notation $x_i = (x_{i1}, ..., x_{iT})$, and $x_{.t} = (x_{1t}, ..., x_{Nt})$. x is the vector of all x_{it}.

Since the equilibrium in a dynamic game is a set of mappings, it is obvious that in many interesting applications, the equilibrium will be very complex. To make it more transparent we do *not* in this paper present the equilibrium strategies, but only the resulting actions. Let s^* be the equilibrium set of strategies. In the first period there is no previous actions, so the strategies are just actions. I.e. $s^*_{.1} = a^*_{.1}$. In the rest of the periods the resulting actions will be defined as $a^*_{.t} = s^*_{.t}(a^*_{.1}, \ldots, a^*_{.t-1})$. The equilibrium is then presented as $a^* = (a^*_{.1}, \ldots, a^*_{.T})$.

A reason for choosing this concept of dynamics is that it corresponds with what we have called "strategic behaviour". By saying that a player in a game behaves "strategically", we mean that he takes into consideration the effect of his actions on the actions of the other players. If the other players are not allowed to use strategies that depend on his actions, then his action has of course no effect on the other players' actions. "Strategic behaviour" is thus eliminated in static games.

5.1 Perfect equilibrium

An important concept in dynamic games is Selten's concept of a perfect equilibrium (Selten 1965). It is possible that there exists Nash equilibria (s_1, s_2) for which s_2 does not satisfy (4) such that: if Player 2 uses strategy s_2 and player 1 chooses his strategy s_1, as in (5) (using s_2 instead of s_2^*), then player 2 will be better off than using s_2^*. The idea is that player 2, by threatening to be irrational, forces player 1 to use a strategy that is good for player 2. But this is not a reasonable solution, because it implies that player 2 will not be maximizing his payoff, given the action of player 1. Player 1 has good reasons to believe that P2 will change to s_2^* when P1 has made his move, and the threat of playing s_2 is thus empty. The idea of perfect equilibrium is to eliminate such solutions.

A strategy in a dynamic game is a total plan for what to do in every possible situation. For a set of such plans to be a perfect equilibrium it must be optimal for the players to stick to the plan in every possible situation.

We will in this paper consider only perfect equilibria.

5.2 Strategic behaviour

Assume we have found a perfect equilibrium s^* with the corresponding actions a^*. In the game presented in this paper the actions a_{it} will be investments. Assume that player i undertakes a large investment in period t. Is he then making this investment simply because the "market is mature", or is it to prevent other players from investing the next period? There are two steps in answering this question, and we define the investment to be strategically motivated if we get an affirmative answer in both the two following tests:

First test : Would other players increased their investments in period $t+1$, if i had not undertaken his investment in period t?.

> This is a well posed question, because $s^*_{j,t+1}$ for $j \neq i$ is a function of previous actions. We then just change a^*_{it} to an action a_{it} implying less investment, and examine whether or not the value of $s^*_{j,t+1}$ is changing. Note that it is sufficient to examine the behaviour in period $t+1$ since, if the actions of the other players are not changed in period $t+1$, player i has the opportunity to delay the investment a_{it} to period $t+1$, without affecting the other players' optimal reply.

Assume that if i had not undertaken the investment, then another player would have undertaken a larger investment the next period. But this does not imply that the market is not mature. It might be the case that as the other players are acting, the market simply becomes mature for investment a_{it} in period t.

The second test is based on the idea that if a player postpones an investment in the optimal solution at least one of the other players will undertake a larger investment the next period.

The question is then, is the player investing to prevent the other players from investing, or would he have done the investment anyway ?

Second test : Would he have made the investment later, if he considered the future actions of the other players to be fixed ?

Let $_ia^*$ be the set of actions of all players except i. What is optimal for i if he knew that the other players would choose the actions $_ia^*$ in all future periods? Let $\nu_i(a_i,_i a^*)$ be i's payoff if he chooses a_i and the others move $_ia^*$. Let $\overline{a}_i$ be defined by $\nu_i(\overline{a}_i,_i a^*) = \max_{a_i} \nu_i(a_i,_i a^*)$. The answer to the posed question is positive if $\overline{a}_i \neq a_i^*$.

To learn more about the impact of dynamics, you can compare a static game to a dynamic one. It seems most natural to compare it with the open loop solution, since this can be considered as the static game version of the same model. However, the solution might be very different from the closed loop solution, and thus it will be difficult to distinguish the different effects working at the same time.

Part III
A DYNAMIC INVESTMENT GAME MODEL

Before describing the theoretical model, we will look at an example that gives the main ideas of this model. We then describe the general model briefly.

6 An example

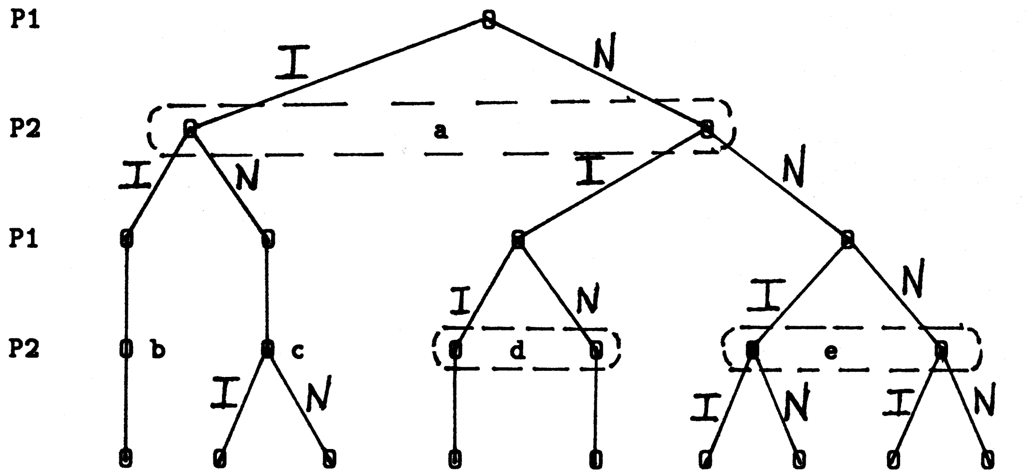
There are two producers both producing at full capacity. Initially they produce one unit of gas per period. Both are able to make one and only one investment that will increase their production capacity by one unit from the period the investment is made. This irreversible investment costs 2. Let $p_t(q)$ be the price at time t when q units are produced. Then suppose that in the first period the prices are: $p_1(2) = 11$, $p_1(3) = 7$ and $p_1(4) = 5$, and in the second period the prices are $p_2(2) = 15$, $p_2(3) = 11$ and $p_2(4) = 6$. The game lasts two periods and in each period the players make their decisions simultaneously.

Let I denote the action "to invest", and N the decision "not to invest". The game with all possible paths of actions is illustrated in figure 2. To illustrate how a payoff is computed, let us look at the case where (N,I) is played in the first period and P1 plays I in the second. This is node 4. The first period P1's production is still 1, since he has not made any investment, while P2 has carried out his investment and produce 2. Total production is 3. Hence P1's payoff the first period is $p_1(3) = 7$ and P2's is $2p_1(3) = 14$. The second period they both produce 2 and both earn $2p_2(4) = 12$. Both have used 2 to invest. This sums up to $(7 + 12 - 2, 14 + 12 - 2) = (17, 24)$.

Note that in the first period P2 does not know the action of P1. Thus his strategy can not depend on this action. This is indicated in figure 2 with a stippled line indicating the information set, within which the strategies are to be constant. An example may make this point clearer: In figure 2 it looks like P2 makes his first move after P1 has made his move. To indicate that this is not so we have drawn the information sets. The point is that P2 is not able to distinguish the two points in a. Thus his strategy must prescribe the same action in both these points. When he makes his second move, however, he can distinguish 4 different sets b-e, and his strategy might depend on witch set he is in.

6.1 The set of possible strategies

To make this game a dynamic game we must allow the players to use strategies that depend on previous actions. The game has only two periods and the players are assumed to move simultaneously. Thus the only way to make it dynamic is to let the strategies for the second period strategies depend on the actions used in the first period.



	1	2	3	4	5	6	7	8	9
St.	(I,I)	(I,I)	(I,N)	(I,I)	(N,I)	(I,I)	(I,N)	(N,I)	(N,N)
Po.1	20	24	34	17	18	21	31	22	26
Po.2	20	17	18	24	34	21	22	31	26

Figure 2: St.-State, Po.i- Payoff for player i . The stippled lines indicate the information structure: The acting player is not able to distinguish the nodes within one information set.

6.2 Perfect equilibrium

As we have pointed out earlier, a strategy is a total plan for what to do in all possible situations. Thus the following would be a strategy in our example: "The first period I do not invest, the second period I invest if the other player has invested otherwise I will not invest." We will not allow this strategy as an equilibrium-strategy! The reason is as follows. If the other player has invested then it is not optimal for me to invest, because this gives me 17 while playing not investing gives me 18. My threat of investing is thus empty. To rule out such empty threats we assume the equilibrium to be perfect.

It is not quite correct to use the word "threat" in this connection, because there is no communication between the players. But the other player is assumed to be able to compute what is my optimal strategy, so there is no need for communication.

6.3 The second period subgames. Nonexistence of Nash-solution

An important consequence of using perfect equilibrium as the solution concept, is that we may solve the game by solving a sequence of the subgames. I.e. we might solve the model by dynamic programming.

Let us consider the case when (N,N) is played the first period. The solution constrained to this subgame must then also form an equilibrium. Let us look at the payoff-matrix of this subgame: (P1's payoff is first in every quadrant)

		P_2: I	P_2: N
P_1	I	21,21	31,22
	N	22,31	26,26

These payoffs are given as node 6-9 in figure 2.

This game has two Nash-equilibria (N,I) and (I,N). (I,N) is the best for P1 and the worst for P2 and vice versa. None of these equilibria gives a satisfying solution of this game. Even with mixed strategy there is no unique Nash-equilibrium in this game.

A possible way out of this problem is to assume that the players in such situation have a maximin behaviour. In our example P1 might argue that "If I do not invest then I get at least 22 while if I invest, I might get only 21. Thus I do not invest." The same argument applied to P2 gives the solution (N,N). The problem with this solution is that if P2 understands that P1 has a maximin behaviour, then he knows that P1 is playing N, the optimal for P2 would thus bee to play I. The same may P1 think.

Still we have chosen to use the maximin solution concept when the Nash-solution is not defined. Note that if we have to use a maximin solution in any of the subgames, the solution of the total game will no longer be a perfect Nash-equilibrium.

The maximin solution will give the payoff (26,26). Thus the players know that if they play (N,N) the first period they will gain a payoff (26,26).

It is interesting to see what would have happened if the payoff matrix above had been:

		P_2: I	P_2: N
P_1	I	21,21	24,18
	N	18,24	26,26

In this case there are also two Nash-equilibria. (I,I) and (N,N). But note that (N,N) is preferred by both players. This is thus a natural solution. If there exists such "best equilibrium" we will implement it as the solution, even if the equilibrium is not unique.

The rest of the second period games are easy to solve.If they played (I,N) the first period, only P2 has a real choice in the second period. We see that it will be optimal for him not to invest in the second period. Similarly with (N,I) in the first period. Thus if only one player invests in the first period, the other player will lose his marketshare for the rest of the game. The final payoff when (I,N) is played in the first period is thus (34,18). Similarly the payoff is (18,34) when (N,I) is played. If (I,I) is played in the first period the payoff is (20,20).

6.4 The first period decision. The reduced game

For every possible outcome in the first period we now know what will happen in the next period and the corresponding payoff. We use this to construct a reduced form of the game. The payoff for the first period reduced game is now known:

		P_2: I	P_2: N
P_1	I	20,20	34,18
	N	18,34	26,26

This is the prisoner's dilemma, and the Nash-solution is (I,I). This solution is Pareto-dominated by (N,N). It is in agreement with the common result in oligopoly theory, that competition forces the players to produce at a higher level than they would in the monopoly case.

6.5 Payoff or periodical profits

In this example we have used the usual game description with the payoff given for each terminal node. For computational reasons we have in our model used another approach. Let us review the subgame that occurs after the players have played (N,N) in the first period. In the first period they have then earned 11 each. The difference in payoff only depends of what they will earn in the second period.

The payoff earned in the second period is :

		P_2: I	P_2: N
P_1	I	10,10	20,11
	N	11,20	15,15

This game is equivalent to the subgame considered earlier. The solution is still (N,N) and the payoff is (15,15). The total payoff from playing (N,N) the first period is then the profit earned that period (11,11) plus the profit earned by continuing playing optimally which is (15,15) . Total profit from playing (N,N) is then (26,26) as we got before.

The reason why this is an equivalent game is that it is only the differences in payoff for different strategies that are important. Thus, adding a constant profit from earlier period to total profit, does not affect the ranking of the different paths. This is easily seen from the following bi-matrix:

		P_2	
		I	N
P_1	I	$10 + k_1, 10 + k_2$	$20 + k_1, 11 + k_2$
	N	$11 + k_1, 20 + k_2$	$15 + k_1, 15 + k_2$

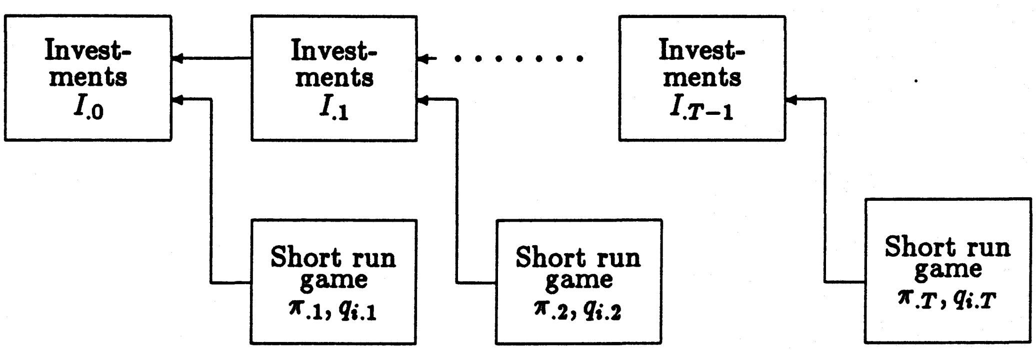
Here k_1 is the profit P1 has earned in previous periods and k_2 is the previous profit for P2. Obviously the values of k_1 and k_2 do not effect the solution of the game.

In the game in figure 2 the state is (I,N) in both terminal node 3 and 7. If these games continue after the second period, the profits would be equal in the subgames starting at 3 and 7, thus the solution would also be equal in the two subgames. In a perfect equilibrium we have only to consider strategies that depend on the state, and not the total history. This simplifies the computations in bigger models considerably.

7 The formal model

We have described an example which captures the main ideas in our model. We then proceed to present the general model.

The model may be presented graphically as:



Each player maximize discounted profits earned in the short run games. We assume that the profit in period t is a function of all previous investments, but only of the last short run game decision:

$$\pi_{i,t} = \pi_{i,t}(I_{.1}, \ldots I_{.t-1}, q_{.t}) \tag{6}$$

The dynamic structure of the game is: Let $K_{.0}$ be the capacities in period 0. The investment decision will determine the capacities next period: $K_{.1} = f(K_{.0}, I_{.0})$. $K_{.1}$ will be an input into the first short run game, and for the next investment decision. Note that what happens in the short run game has no effect on the rest of the game, in the sense that the result is not input for any other decision.

To solve the game we use dynamic programming, and so we have to start in the other end. When all investment decisions has been undertaken, only the last short run game remain. But in this game there is no future reaction to take care of, and the game might be solved as a static game. Note that since the profit is not a function of earlier production, the outcome from previous short run games will have no effect on the outcome in the last one. We now turn to the last investment decision. This investment decision will only affect the last short run game. Thus the payoff from $I_{.T-1}$ is the outcome $\pi_{.T}$ from the last short run game, given that this investment has been undertaken. (This will of course depend on previous investment decisions, but these can be taken as data at this stage.)

For the second last short run game, we have already noted that this has no effect on future decisions. We thus might solve it as a static game. But again the payoff in this game, is not a function of previous production, thus the previous short run games will not affect this game either. Using this argument successively we conclude that all short run games may be treated as static games! This is a parallel to Selten's "chain-store paradox" (R. Selten(1965)).

Let us turn to the second last investment decision. Assume the outcome is $I_{.T-2}$. This is input to the second last short run game, giving outcome $\pi^*_{.T-1}$, as well as input to the last investment decision. The outcome here results in an output $\pi^*_{.T}$. The total payoff from undertaking $I_{.T-2}$ is thus $\pi^*_{.T-1} + \pi^*_{.T}d$. Given the payoff in all possible outcomes at this stage, we are able to solve the game. This is similar to the first period decision in section 6.4. We then repeat the procedure in the third last investment decision, and so on.

Note that since the short run games may be treated as static, any numerical solvable model of the short run market defined by the capacities might be used as short run games.

In this paper it is assumed that each producer operates at full capacity. A theoretical argument for this choice is that production at full capacity is the solution of a game where the players have prices as decision variables. This is known as the Bertrand price-game.

To show this, let p* be the price if all players produce at full capacity, and suppose that p* is higher then variable cost for all players. Each player i, sets the price p_i. Obviously $p_i \geq p*$. We might assume (without loss of generality) that $p_1 \leq p_2$. If $p_1 < p_2$ there are two cases. (i) P2 has zero sales. If this is the case for all $p_2 > p_1$ then $p_1 > p*$, and hence P2 gets positive profit by choosing $p_2 = \frac{1}{2}(p_1 + p*)$. This is then better for P2. (ii) P2 has positive sales. Then 1 might increase his price without letting $p_1 \geq p_2$, and without losing sales. This is obviously better for 1. Thus in equilibrium $p_i = p_j = \overline{p}$ for all i,j. Assume next that $\overline{p} > p^*$, then not all the capacity is used. But then any player might decrease his price infinitesimally and utilize his total capacities which obviously will increase his profit. Thus the only possible equilibrium is that everybody sets $p_i = p^*$ and produces at full capacities.

7.1 A formal description of the model

We have n producers labeled with the index i. Further we assume that the producers make their investment I (possibly zero) and production decision, q at the beginning of every period. Production and capacity are thus assumed to be constant within each period. The production capacity $K_{i,t}$ for a producer is determined by his previous investments and initial capacity

$$K_{i,t} = f_i(K_{i,t-1}, I_{i,t-1}). \tag{7}$$

We denote the vector of capacities $K_{.t}$:

$$K_{.t} = (K_{1,t}, \dots, K_{n,t}). \tag{8}$$

The set S_i of possible strategies for player i is the set of functions from observed capacities to possible investments. One for each period.

$$s_i = (s_{i,1}, \dots, s_{i,T}) \tag{9}$$

When implementing a strategy $s_{i,t}$ we must invest:

$$I_{i,t} = s_{i,t}(K_{.t}) \tag{10}$$

The gas price is a function of the total quantity gas delivered to the European market. $p_t = p_t(Q_t)$, $Q_t = \sum_i q_{i,t}$. The function p_t depends on t. The production cost $C_{i,t}$ is a function of the capacities and production, and is fully known to all participants in the market

$$C_{i,t} = C_{i,t}(q_{i,t}, K_{i,t}). \tag{11}$$

The production is determined in the short run game. In this paper we have used the Bertrand price game which have a particular simple solution:

$$q_{i,t} = K_{i,t} \tag{12}$$

The cash flow is:

$$\pi_{i,t} = (p_t - C_{i,t})q_{i,t} - I_{i,t} \tag{13}$$

The producer maximizes discounted cash flow:

$$Max \sum_{t=0}^{T} \pi_{i,t} d^t \tag{14}$$

Where $d = \frac{1}{1+r}$. Assume that the players use strategies $(s_1^o, \dots, s_n^o)$, and denote the corresponding other variables with the superscript ^{o}.

The payoff for player i in this case will be.

$$V_{i\tau}(s_1^o, \dots, s_n^o) = \sum_{t=\tau}^{T} \pi_{i,t}^o d^t \tag{15}$$

$(s_1^*, \dots, s_n^*)$ is a Nash equilibrium if:

$$V_{i0}(s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots s_n^*) \leq V_{i0}(s_1^*, \dots, s_n^*) \tag{16}$$

for all possible strategies s_i, and for all i.

7.2 The dynamic programming solution of the game

To describe the algorithm we first assume that we have been able to find the solution for the subgames for period $s = t+1, \dots, T$. We then solve the subgame for period t. Let i's profit from period τ to T be $V_\tau(K_\tau)$ when the players behave optimally from period τ to T.

Assume we "stand" at K_t at time t. To go from $K_{i,t}$ to $K_{i,t+1}$ player i has to invest $I_i(K_{i,t}, K_{i,t+1})$. The cash flow in period t is

$$\pi_{i,t}(K_t) = (p_t(Q_t) - C_{i,t})q_{i,t} - I_i(K_{i,t}, K_{i,t+1}) \tag{17}$$

The payoff for player i when all players make the necessary investment so as to increase capacity from K_t to K_{t+1} will thus be:

$$W_{i,t}(K_t, K_{i,t}) = \pi_{i,t}(K_t) + d\,V_{i,t+1}(K_{t+1}) \qquad (18)$$

If we find a Nash solution in this game, we assume that this strategy will be implemented. If not the maximin solution is implemented. Let K^*_{t+1} be the capacities corresponding to the relevant solution. The payoff for player i in position K_t at time t is

$$V_{i,t}(K_t) = W_{i,t}(K_t, K^*_{t+1}) \qquad (19)$$

Thus we see that if we know V_{t+1} we are able to compute V_t, and the optimal decisions at time t. But we know that from time T there will be no more gas production, so then $V_{T+1} = 0$. Thus we might compute V_t starting with t=T, using the result to compute V_t for t=T-1 e.t.c. until t=0.

Now we know what everybody will do in time t=0, and we might compute the state in time t=1. We then know what to do in t=1 and might compute the state in time t=2. In this way we continue until t=T, and the game is solved.

Part IV THE MODEL APPLIED TO THE EUROPEAN GAS MARKET.

The previous chapter describe the theoretical basis of a dynamic game model of the gas market of the European Continent. In this chapter we will apply the model to this market.

The input figures used in the model are from Mathiesen, Roland & Thonstad (L. Mathiesen, K. Roland and K. Thonstad(1986)) and from BP review of Word Gas. (BP (1986))

8 Demand

In the model, the European Continent is regarded as a single homogeneous market. Great Britain is, as earlier mentioned, not included in the marked. We could have included Great Britain by building a model with simultaneous dynamic equilibrium in two markets. This would certainly be interesting, but will not be commented upon in this paper.

We also disregard market power on the demand side. There is no difference in price on imported and domestically produced gas in the model.

Natural gas is assumed delivered in a "centre of gravity" in the demand region. Transportation costs to this point are included in the production costs, and all deliveries of gas are sold to the consumers at the same price (cif).

Demand equation:

$$E(t) = \alpha P(t)^{e_1} Y(t)^{e_2} \qquad (20)$$

		Variable	Value used in model	Unit
E	=	Demand		(bcm)
α	=	Constant	6167.0	bcm
P	=	Price of natural gas		(million $ /bcm)
Y	=	GDP in demand region		
e_1	=	Price elasticity	−0.7	
e_2	=	Income elasticity	0.8	

The demand for natural gas in the model is a function of price and GDP level in the demand region. We could easily have made the function more realistic. We could have included substitution effects through alternative energy prices in the demand equation, making exogenous assumptions about such prices. But the main goal of this model is to show the effects of strategic considerations in a game situation on the players investments. A more complicated demand function would not have contributed to the understanding of this matter.

Price and income elasticities are computed as weighted mean of the elasticities of the countries in the region computed by Mathiesen & al.

Growth in GDP:

$$Y(t) = y_0 e^{\delta t} \tag{21}$$

GDP in the consumer region is assumed to grow with a constant yearly rate. GDP in the base year (1985) is normalized to unity.

		Variable	Value used in model	Unit
y_0	=	Base value	1	
δ	=	Rate of growth	2.5%	

Domestic Production:

$$S(t) = S_0 e^{\gamma t} \tag{22}$$

The amount of remaining recoverable gas within the European Continent, except for Netherlands is relative small. For the sake of simplicity domestic production is assumed to decrease by a constant rate throughout the simulation period.

		Variable	Value used in model	Unit
S_0	=	Domestic production in 1985	109	bcm
γ	=	Rate of decrease	−1.2	

9 Supply

From present state of gas production and distribution we have chosen to consider at most three foreign producers competing for the supply of gas to The European Continent. These producers are the Soviet Union, Norway and Algeria.

Each of these producers can increase their present export capacity by a few large investments. The players choose the investment profile in order to maximize discounted payoff. Thus the decision maker in each producer country faces a limited discrete investment problem.

The present situation and the new possible investments considered in the model with estimated capacity and costs are listed below.

Soviet Union		
Existing export	28.6 bcm	
Excess capacity	27.0 bcm	
Investment projects	Increased capacity	Costs (85 $)
Installation of compressors in existing pipeline	23.0 bcm	$ 200 mill
New pipeline to Western Europe	56.0 bcm	$ 15000 mill
Installation of compressors in new pipeline	23.0 bcm	$ 200 mill

The capacity and cost of installation of compressors in the existing pipeline are estimated on the assumption that no compressors are installed at present.

Norway		
Existing export Excess capacity	13.6 bcm None	
Investment projects	Increased capacity	Costs (85 $)
Field developments		
Sleipner	8.8 bcm	$ 1900 mill
Troll I	24.0 bcm	$ 6500 mill
Troll II	24.0 bcm	$ 6000 mill

The present producing wells in Norway are relatively small. Therefore we have implemented a production profile for these fields based on Wood & Mackenzie & Co (1986). The effect is that production decreases and that existing wells are empty about 2010.

Algeria		
Existing export Excess capacity	17.8 bcm None	
Investment projects	Increased capacity	Costs (85 $)
Restore idle LNG capacity	20 bcm	$ 1000 mill
Installation of compressors in existing pipeline	6 bcm	$ 400 mill
New pipeline to Italy	18 bcm	$ 3500 mill

Variable unit costs : Variable unit costs included transportation are regarded constant, and is estimated to mill $ 90/bcm for Norway and USSR.

Due to the different variable cost of LNG and pipeline gas production and transportation we have calculated the mean variable costs for Algeria after each project is fulfilled.

Algerian project :	Variable unit cost (85 $)
Existing capacity	mill $ 108.4 /bcm
Restore idle LNG capacity	mill $ 119.5 /bcm
Installing compressors	mill $ 111.2 /bcm
New pipeline to Italy	mill $ 105.0 /bcm

The reason for the high variable unit costs for Algeria is the expenses related to LNG production.

Production : The players in the game decide their investment profile so as to maximize discounted payoff. In the model we assume that everybody produces at full capacity, despite the fact that the USSR has idle capacity at present.

Rate of discount : The rate of discount is set to equal 5 % in the three producer countries. We have done some simulations where we assumed higher rate of discount in Algeria based on the assumtion that less developed economies have higher rate of discount. This change had very little impact on the investment decisions in the model.

Order of investments : It is assumed that there is a natural ordering of the investment projects for the Soviet Union and Algeria. You most have a pipeline before you can install compressors, and we assume, due to the cost per unit, that the capacity of an existing pipeline is increased by installing compressors before a new pipeline is built.

The possible Norwegian investments are fields developments. The fields can be explored in any order. A discussion of impact of different investment profiles is given below.

Time horizon and payoff: The producers can undertake new investments until time *t1*, set to 2060 in the model. The production resulting from the investments is assumed constant throughout the simulation period. This is a strong simplification as far as small fields like Sleipner are concerned. The production achieved at time *t1* is assumed to remain constant until time *t2* (2085). As seen from the results below all investments are done long before time *t2* in all cases. We have chosen this long time horizon to be sure that the the investment decisions of the players are not affected by an arbitrary horizon.

These assumptions give unrealisticly high discounted payoff figures, so that the actual value of the discounted payoff over the whole simulation period is not very interesting. We have chosen to comment upon the accumulated discounted cashflow for the first periods.

Full information : We have assumed that all players have access to the same information. Obviously this is not the case in real life. Algeria does not estimate investmentcosts in the North-sea as accuratly as Norway does, and vice versa.

Other assumptions : There is considerable time lag from the time an investment is decided until it is put into effect. This justify a model with long time intervals. In our model the producers can choose an action every 5'th year.

Many of the assumptions in the model can be relaxed. But this would result in a more complex model which is less transparent.

Part V
SIMULATION RESULTS

10 The impact of dynamics

In the dynamic game the investment decisions are affected by previous investments of all actors. This makes ***strategic behaviour*** possible. It seems to be widely accepted that there is a conflict between "fighting for high marked shares", and "fighting for high prices". This conflict is only possible to model in a dynamic game. We thus believe that this model captures important marked forces, not included in other models. But even if this empirical hypothesis should turn out to be incorrect, there is strong normative argument for the model. A producer who is aiming at maximizing NPV of future profits, will behave unoptimal, if he does not take care of the dynamic game element.

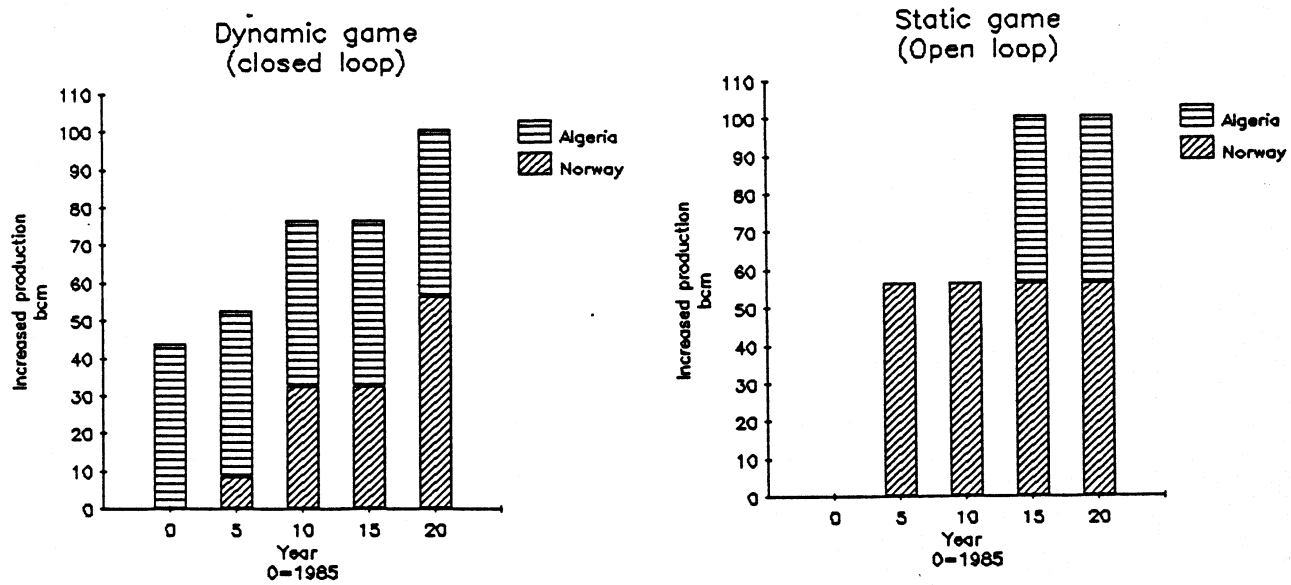


Figure 3: Investment profiles with dynamic and static assumptions

To illustrate some main features of a dynamic game we have simplified the model to a two actor game. USSR is excluded from the game by assuming a fixed market share of 30 % of total consumption. The resulting investment profile of a simulation based on this assumption is shown in figure 3. We can see that Algeria will start all their projects the first period. Norway starts the development of Sleipner in the second period, Troll I in the third, and Troll II in the fifth period.

Test one In order to clarify the impact of dynamic behaviour we perform the test described in section 5.2. In the first part of the "strategic test" we ask the question "What is the effect on the Norwegian investments if Algeria had chosen not to build the pipeline to Italy the first period?". A simulation shows that if Algeria does not build a new pipeline, Norway will start the development of Sleipner and Troll I in the second period. In this case the Algerian pipeline to Italy will be postponed until period 4. The conclusion is : Algeria postpones the second Norwegian investment one period by investing all their projects the first period. This makes the discounted Algerian payoff higher.

Test two The second part of the test is answered if we perform a simulation where we assume that the investment profile of Norway is fixed and equal to the base run profile. This makes Algerian strategic considerations irrelevant. In this case Algeria chooses the investment profile which maximize their payoff given the fixed production of Norway. The result shows that in this non-strategic situation Algeria will invest later than in the base simulation. All investment projects are postponed to the second period. This indicates that the production is higher in a dynamic game than in a static game.

Open loop The fixation of the Norwegian production is of course an unrealistic case. Norway would have invested if Algeria were to postpone their investment. The difference between a static and a dynamic game is best shown if we implement an *open loop* game. In the open loop situation the actors consider the investment possibilities for himself and the other actors at time t_0, and at this time he decides the investment profile which gives him the highest discounted payoff. An equilibrium is obtained if both actors agree upon an investment profile given the investment of the other actors. This game differs from the dynamic game since the actors will follow their plans whatever happens after the game is started. The previous actions of the actors do not affect the investment decisions. Consequently we have a ***dynamic model*** but a ***static game***.

A simulation of the same dynamic model with an open loop game gives the investment profile shown in figure 3. The corresponding price development is shown in figure 4. Investments

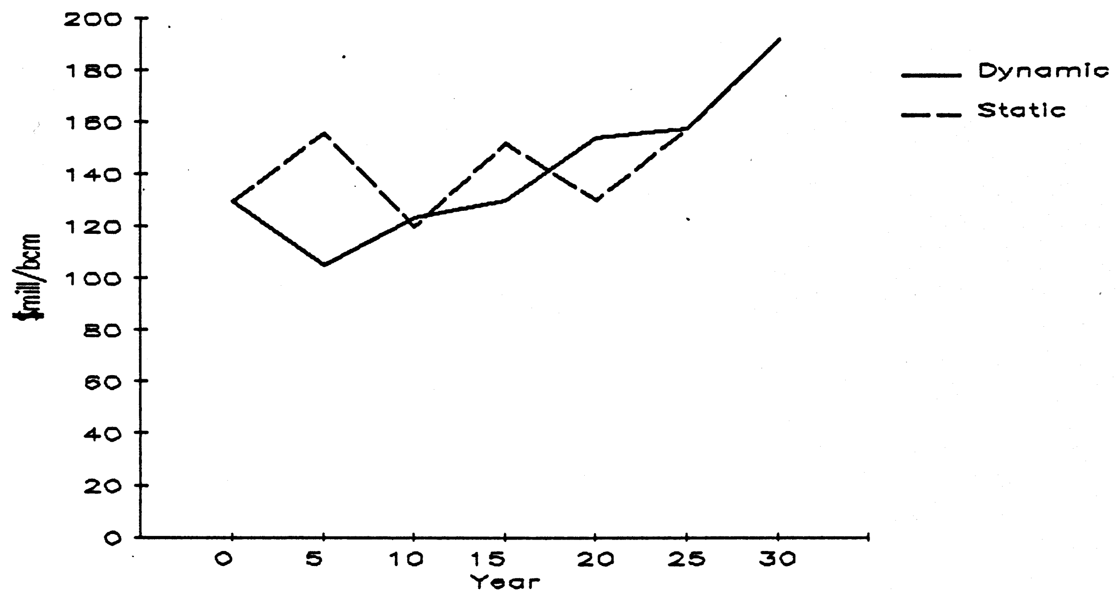


Figure 4: Price development with dynamic and static assumptions

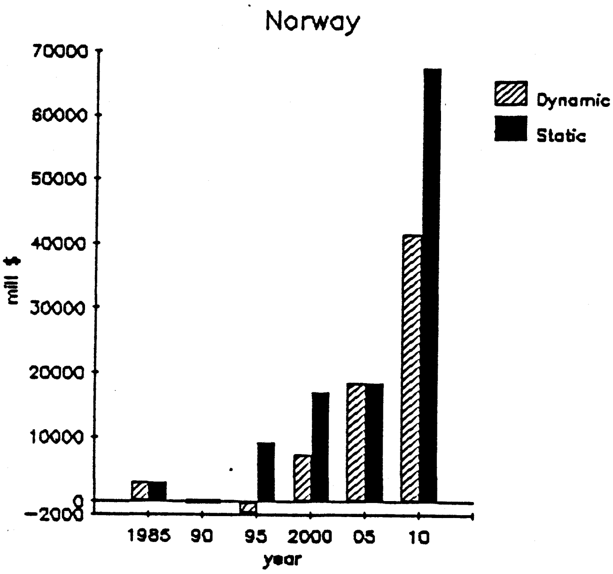
in the first period is considerably higher in the closed loop solution (44 bcm) than in the open loop (0 bcm). This results in lower second period prices in the closed loop case. But already in the second period this difference has vanished. The explanation of this lyes in the number of possible investment project for each player. In the dynamic case Algeria carries out all of its investments in the first period, and has thus no possible investment project in the rest of the game. Hence in the subgame starting in the second period the open loop and closed loop solutions will be identical.

- **The difference between a static and a dynamic game can besummarized by:**

  1. In order to gain higher market shares in the dynamic game one or more players

make one or more investment earlier than in the non-strategic case, in order to affect the investment decisions of the other actors.

2. This results in a higher production in the dynamic game.
3. Higher production makes the gas price decrease.
4. The lower gas price affects the investment decisions. Investments of the other actors become less profitable. And one or more of their investments are postponed.
5. The players making strategic investments gain higher marketshares which give higher discounted payoff.



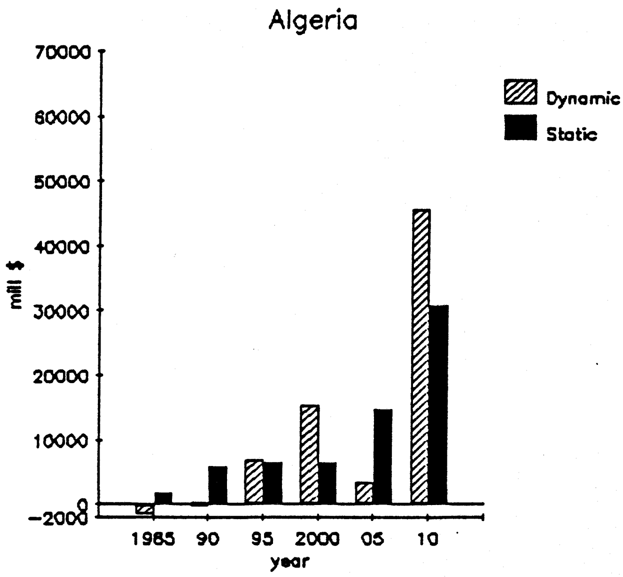


Figure 5: Discounted cashflow under different assumptions

	Norway	Algeria
Dynamic	41,713	45,645
Open loop	67,655 (+62%)	30,832 (-32%)
Norway fixed	45,113 (+ 8%)	47,598 (+ 4%)
Algeria delayed	49,724 (+19%)	44,115 (- 3%)

Table 2: Accumulated payoff of up to 2015. And change from dynamic run in %

Because of the long time horizon in this model the payoff is very large, and the relative difference in the discounted payoff is small. Thus we have shown the discounted cashflow for the two countries the first 5 periods in figure 5. The accumulated discounted cashflow over the same period is listed in table 2. The results indicate that the dynamic game is the worst case for Norway, open loop is the best. Observe that in the case with a fixed Norwegian production both actors will be better of.

11 The significance of market share of the Soviet Union

USSR is highly dependent on Western currency to finance imports. This fact implies that USSR wants a big share of the supply of natural gas to Western Europe, while political considerations in Western Europe suggest minimal dependency on gas deliveries from the communist area.

- We have run the model under four different assumptions:
  1. 20 % of the total consumption is imported from USSR. This is about the present situation.
  2. 30 % of the total consumption is imported from USSR. This is the highest single country import share recommended by EC
  3. 40 % of the total consumption is imported from USSR. A liberal import restriction.
  4. The Soviet Union is an active player in the game.

The different assumptions give different supply of gas from the Soviet Union to the market. Higher market shares give higher total supply.

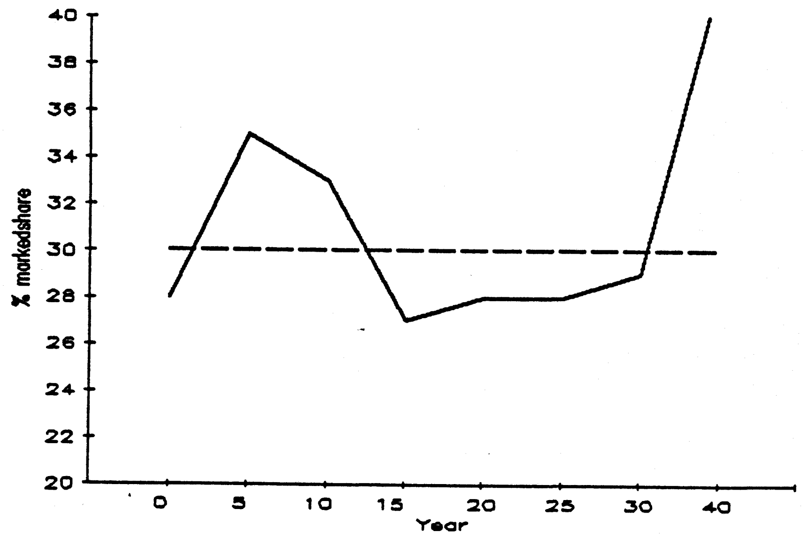


Figure 6: Enodenous market shares of the Soviet Union.

The path of the market share when USSR is an active player is illustrated in figure 6. The market share starts at a level slightly below 30 % and rises to about 40 % by the end of the simulation period due to the fact that Norway and Algeria have completed all their investments. The results indicate that a 30 % aggregate market share restriction will not hit USSR exports hard. But a more disaggregated model where we put the restriction on each country may change this result.

The market share of the Soviet Union affects the total gas supply and thus the gas price. The total production of the three suppliers in the different situations are shown in figure 7.

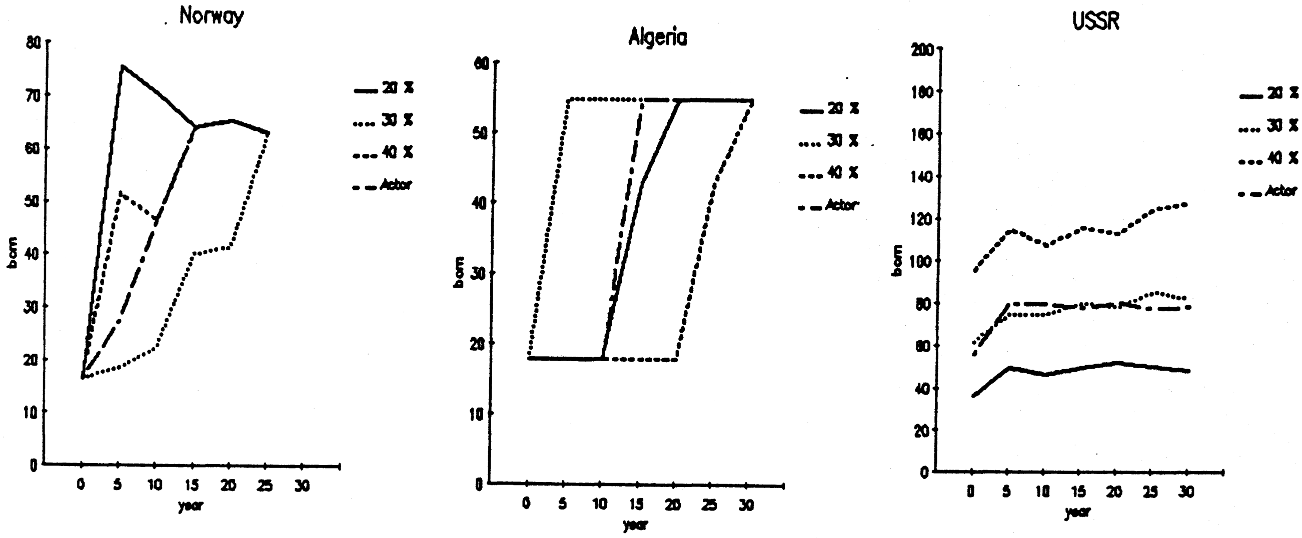


Figure 7: Production with different Russian market shares.

It is obvious that the total supply increases when USSR is allowed higher market shares. The supply from USSR as an active player lies near the 30 % case.

When more gas is supplied the price decreases. The gas price affects the discounted payoff of the actors. The payoff as result of different market situations is illustrated in figure 8.

The results show that a higher market share from the Soviet Union makes the investments of the other players less profitable due to a lower price. Investments are consequently delayed in most cases.

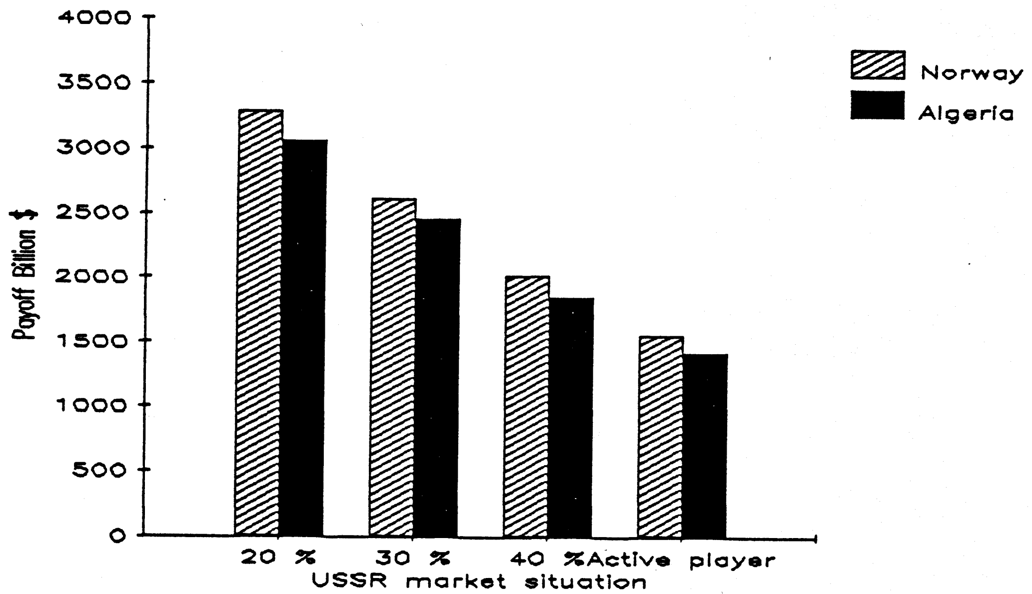


Figure 8: Payoff with different Russian market shares.

USSR as an active player gives the lowest payoff. This is surprising since USSR produces less than in the 40 % of total supply. The reasons for this is that the importance of dynamic considerations increases when USSR is a player in the game. There are now three players fighting for marketshares. The result shows that preventing USSR from investing in a new pipeline gives decreased payoff for Norway and Algeria.

The conclusion is that the Soviet market share strongly affects the situation in the gas market.

12 The price elasticity

A uncertain factor the European gas market is the price elasticity of demand. The value used in the model is calculated as a weighted mean of the elasticities of each country as estimated by Mathiesen & al (L. Mathiesen, K. Roland and K. Thonstad (1986)).

To analyze the effect of a changed price elasticity we remind the reader of the situation in a static Nash-Cournot situation.

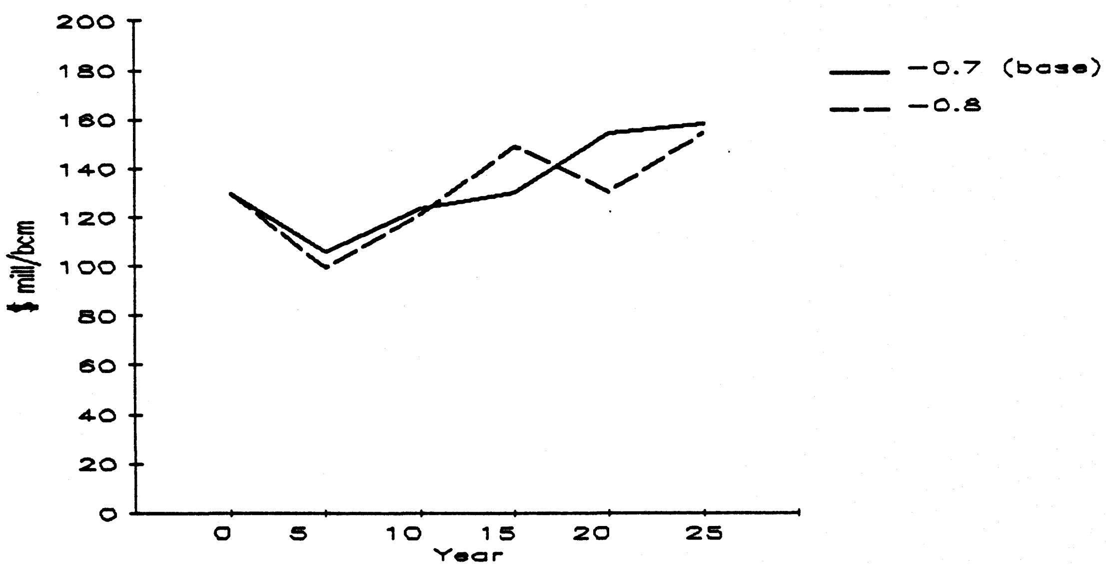


Figure 9: Effect of a higher price elasticity

In a Nash-Cournot game the marginal cost (equals marginal revenue) is : $mc = p(1 + \frac{s}{e})$

s = market share

p = gas price

e = price elasticity

The change in the price due to a change in e_1 is given by (constant marginal cost):

$$\frac{\delta p}{\delta e} = \frac{ps}{e^2 \, (1 + s/e]} > 0 \qquad (23)$$

The short term effect is that if the magnitude of the price elasticity increases the price decreases.

The situation is more complicated in the dynamic game situation. We have done a simulation where we increase the price elasticity from -0.7 to -0.8. The effect on the investment profile is shown in figure 9.

As seen from the investment profile in figure 9 the change in price elasticity alter the order of the investments. Norway completes all their investments before Algeria starts investing. A reasonable explanation to this is that the higher price elasticity results in a lower price (fig.9), and this hits Algeria harder than Norway due to higher Algerian variable costs.æ

13 The ordering of the Norwegian investments

In the model we have assumed a natural ordering of the investment projects in each country. In Algeria and USSR the ordering is given by cost considerations and physical limitations, e.g. you need a pipeline before you install compressors. The existence of such a natural ordering is not so obvious in the case of the development of the Norwegian gas fields Troll and Sleipner.

Order of investments :	
Sleipner ⟶ Troll	2,614,725 mill $
Troll ⟶ Sleipner	2,643,977 mill $
Difference	-29,252 mill $

Table 3: Discounted payoff with different ordering of the Norwegian investments

These fields could be developed in any order, but we have chosen to assume that Sleipner is developed before Troll. The ordering of the fields has an impact on the equilibrium solution and thus the discounted payoff. We have done a simulation with development of both Troll fields before Sleipner.

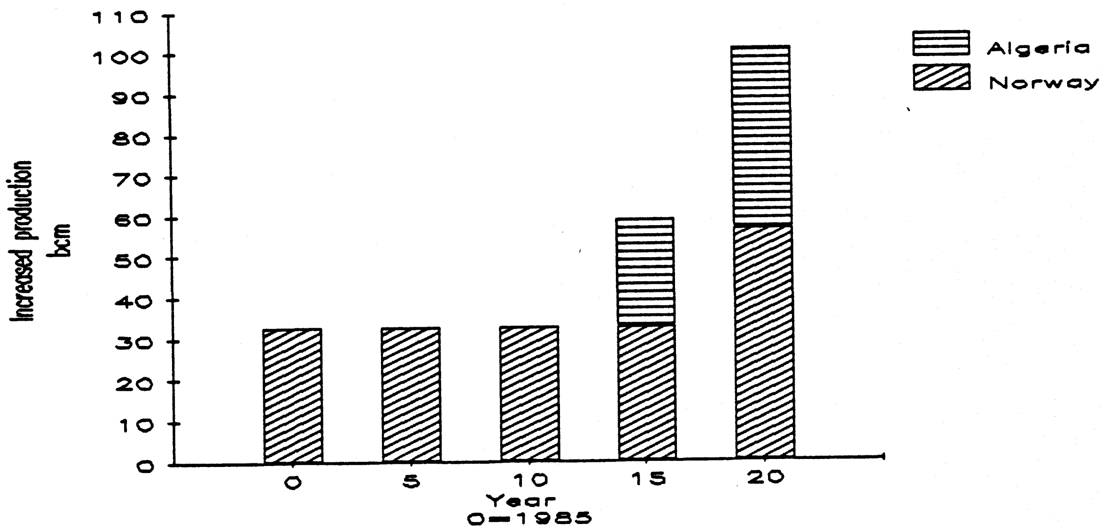


Figure 10: Investment profile with different ordering of the Norwegian investments

From the calculations we can se that Troll-Sleipner investment have a higher payoff than Sleipner-Troll. This seems surprising since the unit cost of Sleipner is less than the unit cost of Troll. The explanation is again found in the strategic aspect of the game. If we change the ordering of the field developments the investment profile is changed (se figure 10). The extra production capacity provided by Sleipner and Troll I is not sufficient to keep Algeria from investing. All of the investments are too expensive due to the resulting fall in prices. Investments Troll I + II are large enough to keep Algeria from investing and the extra supply does not course a radical price fall. This conclusion may be dependent on the price elasticity.

Part VI
CONCLUSIONS

The main purpose of this paper is to describe and present empirical simulations on a dynamic game model of the gas market. The model gives a very simplified picture of the market structure, and we have not incorporated detailed information about costs or technology. The results should thus not be taken as concrete advice as to whether or not undertake specific investments.

Still, we beleve that some important qualitative conclusions can be drawn. The solution in a dynamic game differs considerably and systematically from the results in static games. The main difference is a higher production compared to a static game. We thus believe that static game models will underestimate optimal production.

The most striking example of this effect of the dynamics is the result that Algeria investes in the period 1985-89 even though this investment reduces their cash-flow (excluded investment-costs) in the period 1990-94, to prevent later Norwegian investments. We might say that Algeria sacrifices short run profit to fight for market-shares.

As already pointed out we have made many simplifications. It is possible to expand the model and overcome some of these simplifications, at the cost of a more complicated model.

Some potential generalizations witch can be done are:

- It is possible to increase the realism of the model by introducing a stochastic demand for gas.
- The demand is estimated in a period of high oil prices. It is possible to introduce connections between the gas price and the oil price, and give exogenous price paths for oil.
- We might have assumed that the players moved sequentially, one succeeding the other, and not simultaneously. It is not obvious which assumption is the most natural in the gas market. A sequential model is at least as easy to solve on a computer as the one we have used.

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