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The New Keynesian Phillips Curve revisited

Abstract:
Recently, several authors have questioned the evidence claimed by Galí and Gertler (1999) and Galí, Gertler and López-Salido (2001) that a hybrid version of the New Keynesian Phillips Curve approximates European and US inflation dynamics quite well. We re-examine the evidence using likelihood-based methods. Although including lagged inflation enhances the empirical fit, the improvement is not large enough to yield a model that passes a likelihood ratio test. We also show that the likelihood surface is rather flat, especially in the European case, indicating that the model may be weakly identified as criticised by others using alternative methods.

Keywords: European and US inflation, the New Keynesian Phillips Curve, vector autoregressive models and likelihood ratio tests.

JEL classification: C51, C52, E31.

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1. Introduction

The New Keynesian Phillips Curve, henceforth NKPC, has become increasingly popular in recent years as a theory of inflation, both among researchers and policymakers in inflation targeting central banks. Despite numerous empirical attempts to evaluate its performance, no consensus about the evidence of the model seems to be established in the literature, see Henry and Pagan (2004) for a recent overview. The influential studies by Galí and Gertler (1999), henceforth GG, and Galí, Gertler and López-Salido (2001), henceforth GGL-S, find strong evidence in favour of the model using both European and US post-war data within a conventional GMM framework. Specifically, these studies suggest that European and US inflation dynamics are consistent with a simple hybrid version of the NKPC that relates inflation to expected future inflation, lagged inflation and real marginal costs, of which forward-looking behaviour plays a dominant role in explaining inflation.

Recently, several authors have re-examined this evidence using the same data set and questioned the robustness of the results in GG and GGL-S. For instance, Ma (2002) applies the Stock and Wright (2000) test statistics and casts doubt about the conclusions in GG on the ground that empirical identification of NKPC is weak due to the use of the GMM methodology. Similarly, Mavroeidis (2006) demonstrates by means of the conditional score test of Kleibergen (2005) that the parameters in GG are weakly identified and that US inflation dynamics are coherent with both forward-looking and backward-looking behaviour whereas real marginal costs appear to be an irrelevant determinant of inflation.[1] Also, Martins and Gabriel (2006) address the issue of identification and question the validity of the evidence in GG by applying the recently developed general empirical likelihood procedure based on Kleibergen (2005). Furthermore, Rudd and Whelan (2005) argue that the upward bias of the forward-looking estimates may be large when estimating the structural form of the NKPC by GMM, as GG do, rather than the corresponding closed form solution of the model. As a final example, Bårdsen *et al.* (2004) also show that the estimates in GGL-S most likely are biased in favour of a significant role for expected future inflation since that variable is found negligible in respecified models where variables from the instrument set directly and significantly cause inflation. Galí *et al.* (2005) answer some of the criticism and maintain their conclusion of GG about the importance of the forward-looking behaviour in explaining inflation dynamics.

We may add another potentially important econometric issue to the critiques of GG and GGL-S, namely the neglect of the time series properties of variables involved. If inflation and real marginal costs are non-stationary, we have an additional reason to question the reliability of the evidence in GG and GGL-S. In this paper, we revisit the NKPC using the same data set as in GG and GGL-S within the well-established practice of specifying multivariate time series as vector autoregressive (VAR) models. The VAR approach has several appealing aspects in the present context. First, by specifying the VAR appropriately both stationary and non-stationary time series integrated of order one, so-called $I(1)$ series, can be treated by essentially the same method. Second, the

[1] The issue of weak identification of forward-looking models estimated by GMM is thoroughly discussed in Mavroeidis (2004, 2005) among others.

relationships between the conditional expectations implied by the NKPC hypothesis have a simple form and are equivalent to a set of restrictions on the coefficients of the VAR. Hence, once an appropriate VAR is established, the restrictions can be tested by standard methods such as likelihood ratio, Wald or Lagrange multiplier tests. Such transparency is an advantage compared to the analysis in GG and GGL-S, which forces the assumption of rational expectations in the estimation procedure by a somewhat arbitrary choice of instruments. Finally, by using likelihood ratio tests profile or concentrated likelihoods involving the structural parameters of the NKPC can be computed. The curvature of the profile likelihood surface contains important information on how well the structural parameters are identified. A rather flat surface indicates that it will be difficult to distinguish empirically between different sets of parameter values.

Our empirical findings suggest that a reduced rank VAR and a full rank VAR are well specified models with well-behaved residuals in the case of the post-war European and US data, respectively. The NKPC in the pure forward-looking version as well as in the hybrid version combining both forward-looking and backward-looking price setters is clearly rejected by likelihood ratio tests. We also find that the likelihood surface, especially in the case of European data, is characterised by a rather distinct and flat ridge, confirming the claim of Mavroeidis (2005) that the NKPC specifications entail poor identification of the parameters involved.

The rest of the paper is organised as follows: Section 2 briefly outlines the NKPC models proposed and estimated by GG and GGL-S. Section 3 presents the likelihood based methods used in our context, while Sections 4 and 5 report and contrast empirical results for the European and US data with those of GGL-S and GG, respectively. Section 6 concludes.

2. The NKPC model

As explained by Roberts (1995), there are several routes from a theoretical set up of firm's pricing behaviour that lead to the NKPC model, including the linear quadratic adjustment cost model of Rotemberg (1982) and the models of staggered contracts developed by Taylor (1979, 1980) and Calvo (1983). GG and GGL-S estimate two distinct versions of the NKPC model based on Calvo’s model of staggered nominal pricing in an imperfectly competitive environment with firms producing differentiated products. The first version, which we will refer to as the *baseline model*, assumes only forward-looking price setters, whereas the second version, the *hybrid model*, combines both forward-looking and backward-looking behaviour. The baseline model relates inflation to expected future inflation and real marginal costs in the following way using the same notation as in GG and GGL-S (where lower case letters denote logs of variables involved):

(1) $$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t ,$$

where π_t denotes inflation in period *t*, mc_t represents real marginal costs in period *t* and E_t denotes the conditional expectations given the information at time *t*. The slope parameter $\lambda = \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta[1+\alpha(\varepsilon-1)]}$ depends on the degree of price stickiness (θ), the subjective discount rate (β), the curvature of the underlying production function (α) and the elasticity of demand (ε). GG and GGL-S derive the hybrid version of (1) by allowing a fraction (ω) of the firms to use a backward-looking rule of thumb in their price decisions based on lagged inflation as a predictor. Again using the same notation as in GG and GGL-S, the hybrid version of (1) reads as

$$\text{(2)} \qquad \pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \tilde{\lambda} mc_t,$$

where

$$\gamma_f = \beta\theta\phi^{-1},$$
$$\gamma_b = \omega\phi^{-1},$$
$$\tilde{\lambda} = \frac{(1-\omega)(1-\theta)(1-\beta\theta)(1-\alpha)}{\phi[1+\alpha(\varepsilon-1)]}$$

with $\phi = \theta + \omega[1-\theta(1-\beta)]$. We notice that the hybrid model nests the baseline model as a special case when there are no backward-looking firms present (i.e., $\omega = 0$). Accordingly, if the baseline model is true, then $\gamma_f = \beta$, $\gamma_b = 0$ and $\tilde{\lambda} = \lambda$.

Using quarterly data of the growth in the GDP deflator as a measure of inflation and the labour income share as a proxy for real marginal costs over the sample period 1960*Q*1 – 1997*Q*4 and 1970*Q*1 – 1998*Q*2 in the US and European case, respectively, GG and GGL-S estimate both (1) and (2) by means of GMM.[2] Their empirical findings may be summarised as follows: (*i*) forward-looking behaviour is dominant as estimates of γ_f are broadly speaking twice as large as the estimates of γ_b. (*ii*) since the coefficient γ_b is found to be statistically different from zero, the baseline model is rejected by the data. (*iii*) the labour income share drives inflation as suggested by a positive and significant estimate of $\tilde{\lambda}$. In contrast, alternative forcing variables such as output gap measures do not perform well. Consequently, both GG and GGL-S claim that the hybrid version of the NKPC model can be used as a good first approximation to the European and US inflation dynamics.

[2] For comparison, GGL-S also present results for US over the shorter sample period 1970*Q*1 – 1998*Q*2.

3. Econometric issues

The basic idea behind the procedure in this paper is to start out with a well specified VAR and test, using a likelihood ratio test, the implications of the NKPC on the coefficients of the VAR. Hence, we work out the maximum likelihood estimator of the coefficients, with and without the expectation restrictions imposed, in order to construct a likelihood ratio test. We thus start with a well specified two-dimensional VAR of order *k* having the form

(3) $$X_t = A_1 X_{t-1} + \ldots + A_k X_{t-k} + \Phi D_t + \varepsilon_t, t = k+1,\ldots,T,$$

where $X_t = (\pi_t, mc_t)'$, D_t represent deterministic terms (i.e., constants and trends) and $\varepsilon_{k+1},\ldots,\varepsilon_T$ are independent Gaussian variables with expectation zero and (unrestricted) covariance matrix Ω. The initial observations of $X_1,\ldots,X_k$ are kept fixed. The way the likelihood ratio test is constructed in our context depends on whether the time series involved are stationary, i.e., *I*(0), or nonstationary, i.e., *I*(1). The two situations correspond to whether the impact matrix $-\Pi = I - A_1 - \ldots - A_k$ has full or reduced rank. In the following we assume that the rank is full, corresponding to X_t being stationary, but indicate how the procedure can be modified to suit the nonstationary case.

It is essential how the conditional expectations in (1) and (2) are represented. We proceed by a similar procedure to the one adopted by Baillie (1989) and Hansen and Sargent (1991) and spell out the implications of the NKPC for the model (3). Using vector notation the baseline model may be written in compact form as

(4) $$c_1' E_t X_{t+1} + c_0' X_t + c_{-1}' X_{t-1} = 0,$$

where $c_1 = (\beta, 0)'$, $c_0 = (-1, \lambda)'$ and $c_{-1} = (0,0)'$. In the hybrid model $c_1 = (\gamma_f, 0)'$, $c_0 = (-1, \tilde{\lambda})'$ and $c_{-1} = (\gamma_b, 0)'$. We notice that (4) contains restrictions involving the conditional expected value of the observations *one-step-ahead* and the present and lagged observed values. Having specified the information set, the conditional expectation $E_t X_{t+1}$ can be worked out and the restrictions stated explicitly. Expressing (3) at time *t*+1 and taking conditional expectations after pre-multiplication with c_1' lead to

(5) $$c_1' E_t X_{t+1} - c_1' A_1 X_t - \ldots - c_1' A_{k-1} X_{t-k+1} - c_1' \Phi D_t = 0.$$

Hence, equating (4) and (5) means that the following restrictions on the coefficients of the model (3) must be satisfied in the NKPC case:

$$
\begin{aligned}
&-c_1'A_1 = c_0', \\
(6)\qquad &-c_1'A_2 = c_{-1}', \\
&-c_1'A_j = 0,\ j = 3,\dots,k-1 \text{ and } c_1'\Phi = 0.
\end{aligned}
$$

For fixed values of $\psi = (\beta, \lambda)$ or $\psi = (\gamma_f, \gamma_b, \tilde{\lambda})$, these restrictions may be tested by a Wald, Lagrange multiplier or likelihood ratio test. We shall employ the latter, which is particularly useful in our case as it is independent of the specific parameterisation used. This means that the tests are identical no matter the hybrid model is parameterised by $\gamma_f, \gamma_b, \tilde{\lambda}$ or any three of $\omega, \theta, \beta, \tilde{\lambda}$ provided one of them is fixed.

To see how the likelihod ratio test can be carried out, notice that if we pre-multiply (5) by the non-singular matrix $(c_1, c_{1\perp})$, where $c_{1\perp}$ is a matrix with columns orthogonal to the columns in c_1 taking the restrictions (6) into account, the model (3) decomposes into two parts

$$(7)\qquad c_{1\perp}'X_t = c_{1\perp}'A_1X_{t-1} + \dots + c_{1\perp}'A_kX_{t-k} + c_{1\perp}'\Phi D_t + c_{1\perp}'\varepsilon_t$$

$$(8)\qquad c_1'X_t = -c_0'X_{t-1} - c_{-1}'X_{t-2} + c_1'\varepsilon_t.$$

The error terms $c_{1\perp}'\varepsilon_t$ and $c_1'\varepsilon_t$ are correlated, so the two parts cannot be estimated separately. Conditioning on $c_{1\perp}'X_t$ and the past, model (3) can be expressed as the product of a conditional part and a marginal part, where the marginal part is given in (8). From standard results of a multivariate Gaussian distribution it follows that the error terms in the conditional model are $u_t = c_{1\perp}'\varepsilon_t - c_{1\perp}'\Omega(c_1'\Omega c_1)^{-1}c_1'\varepsilon_t$. Thus, the conditional model is expressed as

$$(9)\qquad c_{1\perp}'X_t = \rho(c_1'X_t + c_0'X_{t-1} + c_{-1}'X_{t-2}) + c_{1\perp}'A_1X_{t-1} + \dots + c_{1\perp}'A_kX_{t-k} + c_{1\perp}'\Phi D_t + u_t,$$

where $\rho = c_{1\perp}'\Omega(c_1'\Omega c_1)^{-1}$. In the marginal model there is only an unknown variance. The unknown parameters in the conditional model can be found by regressing $c_{1\perp}'X_t$ on $c_1'X_t, X_{t-1},\dots, X_{t-k}$ and $c_{1\perp}'D_t$. If *SSC* denotes the mean sum of squares from this regression and *SSM* denotes $\frac{1}{T-k}\sum_{t=k+1}^{T}(c_1'X_t + c_0'X_{t-1} + c_{-1}'X_{t-2})^2$, the maximal value of the likelihood when the restrictions in (6) are imposed, apart from a constant, is given by

$$(10)\qquad L_{\max}(\psi)^{-(T-k)/2} = \frac{SSC \times SSM}{\det(c_1'c_1)\det(c_{1\perp}'c_{1\perp})}.$$

For fixed values of ψ the likelihood ratio test of the restrictions in (6) compares the value in (10) with the corresponding value where the restrictions are not imposed. As is well known, the value of -2 log of the likelihood ratio statistic is asymptotically χ^2-distributed, where the number of degrees of freedom is equal to the difference between the number of parameters in the unrestricted and the restricted model. In the case where the parameters ψ are considered as unknown, the expression $L_{\max}(\psi)$ from (10) can be considered as a profile or concentrated likelihood for ψ. The maximum likelihood estimates can be found as the values which maximise the profile likelihood and the profile likelihood can be studied as a function of ψ. This property can be very useful because the parameters in ψ are those of prime interest in the present paper where in particular the coefficient of the forward-looking term in the NKPC is essential for the economic interpretation of the model. A likelihood ratio test can be carried out as earlier. The additional loss in degrees of freedom equals the number of parameters in ψ that are estimated. For parameter vectors ψ of moderate dimension maximising (10) is quite straightforward and may be done using a numerical optimising procedure.

All the arguments above carry over to the nonstationary $I(1)$ case. The additional feature that has to be accounted for is the reduced rank of the impact matrix $-\Pi$. This is most easily done by reparameterising (3) as a VAR in equilibrium correction form where the impact matrix is explicitly involved. A *necessary* condition for the empirical success of the NKPC in the nonstationary case is that inflation must be cointegrated with real marginal costs. We may see this by reformulating the restrictions in (6) to read as follows:

$$
\begin{aligned}
& c_1'\Pi = -(c_0 + \dots + c_{-k+1})', \\
(11) \qquad & c_1'A_2 = -c_{-1}', \\
& c_1'A_j = c_{-j+1},\ j = 3,\dots,k-1 \text{ and } c_1'\Phi = 0,
\end{aligned}
$$

where $c_{-2} = \dots = c_{-k+2} = 0$. Since the impact matrix $-\Pi$ has reduced rank, the first part of (11) entails restrictions on the adjustment parameters as well as the parameters describing the cointegration space. Hence, if $c_1'\Pi = d' = -(c_1 + \dots + c_{-k+1})'$ is satisfied, then d' belongs to the cointegration space. Since (11) contains several additional restrictions, it is apparent that d' belonging to the cointegration space is only a necessary and not a sufficient condition for the NKPC to hold empirically. For the baseline and hybrid model we have that $d = (1-\beta, -\lambda)$ and $d = (1-\gamma_f - \gamma_b, -\tilde{\lambda})$, respectively. We shall employ the testing procedure suggested by Johansen and Swensen (1999, 2004) in the nonstationary case.[3]

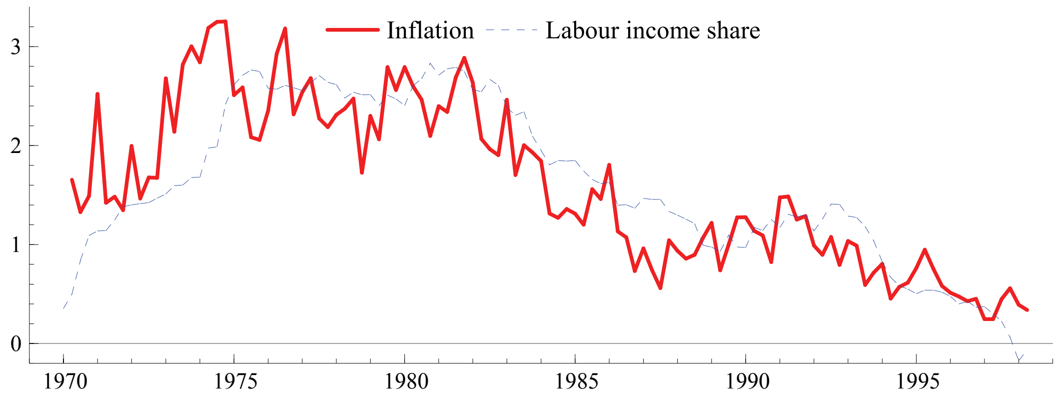
[3] Juselius (2006) uses the same testing procedure to evaluate the NKPC with European and US data. However, Juselius (2006) considers an extended information set that permits testing of the forward-looking IS curve and the NKPC jointly and investigates a different sample period than that of GG and GGL-S. Hence, our study is not directly comparable to Juselius (2006) in this respect. Also, unlike Juselius (2006), we pay particular attention to the possible problem of weak identification by studying the likelihood surface of the estimated models. The present study is also somewhat related to that of Fanelli (2006), which employs a three step method to test the NKPC within a cointegrated VAR using the same data set as in GGL-S. However, the method used in Fanelli (2006) is based on the assumption of strong exogeneity of forcing variables involved in the statistical inference. The

It is clear from the above discussion, that prior information about the cointegration rank is useful to evaluate the empirical performance of the NKPC either by standard likelihood ratio tests for stationary VAR models or for VAR models with $I(1)$ processes as described in Johansen and Swensen (1999, 2004).

4. European inflation dynamics

In this section, we reevaluate the baseline model and the hybrid model of GGL-S on their European data using the likelihood based methods described above. The data are quarterly time series covering the sample period 1970Q1 – 1998Q2. Inflation is measured as the quarterly percentage change in the GDP deflator, whereas real marginal costs are proxied by labour income share constructed as the ratio of compensation to employees to nominal GDP.[4] Figure 1 displays the measures of inflation and real marginal costs.[5]

Figure 1. The European data



It is evident that the two time series move quite closely together during both the high and low inflation periods. Importantly, both inflation and the labour income share exhibit a clear downward trend with no apparent mean reverting property, at least from the mid 1970s, suggesting that π_t and mc_t are nonstationary $I(1)$ series. Therefore, a reduced rank VAR is a candidate as an empirical model. We pursue this hypothesis by fitting the VAR model (3) to the data with an unrestricted constant to reflect the trending behaviour in the level of the series. Initial modelling suggests that $k = 5$ is the appropriate choice of lag length to arrive at a well-specified model in terms of well-

procedure suggested by Johansen and Swensen (1999, 2004) is also applied in an empirical investigation of the NKPC and the forward-looking behaviour of exporters for a small open economy, see Boug *et al.* (2006a, 2006b).

[4] We refer to GGL-S for further details about the data and the construction of variables.

[5] In Figure 1, the scale of the labour income share is adjusted to match that of inflation.

behaved residuals, as indicated by standard diagnostic tests.[6] Then, we apply the Johansen (1995, p. 167) rank test to the model.[7] The test results are reported in Table 1.

Table 1. Johansen's cointegration tests

r	λ_i	λ_{trace}	λ^{a}_{trace}
$r=0$	0.261	33.07 [0.000]**	30.01 [0.000]**
$r \leq 1$	0.003	0.40 [0.528]	0.36 [0.548]

Notes: r denotes the cointegration rank and λ_i are the eigenvalues from the reduced rank regression, see Johansen (1995). The λ_{trace} and λ^{a}_{trace} statistics are the trace statistics without and with degrees-of-freedom-adjustments, respectively. The p-values in square brackets, which are reported in PcGive, are based on the approximations to the asymptotic distributions derived by Doornik (1998). The asterisk ** denotes rejection of the null hypothesis at the 1 per cent significance level.

We observe that the rank should be set to one, indicating existence of one cointegration relationship between inflation and real marginal costs. Also, likelihood ratio tests clearly reject the hypothesis that inflation and real marginal costs are stationary or long run excludable with rank equal to unity. Accordingly, the necessary condition of cointegration for the empirical success of the NKPC model is met in the non-stationary case of the European data. However, it remains to test formally the exact rational expectations restrictions (11) entailed by the NKPC in a cointegrated VAR by means of the procedure suggested by Johansen and Swensen (1999, 2004) so as to judge whether the inflation dynamics indeed is in line with the model.

We first consider taking the restrictions from the baseline model (1) into account. Evaluating the profile likelihood and using a numerical optimising procedure, the maximal value of twice the log likelihood in this case is estimated to 379.25, whereas the corresponding value of the reduced rank VAR without the restrictions imposed is estimated to 429.34.[8] The likelihood ratio statistics is thus calculated to 50.09 with a corresponding p-value of zero given nine degrees of freedom under the null hypothesis that the baseline model is the true model. The model is accordingly strongly rejected. The estimated values of the parameters corresponding to the maximal value of the likelihood are $\beta = 1.06$ and $\lambda = 0.002$. Figure 2 displays the surface of the concentrated log likelihood for the baseline model.[9]

Since the NKPC hypothesis is so clearly rejected it is not reasonable to use the surface for inference, for instance by constructing confidence sets. But it may shed some light over a particular feature discussed in the literature, namely the possibility of weak identification of NKPC models, cf. Mavroeidis (2004, 2005) among others. When estimating the NKPC by GMM, which is often the case, it has been noted that the results are rather sensible to the choice of instruments. Consequently, it is likely that the estimates of the NKPC model are not well identified. As seen from Figure 2, the surface

[6] Noticeably, the instrument set used in GGL-S includes among other variables five lags of inflation. Results from the diagnostic tests, here and below, are available from the authors upon request.

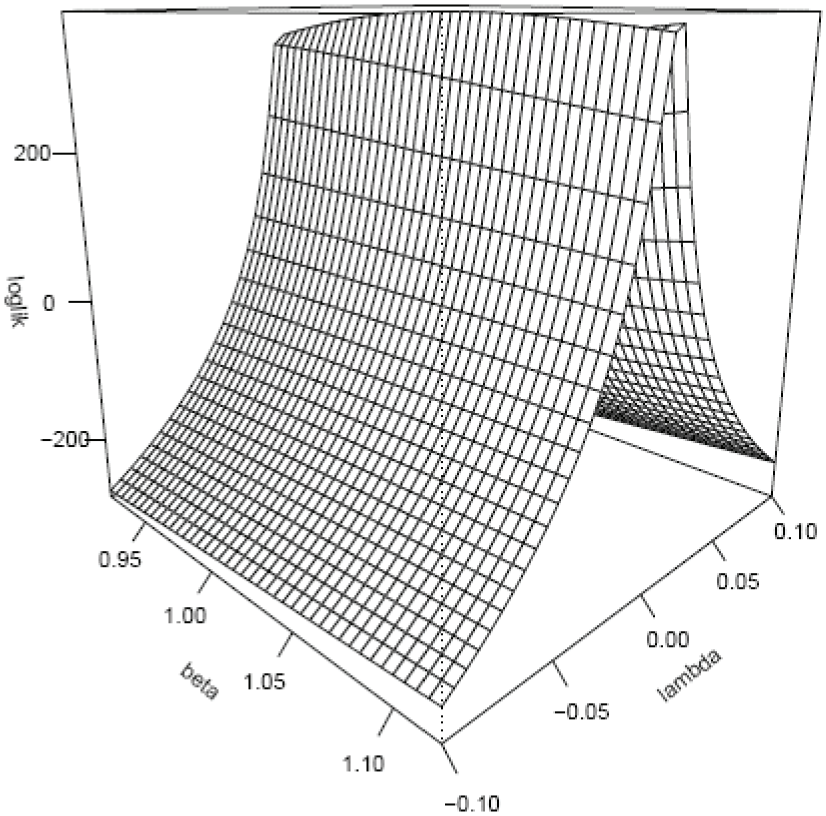
[7] The rank test is performed using PcGive 10.3, see Hendry and Doornik (2001) and Doornik and Hendry (2001).

[8] A program in R, see http://www.r-project.org/, was written in order to evaluate the profile likelihood for fixed values of ψ. The R-procedure `nlm` was used for the numerical optimisation with respect to ψ. The R code used in the testing of the NKPC can be found on http:/www.folk.uio.no/swensen/nkpc.html.

[9] Contour plots corresponding to Figure 2 and the other figures presented below are provided in the Appendix.

is characterized by a distinct and rather flat ridge where the combination of the values of β and λ producing the largest value of the likelihood are situated. Hence, it is difficult to distinguish empirically between different parameter values particularly on or close to the ridge, and it follows that the estimators of β and λ are highly correlated.

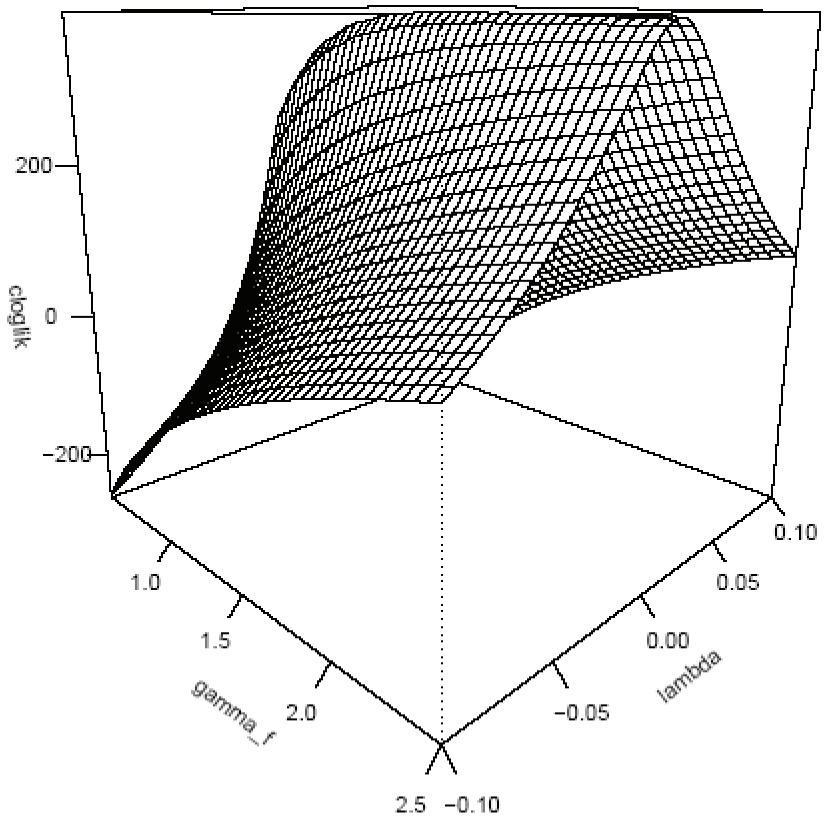
Figure 2. Concentrated log likelihood for the baseline model. Surface plot



Introducing lagged inflation, as in the hybrid model, yields a maximal value of twice the log likelihood of 390.28. The likelihood ratio statistic is in this case calculated to 39.06 with a p-value of zero. Hence, the hybrid model is also clearly rejected even if there is one degree of freedom less due to the fact that the coefficient of lagged inflation is estimated. The values of the parameters corresponding to the maximal value of the likelihood is now estimated to γ_f = 1.50, γ_b = –0.45 and $\tilde{\lambda}$ = 0.002. Figure 3 displays the surface of the concentrated log likelihood for the hybrid model. Again a sharp ridge appears so that the model does not seem to be well identified.

The clear meassage from these findings is that the evidence in GGL-S that the NKPC model with a dominant role for forward-looking behaviour does reasonably well in describing European data must be considered fragile. Our results are in accordance with both Fanelli (2006) and Juselius (2006). An important lesson of the findings here is that although the necessary condition of cointegration between inflation and real marginal costs seems to hold, the overall restrictions implied by the NKPC model are rejected by the data according to likelihood ratio based inference. GGL-S, on the other hand, use a less formal method by just looking at the significance status of the inflation and real marginal costs terms to claim the empirical success of the model. We agree with the point made by Juselius (2006) that GGL-S in essence only estimate the cointegration relationship between inflation and real marginal costs without really testing the rational expectations restrictions implied by the NKPC model as such.

Figure 3. Concentrated log likelihood for the hybrid model. Surface plot



5. US inflation dynamics

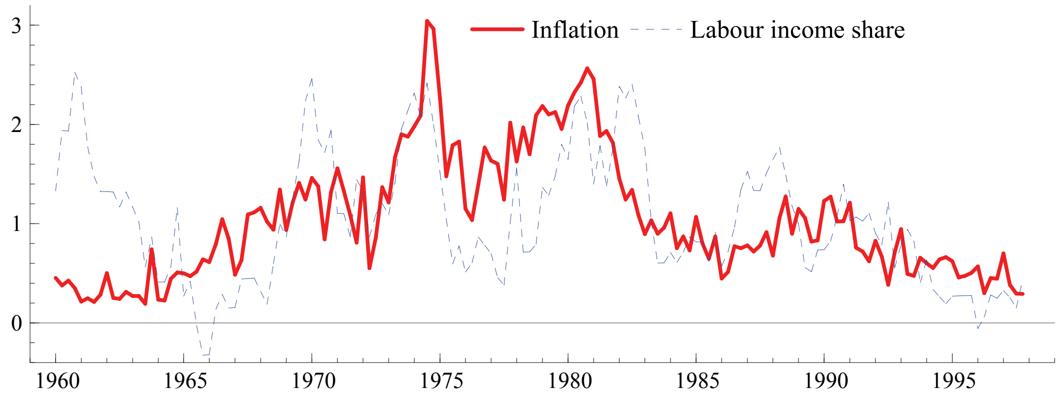
We now present estimates of (1) and (2) and evaluate their empirical performance by likelihood based methods applying the same US data set as in GG. As in the previous section, inflation is measured as the quarterly percentage change in the GDP deflator, whereas real marginal costs are proxied by labour income share. The data covers the sample period 1960Q1 – 1997Q4.[10] Figure 4 displays the measures of inflation and real marginal costs in the case of US data.[11]

As in the European case, there is a close movement between inflation and real marginal costs. However, contrary to the nonstationarity status of the European data, both US inflation and real marginal costs fluctuate around their respective means with no

[10] We refer to GG for further details about the data and the construction of variables.
[11] In Figure 4, the scale of the labour income share is adjusted to match that of inflation.

apparent trending behaviour. One may then suspect, as will be verified formally below, that the US series are better described by a stationary VAR than a nonstationary one.

Figure 4. The US data



Specifying an unrestricted VAR in $X_t = (\pi_t, mc_t)$, we find that $k = 4$ produces a model with no serious misspecification.[12] Indeed, the equation for inflation suffers from non-normal residuals, which is due to some large outliers in the mid 1970s. These outliers may be mopped up by impulse dummies, but doing so does not alter the results from the cointegration analysis, which we now turn to. Table 2 reports the findings from applying the Johansen (1995, p. 167) rank test to the US data based on the unrestricted VAR of order four.

Table 2. Johansen's cointegration tests

r	λ_i	λ_{trace}	λ^{a}_{trace}
$r=0$	0.084	16.87 [0.029]*	15.98 [0.041]*
$r \leq 1$	0.023	3.52 [0.061]	3.34 [0.068]

Notes: r denotes the cointegration rank and λ_i are the eigenvalues from the reduced rank regression, see Johansen (1995). The λ_{trace} and λ^{a}_{trace} statistics are the trace statistics without and with degrees-of-freedom-adjustments, respectively. The p-values in square brackets, which are reported in PcGive, are based on the approximations to the asymptotic distributions derived by Doornik (1998). The asterisk * denotes rejection of the null hypothesis at the 5 per cent significance level.

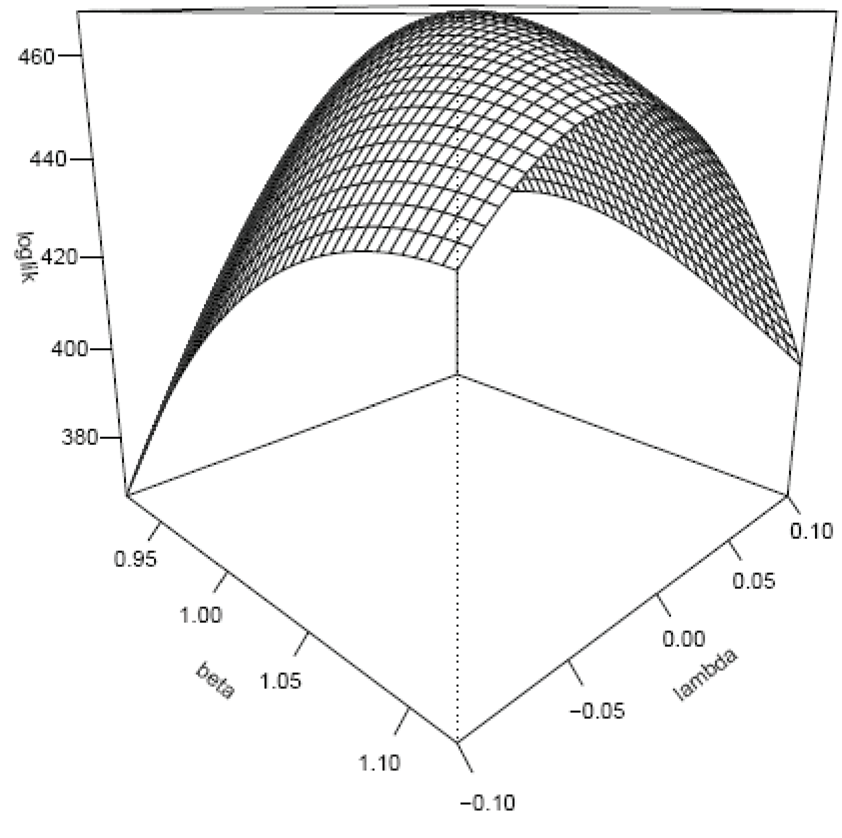
We notice that the hypothesis of no cointegration can be rejected at the 5 per cent significance level, while the hypothesis of at most one cointegrating relationship between inflation and real marginal costs can be rejected at the 6 per cent level. Therefore, we conclude that the impact matrix has full rank so that a stationary VAR in levels seems to fit the US data reasonably well.

Testing the restrictions in (6) by likelihood ratio tests provides a quite similar picture as in the European case when it comes to the empirical failure of the NKPC model. The

[12] The chosen model specification is identical to the one in Table 6 in Mavroeidis (2006). GG also include four lags of inflation in the instrument set underlying the GMM estimation of the NKPC model.

maximal value of twice the log likelihood is estimated to 468.21 in the baseline case under the assumption of a stationary VAR in levels. Compared to the value 495.02, which is the maximal value of twice the log likelihood of the full rank VAR without the rational expectations restrictions imposed, yields a likelihood ratio statistic of 26.81 and a p-value of 0.0004 with seven degrees of freedom. Again, there is overwhelming empirical evidence against the baseline model. The estimates of the parameters corresponding to the maximal value of the likelihood are $\beta = 1.03$ and $\lambda = 0.0003$, which are very close in magnitude to those obtained by the European data. Also for the US data we provide plots of the surface of the concentrated log likelihood. Figure 5 clearly shows that the surface now looks much more quadratic, indicating that the baseline model is less subject to weak identification than what is the case with European data.

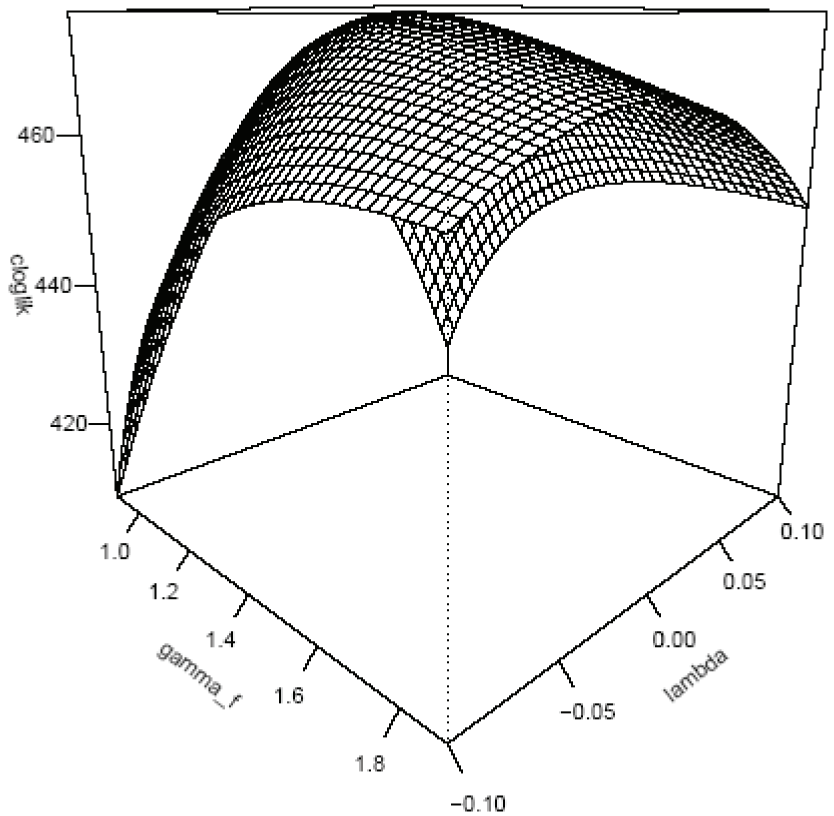
Figure 5. Concentrated log likelihood for the baseline model. Surface plot



Results from testing the hybrid model on US data do not alter the conclusion about the empirical failure of the NKPC. Albeit, the maximal value of the log likelihood of the full rank VAR with rational expectations restrictions imposed is increased somewhat from 468.21 to 475.26, the NKPC is still strongly rejected with a likelihood ratio test statistic of 19.76 and a corresponding p-value of 0.003 (with the loss of one degree of freedom). The values of the parameters corresponding to the maximal value of the likelihood is now estimated to $\gamma_f = 1.31$, $\gamma_b = -0.28$ and $\widetilde{\lambda} = -0.004$, all of which are close to the analogous estimated parameters in the European case.

Figure 6 displays the surface plot of the concentrated log likelihood of the hybrid model. Once more a quadratic pattern becomes visible and the problem of weak identification of the model does not seem to be that pressing as in the European case. Based on the empirical findings in this section, we may again argue that the evidence claimed by GG and GGL-S is not very convincing from a statistical point of view. We provide some discussion about the discrepancy between our results and those of GG and GGL-S and point out one potential reason for the empirical failure of the NKPC model in the next section.

Figure 6. Concentrated log likelihood for the hybrid model. Surface plot



6. Conclusions

We have in this paper re-examined the evidence in GG and GGL-S that both European and US inflation dynamics are in line with the NKPC model. To this end, we have used the same data set and likelihood based methods for testing (exact) rational expectations in VAR models rather than the GMM single equation framework. As in GG and GGL-S we find that the inclusion of a backward-looking term improves the empirical fit of the NKPC. However, the increase in the value of the likelihood is not large enough to yield a well- specified model. We also show that the likelihood surface is rather flat, especially in the European case, indicating that the NKPC model may be weakly identified as claimed by others using alternative methods.

As stated by Galí *et al.* (2005) it is essential for likelihood based methods to work that the overall structure of the underlying model is correctly specified. Generally, VAR models by themselves are based on few assumptions on the data generating process. However, it may be a challenge to find an appropriate specification in the empirical situation at hand. For systems with many variables such a task may be quite demanding or in fact impossible due to the corresponding large number of parameters involved. This problem is less pressing in the present context since the VAR models involve only two variables. It is therefore important to consider what may be the reason for the clear rejection of the NKPC model in both the European and the US case.

As emphasised in the text, the first part of the restrictions in the reduced rank VAR situation involves the long-run parameters and the cointegration relationship between inflation and real marginal costs. These features are accounted for at the outset by specifying the VAR as either $I(0)$ or $I(1)$. Since we find a well-specified cointegration relationship between inflation and real marginal costs in the European case, the long run part of the theory is supported by the data. The rejection of the NKPC model is therefore mainly due to the restrictions imposed on the short run parameters. A reasonable interpretation is thus that the hybrid model with only one lag does not properly reflect the rather complex dynamic structure of US and European inflation as indicated by the well-specified VAR models of order four and five, respectively. It is interesting that GGL-S begin their paper by showing that the traditional Phillips curve (with no forward-looking behaviour) with four and five lags of inflation does a reasonable good job of characterising post-war inflation in the US and the Euro area. An adjacent explanation worth pursuing in the context of the hybrid model is that inflation is highly correlated with its own past values such that inflation is highly persistent.[13]

Indeed, since the value of the likelihood increases when allowing for lagged inflation, expanding the NKPC further in this direction may be helpful to make the model perform better in terms of explaining European and US inflation. A possible method for investigating such a hypothesis is to introduce the rational expectations restrictions entailed by the model sequentially. Doing so has, however, been beyond the scope of the present paper.

[13] Several authors have found evidence of inflation persistence in the sense of positive serial correlation in inflation, see e.g. Fuhrer and Moore (1995), Fuhrer 1997) and Taylor (2000) for US data and Batini (2006) for European data.

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Appendix

Figure A1. Europan data. Concentrated log likelihood for the baseline model. Contour plot

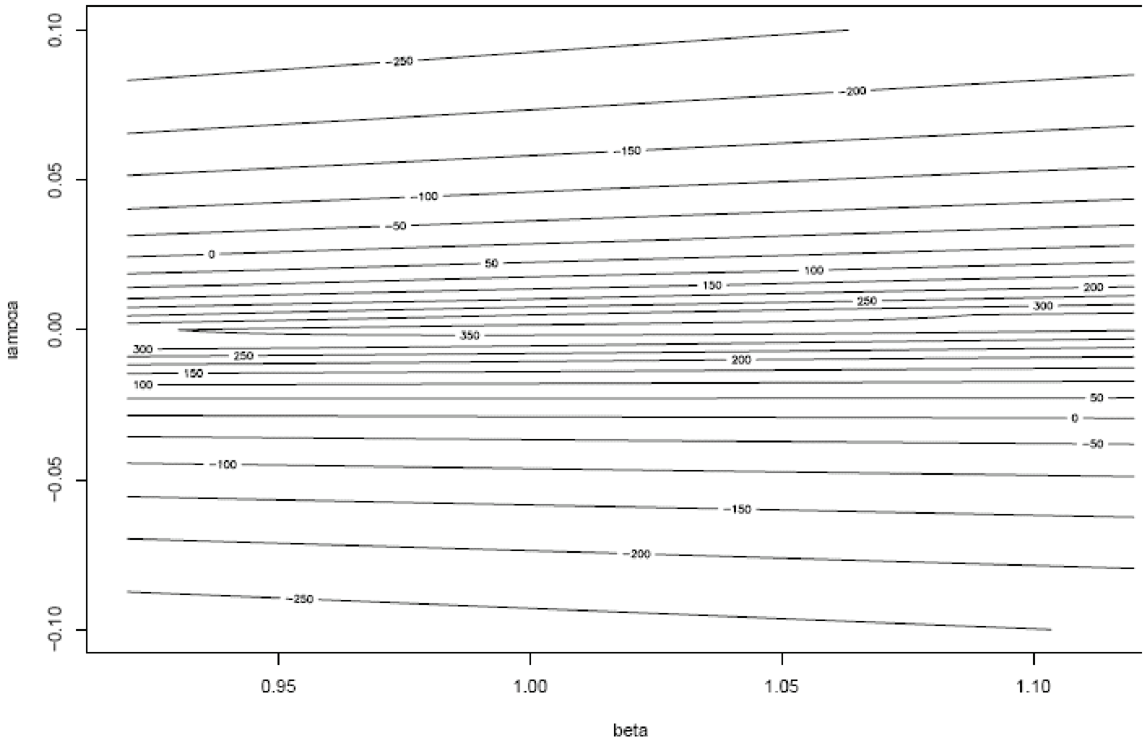

Figure A2. European data. Concentrated log likelihood for the hybrid model. Contour plot

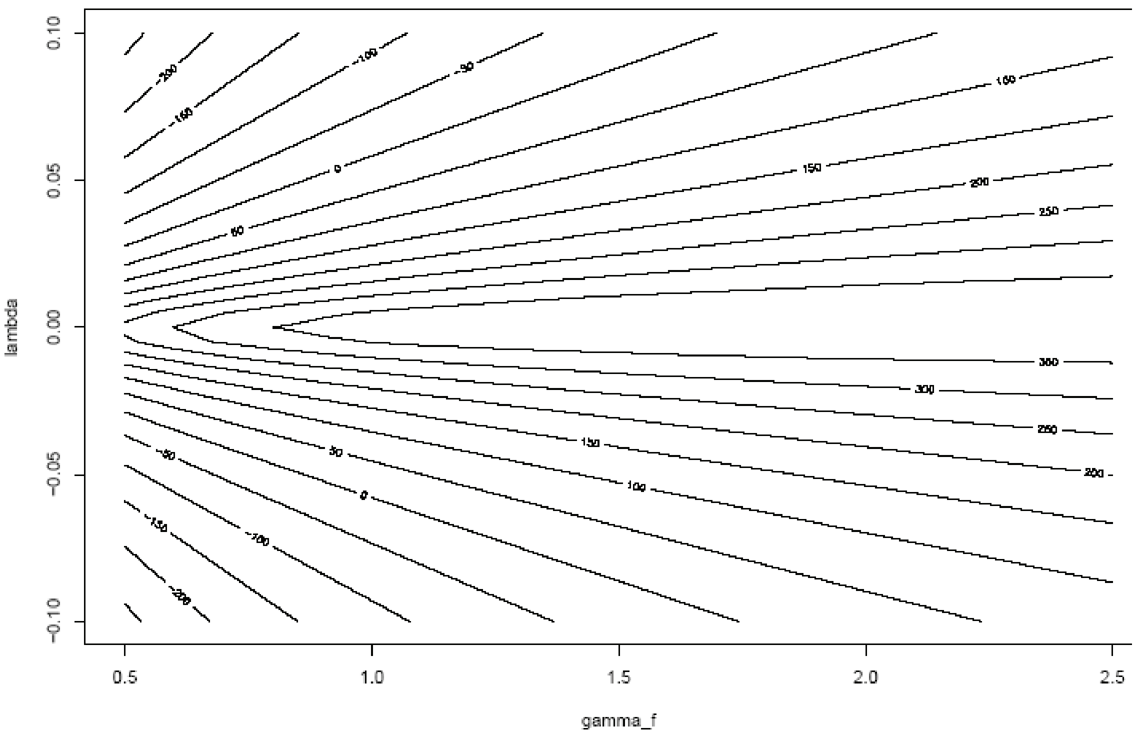

Figure A3. US data. Concentrated log likelihood for the baseline model. Contour plot

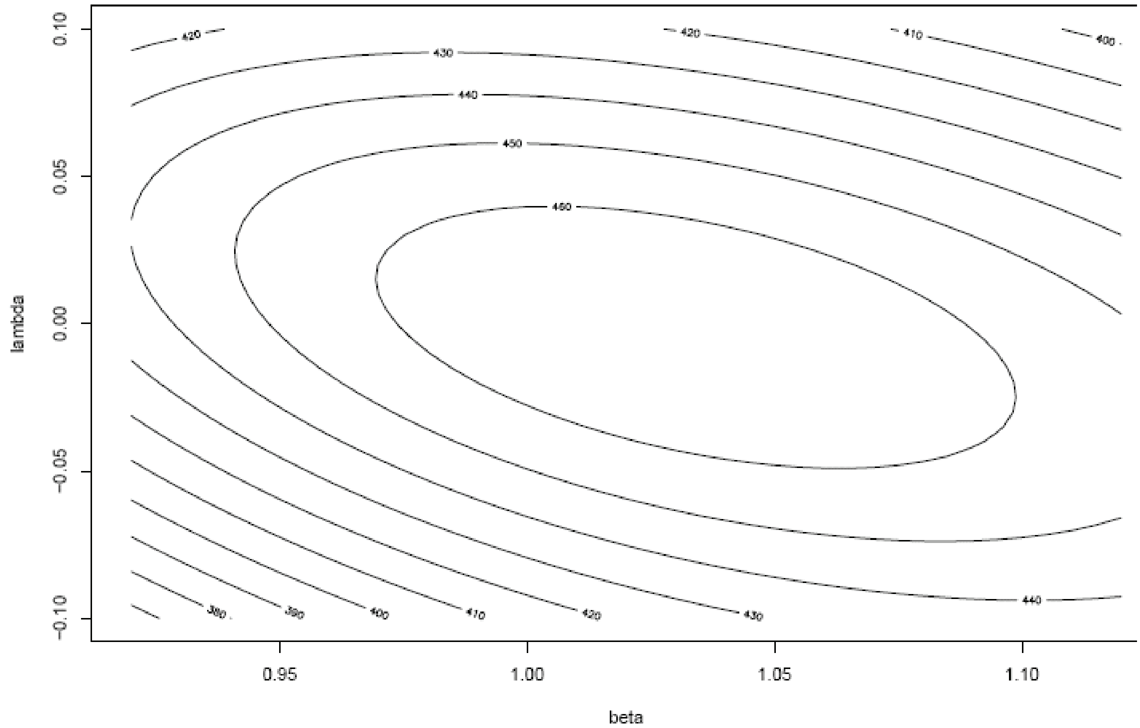

Figure A4. US data. Concentrated log likelihood for the hybrid model. Contour plot