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Arvid Raknerud, Terje Skjerpen and Anders Rygh Swensen

Forecasting key macroeconomic variables from a large number of predictors: A state space approach

Abstract:
We use state space methods to estimate a large dynamic factor model for the Norwegian economy involving 93 variables for 1978Q2–2005Q4. The model is used to obtain forecasts for 22 key variables that can be derived from the original variables by aggregation. To investigate the potential gain in using such a large information set, we compare the forecasting properties of the dynamic factor model with those of univariate benchmark models. We find that there is an overall gain in using the dynamic factor model, but that the gain is notable only for a few of the key variables.

Keywords: Dynamic factor model, Forecasting, State space, AR models

JEL classification: C13, C22, C32, C53

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Address: Arvid Raknerud, Statistics Norway, Research Department. E-mail: rak@ssb.no.

Terje Skjerpen, Statistics Norway, Research Department. E-mail: terje.skjerpen@ssb.no.

Anders Rygh Swensen, Statistics Norway and University of Oslo, Department of Mathematics, E-mail: swensen@math.uio.no.

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Telephone: +47 62 88 55 00
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1 Introduction

Traditionally, forecasting of macroeconomic variables is done within a low dimensional framework, typically using vector autoregressive (VAR) models. However, recent advances in macroeconometric modelling point in another direction, where predictions of key macroeconomic variables are integrated within the analysis of a possibly very large number of other variables that are *not* considered to be of primary interest. An example is when the focus is on predicting inflation and GDP growth, whereas many other variables that are thought to carry information about the future realizations of these key variables are also included in the analysis. To reduce the dimensionality problem, which would cause the number of parameters in a VAR model to increase exponentially with the number of variables, common dynamic factors are currently advocated by many researchers as a parsimonious way of capturing the comovements among different variables – thus attempting to break the curse of dimensionality that arises in VAR models. One specific example is the so-called diffusion index models (see *inter alia* Quah and Sargent, 1993, Forni et al., 2000 and 2001 and Stock and Watson, 2002a, 2002b and 2006). Another variant of these types of models is the factor augmented VAR model, FAVAR (see Bernanke et al., 2005 and Bai and Ng, 2002).

The implementation of large, dynamic models with latent factors is not straightforward. One common approach is to estimate the unobserved factors in a first step using principal component techniques. In this way, information about a few common factors from a large number of interrelated variables can be extracted. Then, in the next step, when the estimated common factors are used to forecast key variables within an AR or VAR model, they are treated "as if" they were observed. More direct approaches, involving parametric specifications of the latent processes, have also been put forward, e.g., subspace algorithms (see Bauer, 2005 and Bauer and Wagner, 2002) and quasi maximum likelihood methods (Doz et al., 2006). These methods can be seen as approximate methods for maximizing the full likelihood implied by a state space model when the observation vector is high dimensional, but the number of latent factors is relatively small.

The performance of various procedures, compared with traditional low dimensional time series modeling is an important topic. The problems we address in this paper concern both efficient estimation and out-of-sample predictive performance. Of course, the issues of estimation and prediction are related. Only if we are able to estimate sufficiently rich models, can we expect that their goodness-of-fit and out-of-sample forecasting properties are satisfactory. Well established univariate methods may provide a useful benchmark for evaluating the more elaborate high-dimensional models. While some papers report satisfactory results when comparing diffusion index models with simple univariate models, these findings may not be robust, especially if the latter type of models are more carefully designed than often is the case in such "competitions", e.g., regarding the question about the number of lags to include. There is some evidence that on data sets dominated by large, irregular components, which is typical for many macroeconomic time series, the performance of diffusion index-type models is disappointing compared with simpler models (Dahl et al., 2005; D'Agostino et al., 2006). This may be explained by the fact that large models with latent factors tend to have simple dynamics, driven by just a few common components. The dynamics related to the individual (variable specific) components are then typically not sufficiently taken into account when the models are estimated and applied for forecasting. An important example of this is the approximate dynamic factor models, where the (idiosyncratic) error terms of each variable are allowed to be weakly correlated over time (and series), but where this correlation structure is not explicitly modeled.

Our approach is to model the individual variables as univariate autoregressive processes, augmented with common dynamic factors to account for the comovements among them.

The number of lags for each of the individual variables and the number of common factors are determined by applications of information criteria in a two-step procedure. More specifically, we shall focus on a situation where a vector of key variables, z_t, is assumed to be an aggregate of an n-dimensional vector $x_t = (x_{t1}, ..., x_{tn})'$ through the deterministic relation $z_t = f(x_t)$. As a special case, some components of x_t and z_t may be identical. Typically, z_t is low dimensional, while x_t is of high dimension. The n endogenous variables x_{it}, $i = 1, ..., n$, are observed for $t = 1, ..., T$. Each variable x_{it} is modeled as an AR(p_i)– process augmented with a small number of common stochastic components (common factors). One question we address is whether there is any gain from predicting the key variables, z_t, using a disaggregated data set, x_t, or whether one obtains equally good, or even better, forecasts from univariate models of the key variables.

Our approach has more in common with the tradition of multivariate structural time series models than with the approximate dynamic factor models mostly favored in the literature. See Harvey (1989) for a general exposition of structural time series models and Proietti (2002) and Harvey (2006) for forecasting within this framework. Our model is formulated using unadjusted values of the variables, and common latent dynamic factors and latent seasonal components are an integral part of the model formulation. Apart from detrending by differencing we perform no preprocessing of the data, such as e.g. seasonal adjustments or corrections for outliers. Estimation of the model is based on the state space formulation. A full information maximum likelihood algorithm using exact (analytical) derivatives is developed, which works well even if the number of unknown parameters is in the range of 1,000–3,000, which is the typical situation in the present study. These parameters comprise, for each of the 93 equations we analyze, AR-parameters, parameters of seasonal dummy variables, loading coefficients of common factors and white noise error variances.

The rest of this paper is organized as follows: Section 2 presents the modelling framework and discusses forecasting. Section 3 describes the data and the estimation method used in the empirical application. The empirical results are presented and discussed in Section 4, while Section 5 concludes.

2 Modeling framework

The n variables x_{it} are assumed to have the following representation:

$$(1-\sum_{j=1}^{p_i}\phi_{ij}L^j)\Delta x_{it}=\beta_i{}'d_t+\theta_i{}'f_t+\lambda_i{}'s_t+e_{it},$$

where L is the lag operator, the ϕ_{ij} are autoregressive parameters, d_t is a 4×1 vector consisting of a constant term and dummy variables for the three first quarters of the calendar year, with corresponding coefficient vector β_i, $f_t=(f_{1t},...,f_{rt})'$ is an $r\times 1$ vector of independent dynamic factors, distributed as Gaussian AR(1) processes:

$$f_t=\Psi f_{t-1}+\eta_t,\eta_t\sim\mathcal{IN}(0,I),$$

with $r\times 1$ loading vector θ_i and Ψ =diag$(\psi_1,...,\psi_r)$.[1] We use the notation 0 and I to denote, respectively, a matrix of zeros and an identity matrix of appropriate dimension. Furthermore, $s_t=(s_{1t},...,s_{kt})'$ is a k-dimensional vector of independent stochastic seasonal components with loading vector λ_i. The seasonal vector process is given by

$$s_t=-s_{t-1}-s_{t-2}-s_{t-3}+\omega_t,\ \omega_t\sim\mathcal{IN}(0,I).$$

Finally, $e_{it}\sim\mathcal{N}(0,\sigma_i^2)$ is an (idiosyncratic) error term with Cov(e_{it},e_{js}) $=0$ if $i\neq j$ or $t\neq s$.

Next, define

$$\Delta x_{it}^*=\left[\begin{array}{ccc}\Delta x_{i,t-1}, & \cdots & ,\Delta x_{i,t-p_i}\end{array}\right]',$$

[1] The (identifying) restriction that Ψ is a diagonal matrix may easily be relaxed, but we find no gain in terms of improved out-of-sample performance by allowing the latent factors to follow a more general VAR process.

and

$$\phi_i = \left[\begin{array}{ccc} \phi_{i1}, & \cdots & , \phi_{ip_i} \end{array}\right]', \quad i = 1, ..., n.$$

Furthermore, let

$$\begin{aligned} \Theta &= \left[\begin{array}{ccc} \theta_1, & \cdots & , \theta_n \end{array}\right]' \\ \Lambda &= \left[\begin{array}{ccc} \lambda_1, & \cdots & , \lambda_n \end{array}\right]'. \end{aligned}$$

To obtain identification, we require the loading matrices $\Theta = \{\theta_{ij}\}_{n\times r}$ and $\Lambda = \{\lambda_{ij}\}_{n\times k}$ to have a lower triangular structure, with $\theta_{ij} = 0$ and $\lambda_{ij} = 0$ if $j > i$. The above model can then be cast in a familiar state space form:

$$\begin{array}{l} y_t = Bd_t + X_t^* \phi + Z\alpha_t + \varepsilon_t \\ \alpha_t = \mathcal{T}\, \alpha_{t-1} + R \left[\begin{array}{cc} \eta_t{}', & \omega_t' \end{array}\right]' \end{array} \qquad t = 1, ..., T, \tag{1}$$

where the observation vector is

$$y_t = \left[\begin{array}{cc} \Delta x_{1t}, & ..., \Delta x_{nt} \end{array}\right]'$$

and the state vector is

$$\alpha_t = \left[\begin{array}{cccc} f_t{}', & s_t{}', & s_{t-1}{}', & s_{t-2}{}' \end{array}\right]'.$$

Furthermore

$$\begin{aligned} \varepsilon_t &= \left[\begin{array}{ccc} \varepsilon_{1t}, & \cdots & , \varepsilon_{nt} \end{array}\right]' \\ B &= \left[\begin{array}{ccc} \beta_1 & \cdots & \beta_n \end{array}\right]' \\ X_t^* &= \left[\begin{array}{cccc} \Delta x_{1t}^*{}' & 0 & 0 & 0 \\ 0 & \Delta x_{2t}^*{}' & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \Delta x_{nt}^*{}' \end{array}\right] \\ \phi &= \left[\begin{array}{ccc} \phi_1{}', & \cdots & , \phi_n{}' \end{array}\right]' \\ Z &= \left[\begin{array}{cccc} \Theta & \Lambda & 0 & 0 \end{array}\right] \\ \mathcal{T} &= \left[\begin{array}{cccc} \Psi & 0 & 0 & 0 \\ 0 & -I & -I & -I \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{array}\right], \quad R = \left[\begin{array}{cc} I & 0 \\ 0 & I \\ 0 & 0 \\ 0 & 0 \end{array}\right]. \end{aligned} \tag{2}$$

The main purpose of the analysis is to predict $z_t = f(x_t)$. Consider the i'th component of z_t, z_{it}. The typical situation is that $z_{it} = \ln Z_{it}$, and that

$$z_{it} = f_i(X_{1t}, ..., X_{nt}).$$

where the econometric model is formulated in terms of $x_{it} = \ln X_{it}$. More specifically, all our aggregates can be represented in the following form:

$$z_{it} = \ln \sum_{j \in n_i} w_{jt} X_{jt}, \tag{3}$$

where n_i denotes the set containing the indices of the components included in the aggregate z_{it}. If a volume aggregate is considered, $w_{jt} = 1$ for all j and t, whereas, if an aggregate price index is considered, the w_{jt} are time dependent and $\sum_{i \in n_i} w_{jt} = 1$ (w_{jt} will then also depend on i, but for simplicity we have omitted the i-subscript here). The typical variable of interest to predict is the relative growth from t to $t+h$, i.e., $\Delta_h z_{i,t+h}$, where, for any series X_t, $\Delta_h X_t \equiv X_t - X_{t-h}$. The optimal predictor would then be

$$E_t(\Delta_h z_{i,t+h}) = E_t \left(\Delta_h \ln \sum_{j \in n_i} w_{jt} X_{j,t+h} \right),$$

where $E_t(\cdot)$ denotes the expectation given the information set I_t including observations up until period t: $I_t = \{X_s\}_{s \leq t}$ This calculation is obviously complicated and depends critically on distributional assumptions. A simpler alternative, which is the one we pursue, is to linearize $z_{i,t+h}$ around the current value z_{it} as follows:

$$\begin{aligned} z_{i,t+h} &= \ln Z_{it} + \ln \left(1 + \frac{\Delta_h Z_{i,t+h}}{Z_{it}} \right) \\ &= z_{it} + \ln \left(1 + \sum_{j \in n_i} \frac{w_{jt} X_{jt}}{Z_{it}} \frac{\Delta_h X_{j,t+h}}{X_{jt}} \right) \\ &\simeq z_{it} + \sum_{j \in n_i} \alpha_{jt} \Delta_h x_{j,t+h} \end{aligned}$$

where

$$\alpha_{jt} \equiv \left(\frac{w_{jt} X_{jt}}{\sum_{k \in n_i} w_{kt} X_{kt}} \right), \; x_{jt} \equiv \ln X_{jt},$$

and we have used the approximations $x \simeq \ln(1+x)$ and $\Delta_h X_{t+h}/X_t \simeq \Delta_h x_{t+h}$ (see Appendix B for two concrete examples of how α_{jt} is constructed). Hence

$$\Delta_h z_{i,t+h} \simeq \sum_{j \in n_i} \alpha_{jt} \Delta_h x_{j,t+h} = \sum_{j \in n_i} \alpha_{jt} \left(\sum_{s=t+1}^{t+h} \Delta x_{js} \right)$$

and

$$E_t(\Delta_h z_{i,t+h}) \simeq \sum_{j \in n_i} \sum_{s=t+1}^{t+h} \alpha_{jt} E_t(\Delta x_{js}). \tag{4}$$

The approximation (4) should work well when the terms $E_t(\Delta x_{js})$ are small over the forecasting horizon $s \in [t+1, t+h]$. Well-known prediction methods for state space models can be used to obtain $E_t(\Delta x_{js})$, for given parameter values. To make explicit the dependence of the forecasts on the parameter values, let ϑ be a vector of unknown parameters and let $\widehat{\vartheta}_\tau$ denote the ML estimator of ϑ using all the data up until (and including) time period τ, i.e., $\{x_t\}_{t \leq \tau}$. We then use the notation $E_t(z_{i,t+h}|\widehat{\vartheta}_\tau)$ to denote the forecast of $z_{i,t+h}$ given the information set I_t and the parameter estimate $\widehat{\vartheta}_\tau$:

$$E_t(z_{i,t+h}|\widehat{\vartheta}_\tau) = z_{it} + E_t(\Delta_h z_{i,t+h}|\widehat{\vartheta}_\tau). \tag{5}$$

3 Data and estimation

We mainly use quarterly data from the Norwegian national accounts. In addition we use time series for household wealth, housing prices, the money market interest rate, the unemployment rate and the import weighted exchange rate where the data source is either Statistics Norway or the Central Bank of Norway. Altogether, not counting deduced variables, data for 93 variables are utilized. The time series start in 1978Q2 and end in 2005Q4. Table A1 gives an exhaustive overview over the variables, which we denote by X_i. Note that nominal variables, price indices and hourly wage rates have been divided by the consumer price index, as indicated in the footnotes of Table A1. In conjunction with the econometric analysis, the variables are log transformed. For the money market interest rates and the unemployment rate, we apply the transformations given below:

$$\begin{aligned} x_5 &= \ln(1 + X_5) \\ x_8 &= \ln(1 + (X_8/100)). \end{aligned}$$

Table A2 shows how the 22 key variables, referred to as Z-variables, are derived from those in Table A1. In Table 1 we list the key variables and the transformation undertaken for each of them. We also introduce short labels for the transformed key variables, that we are going to predict.

Referring to the state space formulation (1), let A_t =E$(\alpha_t|y_1, ..., y_{t-1})$, V_t =Var$(\alpha_t|y_1, ..., y_{t-1})$, $\Sigma = diag[\sigma_1^2, \sigma_2^2, ..., \sigma_n^2]$ and let D_t be the one-step ahead prediction error covariance matrix for y_t:

$$D_t = E\left([y_t - E(y_t|y_1, ..., y_{t-1})]\left[(y_t - E(y_t|y_1, ..., y_{t-1}))\right]'\right). \qquad (6)$$

As above, ϑ denotes the vector of unknown parameters. Then the log-likelihood function takes the standard form

$$l(\vartheta) = -\frac{1}{2}\sum_{t=1}^{T} \left[\ln|D_t| + (y_t - Bd_t - X_t^*\phi - ZA_t)' D_t^{-1}(y_t - Bd_t - X_t^*\phi - ZA_t)\right],$$

where A_t and D_t are calculated by means of the Kalman filter, as follows:

$$
\begin{aligned}
A_1 &= 0 \\
V_1 &= 0 \\
\text{For } t &= 2, ..., T: \\
e_t &= y_t - Bd_t - X_t^* \phi - ZA_t \\
D_t &= ZV_tZ' + \Sigma \\
K_t &= \mathcal{T} V_t Z' D_t^{-1} \\
A_{t+1} &= \mathcal{T} A_t + K_t e_t \\
V_{t+1} &= (\mathcal{T} - K_t Z) V_t \mathcal{T}' + RR'.
\end{aligned} \tag{7}
$$

The dimensionality problem associated with this model is related to the observation vector y_t. Although the Kalman filter requires inversion of the $n \times n$ matrix D_t, this can be carried out by using the matrix inversion lemma:

$$
[ZV_tZ' + \Sigma]^{-1} = \Sigma^{-1} - \Sigma^{-1} Z (V_t^{-1} + Z' \Sigma^{-1} Z)^{-1} Z' \Sigma^{-1}. \tag{8}
$$

Because Σ is diagonal, the use of (8) simplifies considerably.

Partial optimization of the likelihood function with respect to the regression parameters (ϕ_i', β_i'), $i = 1, ..., n$, is automatically obtained, in closed form, by the augmented Kalman filter, see de Jong (1991). Full maximum likelihood estimation with respect to all the parameters of the model, i.e., also including the factor loadings and variance parameters, $(\theta_i', \lambda_i', \sigma_i)$, $i = 1, ..., n$; and the autoregressive coefficients of the latent factors, $\psi_1, ..., \psi_r$, is more cumbersome. Most papers that use likelihood methods, e.g., Doz et al., 2006, rely on the EM algorithm. However, because of its linear convergence properties, this method is not practical when the number of parameters is very large. For example, Doz et al. (2006), using seasonally adjusted data, do not attempt to maximize the log-likelihood function but just perform a few iterations of the EM algorithm. In our model, the number of parameters is given by $\sum_{i=1}^{n} p_i + n(r + k + 5)$, except for the correction following from identifying restrictions on the loading factors. The parameters consist of AR-parameters, factor loadings, dummy variables and the variance of the genuine error term. The median value of p_i is 3 and, for example if $r = k = 3$, the total number of parameters is around 1,500.

A property of the EM algorithm, which is seldom utilized in practice, is that it can be used to obtain exact derivatives of the log-likelihood function; see Koopman and Shephard (1992). To obtain $\frac{\partial l}{\partial \vartheta}$ via the EM-algorithm, the following result is useful:

$$\left.\frac{\partial l(\vartheta)}{\partial \vartheta}\right|_{\vartheta=\vartheta_0} = \left.\frac{\partial M(\vartheta|\vartheta_0)}{\partial \vartheta}\right|_{\vartheta=\vartheta_0}, \tag{9}$$

with

$$M(\vartheta|\vartheta_0) = \int \ln g(Y, \alpha; \vartheta)\, g(\alpha|\, Y; \vartheta_0)\, d\alpha, \tag{10}$$

where $g(\cdot|\cdot)$ is a conditional probability density, $Y = \{y_t\}_{t=1}^{T}$ are the observed variables, and $\alpha = \{\alpha_t\}_{t=1}^{T}$ are the latent variables. While direct differentiation of the log-likelihood function will break down because the number of computations involved in the derivative of the covariance matrix $[ZV_tZ' + \Sigma]$ is of order $O(n^4)$, indirect differentiation of the log-likelihood function using (9) is of order $O(n^2)$, as we show in Appendix A, and hence quite feasible even for large n. Some background for (9) and (10) for the general Gaussian state space model is given in Dempster et al. (1977). See also Fahrmeir and Tutz (1994).

4 Empirical results

To evaluate the out-of-sample forecasting properties of the dynamic factor model and the benchmark AR model, 16 observations were retained. We refer to these observations, i.e., $t \in [T + 1, T + 16]$, as the out-of-sample period. The in-sample period, $[1, T]$, is used for estimation and model selection only. It is the change using a logarithmic scale, i.e., relative change, which is predicted, not the (nominal) levels of the variables.

To choose p_i, r and k in the dynamic factor model, a two-step model selection procedure was used. In the first step, the Akaike information criterion was applied to determine the number of lags (p_i) in each of the 93 equations of the dynamic factor model. The chosen lag lengths vary between one and eleven quarters. The maximum number of lags allowed was 12. In the second step, given the number of lags p_i in each equation determined in the first step, the numbers r and k were jointly determined by the use of information criteria. Let DFM(r, k) denote the resulting dynamic factor model with r non-seasonal dynamic factors and k stochastic seasonal components. This two-step procedure has two main benefits. First, it is computationally simple. In contrast, joint optimization with respect to $(p_1, ..., p_n, r, k)$ is not computationally feasible – for obvious reasons. Second, when $r = k = 0$ we obtain as a special case a set of optimal (according to Akaike's criterion) univariate AR models, which are natural benchmarks to which we may compare the forecasting properties of the dynamic factor model.

We consider three types of criteria in the second step: Akaike (AIC), Bayes (BIC) and a criterion proposed by Bai and Ng (2002), denoted IC_{p1}. Formally, these three information criteria are defined as

$$\begin{aligned} AIC &= \ln|\widehat{D}_\infty| + (r+k)n\frac{2}{T} \\ BIC &= \ln|\widehat{D}_\infty| + (r+k)n\frac{\ln(T)}{T} \\ IC_{p1} &= \ln\frac{\mathrm{tr}(\widehat{D}_\infty)}{n} + (r+k)\frac{n+T}{nT}\ln(\frac{nT}{n+T}). \end{aligned}$$

As pointed out by Reinsel (1993, p. 92), $\widehat{D}_\infty$ is the estimated covariance matrix of the one-step ahead prediction error covariance matrix D_t when $t \to \infty$, T is the number of observations and $(r + k)n$ is the total number of factor loading parameters (including also the zeros imposed to achieve identification). To estimate the parameters, 81 quarterly observations from 1978Q2 to 2001Q4 were used. The AIC criterion is standard for VARMA models, while the BIC is similar to the AIC except that the penalty factor $2/T$

is replaced by the heavier penalty factor $\ln(T)/T$. The criterion IC_{p1} is one of several criteria suggested by Bai and Ng (2002), that are tailored to dynamic factor models with both large n and T. Their criteria have in common that they are based on the trace of $\widehat{D}_\infty$ – instead of the generalized prediction error variance, $|\widehat{D}_\infty|$.

Table 2 presents the results for the AIC, BIC and IC_{p1} for some combinations of r and k in the neighborhood of the optimal solutions. The optimal solution based on the AIC is characterized by $r = 5$ and $k = 3$, i.e., it leads to $\mathrm{DFM}(5, 3)$, the BIC leads to the model choice $\mathrm{DFM}(1, 1)$, while IC_{p1} degenerates into the case with zero factors, $\mathrm{DFM}(0, 0\,)$. In the case of the Bai and Ng (2002) criterion, the reason for the degenerate outcome is that $\mathrm{tr}(\widehat{D}_\infty)$ changes little across the different models. It is especially interesting that the optimal model according to the BIC, $\mathrm{DFM}(1, 1)$, has a higher value of $\mathrm{tr}(\widehat{D}_\infty)$ than the degenerate model $\mathrm{DFM}(0, 0\,)$, which is optimal according to the IC_{p1}. On the other hand, the results in Table 2 do tell us that by including more common factors, a decrease in the generalized variance, $|\widehat{D}_\infty|$, of the whole system of equations is always attained. Because Bai and Ng (2002) do not take the off-diagonal elements of $\widehat{D}_\infty$ into account, their criterion does not appear to be appropriate in the present context: It is the off-diagonal elements of $\widehat{D}_\infty$ that are most affected by the common factors. Thus the potential for reducing $tr(\widehat{D}_\infty)$ in the second stage of the model selection procedure seems small, given that the AIC was used to select the number of AR terms in the first stage.

We shall now compare the forecasting properties of four different models: (i) 22 univariate "benchmark" AR models, where the number of lags in each equation is determined by means of Akaike's Information Criterion, (ii) $\mathrm{DFM}(5, 3)$, (iii) $\mathrm{DFM}(1, 1)$ and (iv) the degenerate case $\mathrm{DFM}(0, 0)$, which is a system of $n = 93$ univariate AR models. In cases (ii)–(iv), forecasts of the key variables are obtained by aggregation, as outlined in Section 2.

For the four models above, Table 3 shows the root mean square forecasting error

(RMSE) for each of the 22 key variables, all of which are measured using a logarithmic scale (cf. (3)). The results in the table refer to the out-of-sample period $[T+1, T+16]$. At the end of period T, we carried out 1-, 2- , 3- and 4-quarters ahead forecasts using (5). Then the information set was updated by including data up until $T+4$. The models were then reestimated, with the new data appended, and corresponding 1-, 2- , 3- and 4-quarters ahead forecasts were calculated, etc. The results for out-of-sample RMSE presented in Table 3 are therefore based on the 16 forecasts $E_{T+4l}(z_{i,T+4l+h}|\widehat{\vartheta}_{T+4l})$ for $l = 0, 1, 2, 3$ and $h = 1, 2, 3, 4$. The resulting forecast errors $z_{i,T+4l+h} - E_{T+4l}(z_{i,T+4l+h}|\widehat{\vartheta}_{T+4l})$ for each of the 22 key variables produced by the models were used to calculate the RMSEs. Results from similar calculations using the mean absolute error (MAE) are presented in Table 4 for the benchmark AR models and DFM$(5, 3)$.

Let us first look at the results for the model DFM$(5, 3)$ in Tables 3 and 4. The columns labeled "Relative" refer to RMSE or MAE for the dynamic factor model relative to the AR benchmark. We see that the dynamic factor model is improving somewhat upon the out-of-sample forecasts of the benchmark AR model. The gain in terms of both reduced RMSE and MAE is about 20 per cent when averaging the results for all the 22 key variables. In terms of the median, the difference between the two models is somewhat smaller, slightly exceeding 10 per cent. The gain is most notably related to the forecasts for aggregate manufacturing investments (INVM), which are improved substantially by using the dynamic factor model compared with the univariate AR models. For the other variables, the results are mixed. In most cases, however, the factor model is at least as good as the AR model; that is for 17 and 19 of the 22 key variables, according to the RMSE and MAE, respectively.

Comparing the RMSEs for the model DFM$(5, 3)$ with the degenerate model DFM$(0, 0)$, i.e., the AR models for the 93 disaggregated variables, we get similar results as for the benchmark AR model, except that the RMSE for INVM in the latter model (.105) is much smaller than in the AR model (.162). The same is even more pronounced for DFM$(5, 3)$ (.070). Compared with the disaggregate AR model, both the mean and median RMSE for DFM$(5, 3)$ is 15 per cent lower. For only one variable, unemployment (UNEMP), is the RMSE of DFM$(5, 3)$ higher than for DFM$(0, 0)$. Moreover, compared to the figures reported in study by Artis et al. (2005) for UK, we find that the optimal dynamic factor model according to the AIC criterion performs substantially better relative to the AR models (regardless of whether one applies the aggregated or the disaggregated data). Artis et al. report an overall gain in terms of the MSE of 10–20 per cent, which corresponds to only 5–10 per cent in terms of the RMSE. On the other hand, the optimal dynamic factor model according to the BIC, i.e., DFM$(1, 1)$, generally performs poorer than the other models reported in Table 3, for example with a median RMSE about 20 per cent higher than the benchmark AR model. This may be because the model DFM$(1, 1)$ has the highest estimated in-sample MSE of all the specifications in Table 2, as seen from $\ln(\mathrm{tr}(\widehat{D}_\infty))$. Thus, it appears that neither the BIC nor the IC_{p1} are appropriate criteria for choosing the number of dynamic factors in our two step procedure.

In Figures 1–3 we display, as examples, the forecast errors of the key variables INVM, man-hours in manufacturing (MM) and the price index of traditional imports (PRIM), respectively. Each figure consists of four parts corresponding to the four different horizons employed. For each horizon, there are four forecast errors. Figure 1 reinforces that the out-of-sample forecasts are better for the dynamic factor model than for the AR model in the case of INVM. The same feature, but to a lesser extent, is also evident for MM, whereas the forecast errors from the two models are close to equal in the case of PRIM.

We have compared our results with the model DFM(5,3), where we also have included two exogenous regressors: (i) the relative change of the real oil price (Brent spot deflated by the consumer price index) and (ii) the relative change in a foreign market indicator. The out-of-sample forecasts of the exogenous variables themselves were obtained using univariate AR models, with lag lengths determined by means of the AIC. While the in-sample forecasts for our key macroeconomic variables were somewhat improved, the out-of-sample results are disappointing. The RMSE was almost 50 per cent higher on average than for the factor model without exogenous regressors. Our interpretation of this result is that the relevant information for our key variables that is contained in these variables are already incorporated in the dynamic factors, so that adding them directly as regressors does not convey any additional information. On the contrary, idiosyncratic components of these variables, that are not informative about the key variables, seem to contaminate the forecasts in a way that leads to substantially worse out-of-sample performance.

5 Conclusions

In this paper, we have estimated a dynamic factor model using a quarterly data set of 93 unadjusted variables for the Norwegian economy. The model is formulated in the relative changes of the variables. In the final specification we include, after having performed model selection using Akaike's information criterion, five common stationary latent components and three common nonstationary latent components related to seasonality.

The main aim of the paper has been to compare forecasts for 22 derived key variables using a dynamic factor model with forecasts based on univariate autoregressive models augmented with seasonal dummy variables. We consider forecasts up to four periods using root mean square error and mean absolute error and find that there is an overall gain in employing the dynamic factor model. The improvement is notable only for a small set of variables. However, for most of the variables, the dynamic factor model does not perform worse than the univariate model. Our results seem to be in line with other analyses in this line of research.

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Appendix A. Derivatives of the log-likelihood function

For general random vectors z and α, let Y denote the observed data and α the "missing" data, i.e., all of the latent variables. Furthermore, let $g(Y, \alpha; \vartheta)$ be their joint density (i.e. the "complete data" density), and $g(\alpha| Y; \vartheta)$ the conditional density of α given Y. The ML estimator, $\widehat{\vartheta}$, is the maximum of the log-likelihood $l(\vartheta)$ of the observed data, where

$$l(\vartheta) = \ln g(Y; \vartheta). \tag{11}$$

Because

$$g(Y; \vartheta) = \frac{g(Y, \alpha; \vartheta)}{g(\alpha| Y; \vartheta)},$$

(11) can be rewritten as

$$l(\vartheta) = \ln g(Y, \alpha; \vartheta) - \ln g(\alpha| Y; \vartheta). \tag{12}$$

Taking the expectation on both sides in (12) with respect to the conditional density $g(\alpha|Y; \vartheta_0)$ for any arbitrary value ϑ_0 gives

$$\begin{aligned} l(\vartheta) &= M(\vartheta|\vartheta_0) - H(\vartheta| \vartheta_0) \qquad (13) \\ \left.\frac{\partial l(\vartheta)}{\partial \vartheta}\right|_{\vartheta=\vartheta_0} &= \left.\frac{\partial M(\vartheta|\vartheta_0)}{\partial \vartheta}\right|_{\vartheta=\vartheta_0} \end{aligned}$$

where

$$M(\vartheta|\vartheta_0) = \mathsf{E}[\ln g(Y, \alpha; \vartheta)|Y, \vartheta_0]$$
$$H(\vartheta|\vartheta_0) = \mathsf{E}[\ln g(\alpha|Y; \vartheta)|Y, \vartheta_0]$$

and the expectation is with respect to the conditional density of α given the observed data Y, evaluated at ϑ_0. Let $\alpha_t^{(1)}$ denote the subvector of α consisting of the first r components, i.e., $\alpha_t^{(1)} = f_t$, for $t = 1, ..., T$. Using (1) and (2), we can write

$$M(\vartheta| \vartheta_0) = M_1(B, \phi, Z, \Sigma|\vartheta_0) + M_2(\Psi|\vartheta_0), \tag{14}$$

where

$$\begin{aligned} M_1(B, \phi, Z, \Sigma|\vartheta_0) = &-\frac{T}{2}\sum_{i=1}^{n} \ln \sigma_i^2 \\ &- \frac{1}{2}\mathsf{E}\{\sum_{t=1}^{T}(y_t - Bd_t - X_t^*\phi - Z\alpha_t)'\Sigma^{-1}(y_t - Bd_t - X_t^*\phi - Z\alpha_t) \,|\, Y; \vartheta_0\} \end{aligned} \tag{15}$$

and

$$M_2(\Psi|\vartheta_0) = \\ -\frac{1}{2}\mathsf{E}\{\sum_{t=2}^{T}(\alpha_t^{(1)} - \Psi\alpha_{t-1}^{(1)})'(\alpha_t^{(1)} - \Psi\alpha_{t-1}^{(1)}) \,|\, Y;\vartheta_0\}. \tag{16}$$

Note that many elements of B, ϕ, Z and Σ are zeros (for notational simplicity we do not make this explicit) and that $\mathcal{T}$ depends on Ψ through (2). In (15)-(16), the expectation is with respect to the latent variables $(\alpha_1, ..., \alpha_T)$, conditional on the data Y, and with ϑ evaluated at ϑ_0.

Because $M(\vartheta|\,\vartheta_0)$ is quadratic in $(\alpha_1, ..., \alpha_T)$, to evaluate the expectations in (15)-(16) we only need to calculate the conditional expectations

$$a_{t|T} = \mathsf{E}\{\alpha_t \,|\, Y;\vartheta_0\}, \tag{17}$$

and the covariance matrices

$$V_{t\,|T} = \mathsf{E}\{(\alpha_t - a_{t|T})(\alpha_t - a_{t|T})' \,|\, Y;\vartheta_0\}. \tag{18}$$

Note that $a_{t|t-1} \equiv A_t$ and $V_{t|t-1} \equiv V_t$ (cf. (7)), while the required conditional expectations $a_{t|T}$ and covariance matrices $V_{t|T}$ are obtained by the backward Kalman-smoothing recursions (see e.g. Harvey, 1989):

Kalman smoothing

For $t = T, ..., 2$:

$$a_{t-1|T} = a_{t-1|t-1} + B_t(a_{t|T} - a_{t|t-1})$$

$$V_{t-1|T} = V_{t-1|t-1} + B_t(V_{t|T} - V_{t|t-1})B_t',$$

where

$$B_t = V_{t-1|t-1}'\mathcal{T}'V_{t|t-1}^{-1}. \tag{19}$$

Let us first consider the differentiation of $M_2(\Psi|\vartheta)$ with respect to the elements of Ψ:

$$\frac{\partial M_2(\Psi|\vartheta_0)}{\partial \Psi} = \left(\sum_{t=2}^{T} a_{t|T}^{(1)} a_{t-1|T}^{(1)}{}' + (V_{t|T}'B_t')^{(1,1)}\right) - \Psi\left(\sum_{t=2}^{T} a_{t-1|T}^{(1)} a_{t-1|T}^{(1)}{}' + V_{t-1|T}^{(1,1)}\right), \tag{20}$$

where the $r \times 1$ vector $a_{s|t}^{(1)}$ is the first elements of $a_{s|t}$, $V_{t-1|T}^{(1,1)}$ is the corresponding upper left block of the matrix $V_{t-1|T}$, and we have utilized that

$$\mathsf{E}(\alpha_t\alpha_{t-1}'|\,Y;\vartheta) = a_{t|T}a_{t-1|T}{}' + V_{t|T}B_t',$$

with B_t defined in (19) (see Fahrmeir and Tutz, 1994, p. 269; and Schneider, 1986, for a proof). Note that all the matrices $a_{t|T}$, $V_{t|T}$, and B_t are outputs from the Kalman filtering and smoothing algorithms, with ϑ evaluated at ϑ_0. Next consider $M_1(B, \phi, Z, \Sigma|\vartheta_0)$. Some straightforward calculations yield

$$M_1(B, \phi, Z, \Sigma|\vartheta_0) = -\frac{T}{2}\sum_{i=1}^{n} \ln \sigma_i^2$$
$$-\frac{1}{2}\sum_{t=1}^{T} \operatorname{tr}\left(\Sigma^{-1}\left(\left[y_t - Bd_t - X_t^*\phi - Za_{t|T}\right]\left[y_t - Bd_t - X_t^*\phi - Za_{t|T}\right]' + ZV_{t|T}Z'\right)\right)$$

The number of unknown parameters in our model is of order $O(n)$ (see the discussion following (8)). Moreover, X_t^* is sparse: from (2), we see that it contains $O(n^2)$ elements, of which there are $O(n)$ non-zero elements. Hence it is easily seen that differentiation with respect to (the non-zero elements of) B, ϕ, Z and Σ requires $O(n^2)$ operations.

Appendix B. Construction of weights. Two specific examples

Here, we consider two particular examples of aggregate variables. Recall that the upper case X and Z variables are defined in Tables A1 and A2, respectively. Let w_{rjt} and α_{rjt} denote w_{jt} and α_{jt}, respectively, when applied to the (aggregate) variable z_r, $r = 1, ..., 22$ (in Section 2 the r-index was suppressed for simplicity). First, we look at z_9, the log of traditional exports. Then

$$w_{9jt} = 1 \text{ and } \alpha_{9jt} = \frac{X_{jt}}{\sum_{k \in n_9} X_{kt}} \text{ for } j \in n_9 = \{9, 10, ..., 13\}.$$

Let us next consider z_{10}, the log of the price of the traditional export aggregate deflated by the consumer price index. Then

$$w_{10jt} = \alpha_{9,j-5,t} \text{ and } \alpha_{10jt} = w_{10jt}\frac{X_{jt}}{Z_{10t}} \text{ for } j \in n_{10} = \{14, 15, ..., 18\}.$$

Tables and figures

Table 1: Key variables, transformations and short labels

Symbol	Short label	Short description	Transformation
z_1	INC	Income	$\ln(Z_1)$
z_2	WTH	Wealth	$\ln(Z_2)$
z_3	CPI	Consumer price	$\ln(Z_3)$
z_4	HPI	Housing price	$\ln(Z_4)$
z_5	INTR	Interest rate	$\ln(1 + Z_5)$
z_6	GOV	Government consumption	$\ln(Z_6)$
z_7	EXCR	Exchange rate	$\ln(Z_7)$
z_8	UNP	Unemployment rate	$\ln(1 + (Z_8/100))$
z_9	EXP	Traditional exports	$\ln(Z_9)$
z_{10}	PREX	Price index of trad. exports	$\ln(Z_{10})$
z_{11}	IMP	Traditional imports	$\ln(Z_{11})$
z_{12}	PRIM	Price index of trad. imports	$\ln(Z_{12})$
z_{13}	CON	Private consumption	$\ln(Z_{13})$
z_{14}	MM	Man-hours in manufact.	$\ln(Z_{14})$
z_{15}	WM	Wage in manufact.	$\ln(Z_{15})$
z_{16}	VAM	Value added manufact.	$\ln(Z_{16})$
z_{17}	INVM	Investment manufact.	$\ln(Z_{17})$
z_{18}	VAS	Value added services	$\ln(Z_{18})$
z_{19}	INVS	Investment services	$\ln(Z_{19})$
z_{20}	MS	Man-hours in services	$\ln(Z_{20})$
z_{21}	WAS	Wage services	$\ln(Z_{21})$
z_{22}	VAML	Value added "mainland"	$\ln(Z_{22})$

Table 2: Information criteria for different dynamic factor models

DFM(r, k)		$\ln(\mathrm{tr}(\widehat{D}_\infty))$	$\ln(\lvert\widehat{D}_\infty\rvert)$	$(r + k)2n/T$	AIC	BIC	IC$_{p1}$
r	k						
0	0	-630.9	-630.9	0	-630.9	-630.9	-4.39
1	0	-631.7	-661.2	2.3	-658.8	-655.9	-4.30
1	1	-624.0	-670.2	4.7	-665.4	-659.7	-4.20
1	2	-627.8	-674.3	7.1	-667.1	-658.5	-4.16
1	3	-630.8	-678.3	9.5	-668.8	-657.4	-4.09
2	3	-640.4	-682.0	11.9	-670.1	-655.8	-4.03
3	3	-642.1	-685.6	14.2	-671.3	-654.2	-3.96
4	3	-642.2	-689.0	16.6	-672.4	-652.5	-3.87
5	3	-642.6	-692.3	18.9	-673.3	-650.7	-3.78
6	3	-642.3	-693.8	21.2	-672.5	-647.1	-3.68

Table 3: Out-of-sample root mean squared error (RMSE) and relative RMSE

Variable	Benchmark AR		Dynamic factor models					
			DFM(5,3)		DFM(1,1)		DFM(0,0)	
	RMSE	#lag	RMSE	Relative	RMSE	Relative	RMSE	Relative
INC	0.044	3	0.034	0.767	0.043	0.961	0.042	0.936
WTH	0.033	3	0.028	0.852	0.035	1.048	0.033	1.000
CPI	0.010	4	0.008	0.755	0.010	0.992	0.009	0.912
HPI	0.032	7	0.032	1.002	0.039	1.210	0.034	1.058
INTR	0.003	1	0.003	0.889	0.003	0.991	0.003	1.022
GOV	0.026	3	0.019	0.700	0.030	1.141	0.025	0.961
EXC	0.055	4	0.054	0.988	0.060	1.085	0.060	1.092
UNP	0.002	4	0.003	1.313	0.002	0.946	0.002	1.012
EXP	0.023	1	0.027	1.170	0.031	1.311	0.027	1.138
PREX	0.051	1	0.048	0.933	0.050	0.974	0.050	0.969
IMP	0.044	1	0.030	0.670	0.046	1.046	0.041	0.938
PRIM	0.035	1	0.033	0.951	0.035	0.988	0.035	0.991
CON	0.015	5	0.010	0.664	0.013	0.915	0.012	0.856
MM	0.041	11	0.031	0.765	0.044	1.063	0.038	0.912
WM	0.025	2	0.026	1.016	0.039	1.547	0.026	1.037
VAM	0.032	5	0.032	0.993	0.042	1.311	0.034	1.079
INVM	0.162	3	0.070	0.432	0.100	0.619	0.105	0.648
VAS	0.018	5	0.016	0.926	0.025	1.418	0.017	0.991
INVS	0.054	5	0.044	0.813	0.052	0.962	0.050	0.927
MS	0.035	6	0.027	0.772	0.041	1.173	0.032	0.899
WAGS	0.027	3	0.024	0.911	0.038	1.448	0.025	0.939
VAML	0.022	1	0.024	1.116	0.030	1.370	0.026	1.187
mean	0.036		0.028	0.788	0.037	1.021	0.033	0.919
median	0.032		0.028	0.873	0.039	1.209	0.032	1.016

Table 4: Out-of-sample mean absolute error (MAE)

Variable	DFM(5, 3)	Benchmark AR	Relative MAE
INC	0.028	0.036	0.778
WTH	0.025	0.030	0.837
CPI	0.006	0.008	0.698
HPI	0.027	0.027	0.987
INTR	0.002	0.002	0.869
GOV	0.015	0.023	0.660
EXC	0.040	0.041	0.979
UNP	0.002	0.002	1.250
EXP	0.022	0.016	1.327
PREX	0.041	0.044	0.935
IMP	0.025	0.035	0.702
PRIM	0.030	0.032	0.958
CON	0.007	0.013	0.592
MM	0.027	0.036	0.754
WM	0.021	0.022	0.958
VAM	0.026	0.028	0.946
INVM	0.056	0.147	0.382
VAS	0.012	0.015	0.832
INVS	0.035	0.042	0.823
MS	0.024	0.032	0.740
WAGS	0.019	0.020	0.943
VAML	0.020	0.018	1.140
mean	0.023	0.030	0.764
median	0.024	0.027	0.881

Table A1: Overview of variables in the dynamic factor model[1]

Variable	Description	Unit of measurement
X_1	Households' disposable income (*)	In million 2003 NOK
X_2	Households' wealth (*)	In million NOK
X_3	The Norwegian consumer price index	1 in 2003 (average)
X_4	Housing price (*)	1 in 2003 (average)
X_5	Money market interest rate	Quarterly interest rate
X_6	Governmental consumption	In million 2003 NOK
X_7	Import weighted exchange rate (*)	
X_8	Unemployment rate	In per cent
X_9	Exports of manufactured agricultural and fish products	In million 2003 NOK
X_{10}	Exports of different manufactured products	In million 2003 NOK
X_{11}	Exports of pulp and paper products	In million 2003 NOK
X_{12}	Exports of machinery	In million 2003 NOK
X_{13}	Exports of other traditional goods	In million 2003 NOK
X_{14}	Price index of exports of manufactured agricultural and fish products (*)	1 in 2003 (average)
X_{15}	Price index of exports of different manufactured products (*)	1 in 2003 (average)
X_{16}	Price index of exports of pulp and paper products (*)	1 in 2003 (average)
X_{17}	Price index of exports of machinery (*)	1 in 2003 (average)
X_{18}	Price index of exports of other traditional goods (*)	1 in 2003 (average)
X_{19}	Imports of manufactured agricultural and fish products	In million 2003 NOK
X_{20}	Imports of different manufactured products	In million 2003 NOK
X_{21}	Imports of pulp and paper products	In million 2003 NOK
X_{22}	Imports of machinery	In million 2003 NOK
X_{23}	Imports of other traditional goods	In million 2003 NOK
X_{24}	Price index of import of manufactured agricultural and fish products (*)	1 in 2003 (average)
X_{25}	Price index of import of different manufactured products (*)	1 in 2003 (average)
X_{26}	Price index of imports of pulp and paper products (*)	1 in 2003 (average)
X_{27}	Price index of imports of machinery (*)	1 in 2003 (average)
X_{28}	Price index of imports of other traditional goods (*)	1 in 2003 (average)
X_{29}	Domestic consumers' consumption of food	In million 2003 NOK
X_{30}	Domestic consumers' consumption of beverages	In million 2003 NOK
X_{31}	Domestic consumers' consumption of tobacco	In million 2003 NOK

Table A1: (Continued)

Variable	Description	Unit of measurement
X_{32}	Domestic consumers' consumption of electricity	In million 2003 NOK
X_{33}	Domestic consumers' consumption of fuel etc.	In million 2003 NOK
X_{34}	Domestic consumers' running expenses on own vehicles	In million 2003 NOK
X_{35}	Domestic consumers' consumption of other non-durables	In million 2003 NOK
X_{36}	Domestic consumers' purchase of cloth	In million 2003 NOK
X_{37}	Domestic consumers' purchase of own transport equipment	In million 2003 NOK
X_{38}	Domestic consumers' purchase of other durables	In million 2003 NOK
X_{39}	Domestic consumers' consumption of housing services	In million 2003 NOK
X_{40}	Domestic consumers' consumption of other services	In million 2003 NOK
X_{41}	Domestic consumers' consumption of transport services	In million 2003 NOK
X_{42}	Domestic consumers' consumption of health services	In million 2003 NOK
X_{43}	Domestic consumers' consumption abroad	In million 2003 NOK
X_{44}	Man-hours in manufacturing of consumption goods	In 1000
X_{45}	Man-hours in manufacturing of materials and investment goods	In 1000
X_{46}	Man-hours in manufacturing of raw materials	In 1000
X_{47}	Man-hours in manufacturing of machinery etc.	In 1000
X_{48}	Man-hours in manufacturing of ships and transport equipment	In 1000
X_{49}	Wage per man–hour in manufacturing of consumption goods (*)	In 2003 NOK
X_{50}	Wage per man–hour in manufacturing of materials and investment goods (*)	In 2003 NOK
X_{51}	Wage per man–hour in manufacturing of raw materials (*)	In 2003 NOK
X_{52}	Wage per man–hour in manufacturing of machinery etc. (*)	In 2003 NOK
X_{53}	Wage per man-hour in manufacturing of ships and transport equipment (*)	In 2003 NOK
X_{54}	Value added in manufacturing of consumption goods	In million 2003 NOK
X_{55}	Value added in manufacturing of materials and investment goods	In million 2003 NOK
X_{56}	Value added in manufacturing of raw materials	In million 2003 NOK
X_{57}	Value added in manufacturing of machinery etc.	In million 2003 NOK
X_{58}	Value added in manufacturing of ships and transport equipment	In million 2003 NOK
X_{59}	Acq. of new tang. fixed assets in manufacturing of consumption goods	In million 2003 NOK
X_{60}	Acq. of new tang. fixed assets in manufacturing of materials and investment goods	In million 2003 NOK
X_{61}	Acq. of new tang. fixed assets in manufacturing of raw materials	In million 2003 NOK
X_{62}	Acq. of new tang. fixed assets in petroleum refining	In million 2003 NOK

Table A1: (Continued)

Variable	Description	Unit of measurement
X_{63}	Acq. of new tang. fixed assets in manufacturing of machinery etc.	In million 2003 NOK
X_{64}	Acq. of new tang. fixed assets in manufacturing of ships and transport equipment	In million 2003 NOK
X_{65}	Value added in construction	In million 2003 NOK
X_{66}	Value added in finance and insurance	In million 2003 NOK
X_{67}	Value added in production of electricity	In million 2003 NOK
X_{68}	Value added in domestic production	In million 2003 NOK
X_{69}	Value added in wholesale and retail trade	In million 2003 NOK
X_{70}	Value added in housing services	In million 2003 NOK
X_{71}	Value added in other private services	In million 2003 NOK
X_{72}	Acq. of new tang. fixed assets in construction	In million 2003 NOK
X_{73}	Acq. of new tang. fixed assets in finance and insurance	In million 2003 NOK
X_{74}	Acq. of new tang. fixed assets in production of electricity	In million 2003 NOK
X_{75}	Acq. of new tang. fixed assets in prod. of domestic transportation services	In million 2003 NOK
X_{76}	Acq. of new tang. fixed assets in wholesale and retail trade	In million 2003 NOK
X_{77}	Acq. of new tang. fixed assets in production of housing services	In million 2003 NOK
X_{78}	Acq. of new tang. fixed assets in production of other services	In million 2003 NOK
X_{79}	Man-hours in construction	In 1000
X_{80}	Man-hours in finance and insurance	In 1000
X_{81}	Man-hours in production of electricity	In 1000
X_{82}	Man-hours in production of domestic transportation services	In 1000
X_{83}	Man-hours in wholesale and retail trade	In 1000
X_{84}	Man-hours in production of housing services	In 1000
X_{85}	Man-hours in production of other private services	In 1000
X_{86}	Wage in construction (*)	In NOK
X_{87}	Wage in finance and insurance (*)	In NOK
X_{88}	Wage in production of electricity (*)	In NOK
X_{89}	Wage in production of domestic transportation services (*)	In NOK
X_{90}	Wage in wholesale and retail trade (*)	In NOK
X_{91}	Wage in production of housing services (*)	In NOK
X_{92}	Wage in production of other services (*)	In NOK
X_{93}	Value added in production and pipeline transport of oil and gas etc.	In million 2003 NOK

[1]The star in parenthesis signifies that the nominal variable or nominal price index has been divided by the consumer price index.

Table A2: Overview of deduced key macroeconomic variables

Deduced key variable	Expressions in terms of X and functions of X
Z_1	Households' disposable income: X_1
Z_2	Households' wealth: X_2
Z_3	The Norwegian consumer price index: X_3
Z_4	Housing prices: X_4
Z_5	Money market interest rate: X_5
Z_6	Governmental consumption: X_6
Z_7	Import weighted exchange rate: X_7
Z_8	Unemployment rate: X_8
Z_9	Traditional exports: $X_9 + X_{10} + X_{11} + X_{12} + X_{13}$
Z_{10}	Price index of traditional exports: $(X_{14} * X_9 + X_{15} * X_{10} + X_{16} * X_{11} + X_{17} * X_{12} + X_{18} * X_{13})/Z_9$
Z_{11}	Traditional imports: $X_{19} + X_{20} + X_{21} + X_{22} + X_{23}$
Z_{12}	Price index of traditional imports: $(X_{24} * X_{19} + X_{25} * X_{20} + X_{26} * X_{21} + X_{27} * X_{22} + X_{28} * X_{23})/Z_{11}$
Z_{13}	Private consumption: $X_{29} + X_{30} + X_{31} + X_{32} + X_{33} + X_{34} + X_{35} + X_{36} + X_{37} + X_{38} + X_{39} + X_{40} + X_{41} + X_{42} + X_{43}$
Z_{14}	Man-hours worked by employees in the manufacturing sector: $X_{44} + X_{45} + X_{46} + X_{47} + X_{48}$
Z_{15}	Wages per man-hour for employees in the manufacturing sector: $(X_{49} * X_{44} + X_{50} * X_{45} + X_{51} * X_{46} + X_{52} * X_{47} + X_{53} * X_{48})/Z_{14}$
Z_{16}	Value added in the manufacturing sector: $X_{54} + X_{55} + X_{56} + X_{57} + X_{58}$
Z_{17}	Acq. of new tang. fixed assets in the manufacturing sector: $X_{59} + X_{60} + X_{61} + X_{62} + X_{63} + X_{64}$
Z_{18}	Value added in the production of private services: $X_{65} + X_{66} + X_{67} + X_{68} + X_{69} + X_{70} + X_{71}$
Z_{19}	Acq. of new tang. fixed assets in the private services sectors: $X_{72} + X_{73} + X_{74} + X_{75} + X_{76} + X_{77} + X_{78}$
Z_{20}	Man-hours worked by employees in the private service sectors: $X_{79} + X_{80} + X_{81} + X_{82} + X_{83} + X_{84} + X_{85}$
Z_{21}	Wages per man-hour for employees in the private service sectors: $(X_{86} * X_{79} + X_{87} * X_{80} + X_{88} * X_{81} + X_{89} * X_{82} + X_{90} * X_{83} + X_{91} * X_{84} + X_{92} * X_{85})/Z_{20}$
Z_{22}	Value added in the manufacturing sector, in private services and in production and pipeline transport of oil and gas: $Z_{16} + X_{18} + X_{93}$

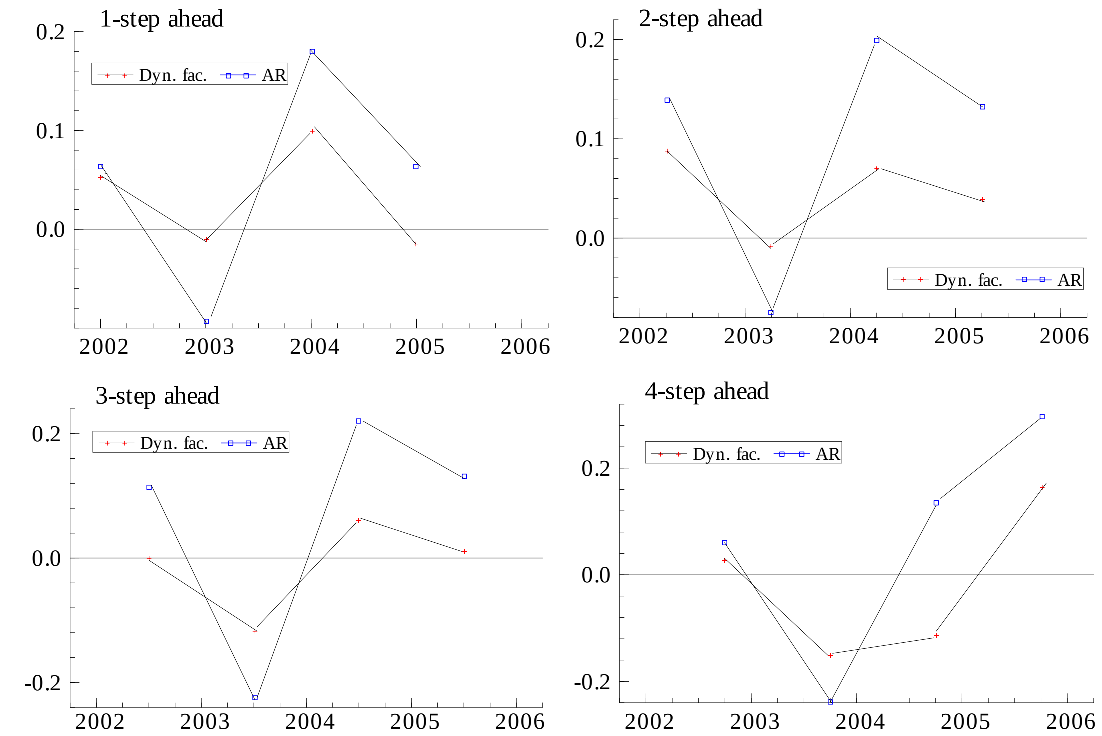

Figure 1: Forecasting errors at different horizons for INVM (log scale)

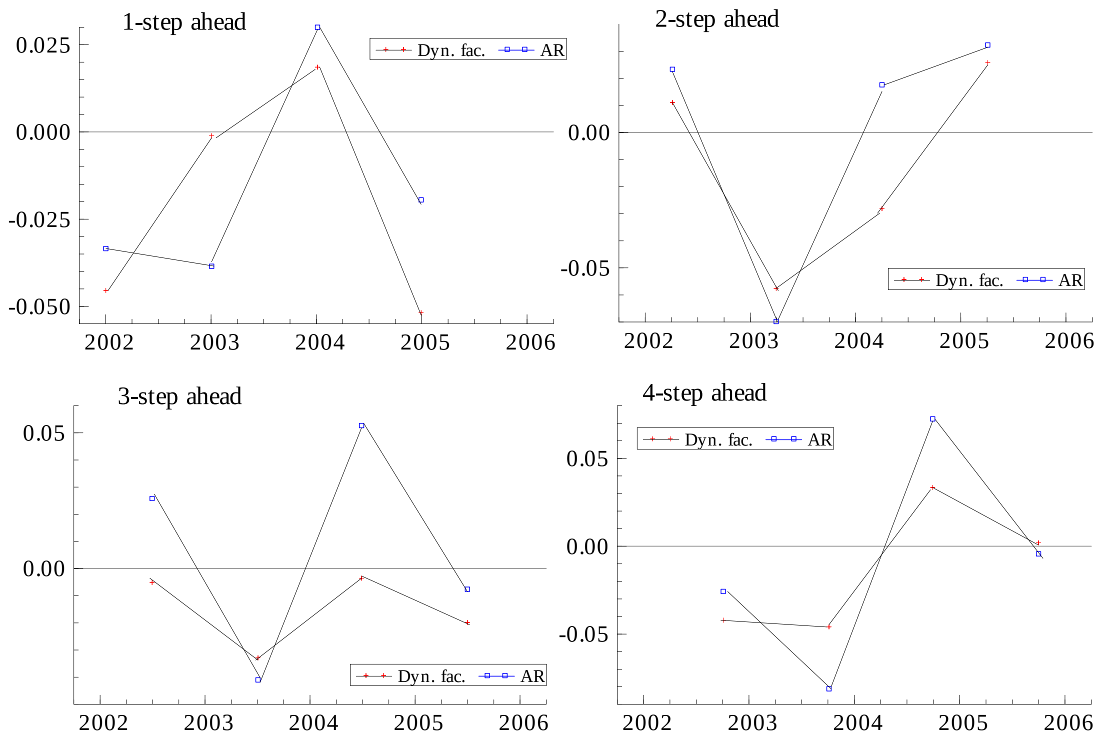

Figure 2: Forecasts errors at different horizons for MM: Benchmark AR vs. DFM(5,3) (log scale)

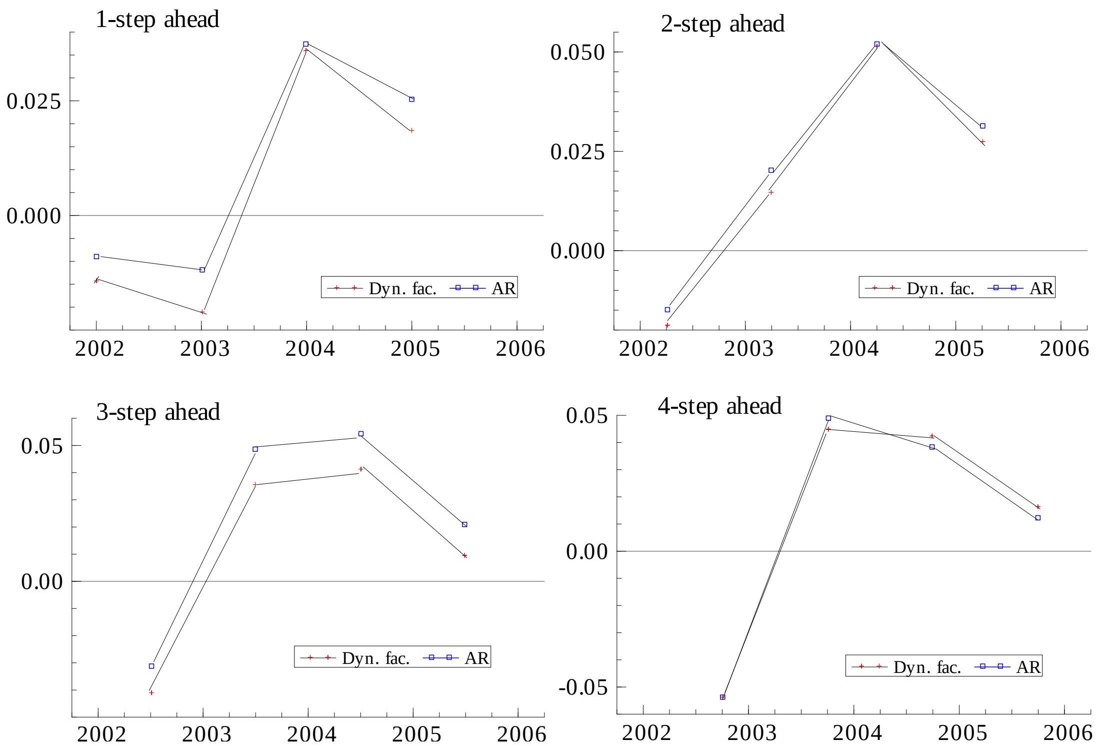

Figure 3: Forecast errors at different horizons for PRIM: Benchmark AR vs. DFM(5,3) (log scale)