

*Rolf Aaberge and Magne Mogstad*

## **On the Definition and Measurement of Chronic Poverty**

**Abstract:**

As an alternative to the conventional methods for measuring chronic poverty, this paper proposes an interpersonal comparable measure of permanent income as a basis for defining and measuring chronic poverty. This approach accounts for the fact that individuals regularly undertake inter-period income transfers. Moreover, the approach allows for individual-specific interest rates on borrowing and saving as well as for the presence of liquidity constraints. Due to the general nature the proposed method proves useful for evaluating the theoretical basis of the standard methods for measuring chronic poverty.

**Keywords:** Intertemporal choice, liquidity constraints, permanent income, chronic poverty.

**JEL classification:** D71, D91, I32.

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**Address:** Rolf Aaberge, Statistics Norway, Research Department. E-mail: [rolf.aaberge@ssb.no](mailto:rolf.aaberge@ssb.no)

Magne Mogstad, Statistics Norway, Research Department.  
E-mail: [magne.mogstad@ssb.no](mailto:magne.mogstad@ssb.no)

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# 1. Introduction

Over the last decades, increasing discontent has been expressed with poverty snapshots based on observations of income for a single year. The reason behind this discontent is that annual incomes fluctuate due to transitory shocks and institutional factors such as the accounting and tax rules. This variation, however, may not fully reflect changes in the economic resources available for consumption and economic welfare. As a consequence, poverty scholars have worked on broadening the perspective of distributional analysis to study the chronically poor, i.e. those with persistent inability to pursue welfare due to lack of economic means. A typically applied method for analysing chronic poverty, the so-called spell-based approach pioneered by Bane and Ellwood (1986), distinguishes the chronically poor from the transient poor according to the time span the individuals have endured low annual income. An underlying assumption for the validity of this approach is that income is perfectly transferable within the year that it is earned, but not transferable between years. This assumption is, however, in conflict with empirical evidence, which shows that households undertake intertemporal income transfers to smooth consumption.<sup>1</sup> Thus, the spell-based approach pays excessive attention to the time-patterns of income, but is rather insensitive to the magnitude of the actual income streams. Indeed, a person who solely experiences annual income above the poverty threshold for a single year will not be defined as chronically poor, although his income stream may entail less consumption potential than the income stream of a chronically poor who has annual income below the poverty threshold in each year. This method also suffers from the additional deficiency that, by construction, it is unable to reflect the depth of poverty as well as the inequality in the distribution of income among the poor.

As a response to the excessive sensitivity of spell-based estimates of chronic poverty to fluctuations in annual incomes, alternative methods for measuring chronic poverty based on an extended accounting period of income are called for. One such method for measuring chronic poverty simply adds up real annual incomes over a set of consecutive years. Accordingly, the chronically poor are identified as those with average real income below the average poverty threshold, where incomes from different years are made comparable by accounting for price changes.<sup>2</sup> Making incomes from different periods comparable is, however, not merely a question of accounting for changes in the price of goods; it is

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<sup>1</sup> See Browning and Crossley (2001) for a survey of income and consumption smoothing among households.

<sup>2</sup> Duncan and Rodgers (1991) and Aaberge et al. (2000) demonstrate that poverty estimates are rather sensitive to whether chronic poverty estimates rely on information about the time span individuals have endured low annual income or are based on the average real income over the given sequence of years.

also necessarily to take the price of consumption into account. The price of consumption depends on the real interest rates, which determine how much future consumption an individual must give up for being able to consume more today. By exclusively adjusting for changes in the prices of goods when aggregating income over time, only part of the potential costs and/or benefits of receiving income at different times is considered.

As pointed out by Rodgers and Rodgers (1993), the concept of permanent income emerging from the theory of intertemporal choice may form a more relevant basis for analysing chronic poverty. Specifically, they propose a permanent income measure defined as the maximum sustainable annual consumption level an individual can achieve for a given real income stream. An underlying assumption of the maximum sustainable consumption method (MSC) is that carrying out equalizing income transfers at the prevailing interest rates to achieve exactly constant consumption level over time may be preferable. For it to be optimal to have constant consumption levels over time, it is, however, necessary to rely on rather restrictive intertemporal preferences. Thus, it may seem that the MSC approach lacks the strong theoretical basis of intertemporal optimisation that it claims to have. While the standard theory of intertemporal choice assumes that individuals can, if they prefer, make consumption-equalising income transfers, the MSC approach insists that they strictly prefer to make such transfers. Indeed, it is important to emphasize that within the intertemporal choice framework, inter-period income transfers are carried out to ensure that the marginal utility of consumption is constant over time, which often will result in consumption levels that differ between time periods. To abandon the assumption of constant consumption levels over time we introduce a more flexible representation of permanent income that is compatible with a more general preference structure and, moreover, can be considered as a money-metric measure of the welfare level associated with the income stream for a given individual. Specifically, this measure of permanent income, denoted the equally-allocated equivalent income (EAEI), is defined as *the minimum annual income the individual would need in order to achieve the same welfare level as he could by undertaking inter-period income transfers subject to his budget constraint.*

This paper is organised as follows. Section 2 provides a brief discussion of the importance of drawing on the theory of intertemporal choice as a basis for defining and measuring chronic poverty. Furthermore, we introduce and derive an expression for the permanent income measure EAEI. Section 3 uses the EAEI framework for evaluating and interpreting the theoretical basis of some standard approaches for measuring chronic poverty. Although the justification of the MSC approach relies on the assumption of constant annual consumption levels over time, we prove that the MSC-related

measure of permanent income can be used as a measure of the welfare level associated with a given income stream even in situations where the optimal consumption levels vary over time. Thus, the MSC permanent income forms an appropriate basis for measuring chronic poverty in a broader context than that suggested by Rodgers and Rodgers (1993). Moreover, it is demonstrated that the EAEI method encompasses the MSC approach as well as the method using average real income as the basis for measuring chronic poverty. However, as will be demonstrated in Section 4, it is solely the EAEI approach that accounts for liquidity constraints as well as for the effect of individual-specific interest rates on savings and borrowing. If chronic poverty is essentially about identifying those with persistent inability to pursue welfare due to lack of economic means, it is important to use methods that are capable of capturing the fact that some individuals face constraints as borrowers in the credit market. In Section 5, we illustrate the sensitivity of poverty estimates to the choice between the EAEI method and the conventional methods for measuring chronic poverty. Concluding remarks are provided in Section 6.

## **2. Permanent income and chronic poverty**

*"The identification of low measured income with 'poor' and high measured income with 'rich' is justified only if measured income can be regarded as an estimate of expected income over a lifetime or a large fraction thereof" (Friedman, 1957, p38).*

Acknowledging that income is a good that does not have an intrinsic value, but is important merely as an instrument for individuals to pursue economic welfare, it follows that defining a person with relatively low income as poor requires that measured income can be regarded as an empirically sound estimate of the economic resources available to achieve welfare. On the basis of the theory of intertemporal choice, one may argue that studies based on annual income provide a misleading picture of the consumption possibilities of individuals and, consequently, also the extent of poverty in a society. The underlying behavioural assumption in the mainstream theory of intertemporal choice is that individuals optimise their choices intertemporally given their budget constraints, by undertaking inter-period income transfers if it is to their advantage. Thus, consumption today is a function of expected income over the lifetime or at least a large fraction thereof. Accordingly, annual incomes, which may heavily fluctuate due to transitory shocks, do not provide a meaningful basis for identifying those with insufficient economic resources to achieve an acceptable welfare level. Furthermore, individuals may themselves behave such that annual incomes vary substantially over time without reflecting changes in economic resources available for consumption. In particular,

institutional factors such as the accounting and tax rules for income from self-employment and financial assets may have strong impact on the assessment of annual income.<sup>3</sup>

In this regard, it is of interest that most individuals in the OECD-countries actually carry out intertemporal income transfers, largely by means of education loans, housing mortgage and pension schemes.<sup>4</sup> This should lead to greater prevalence of transitory components in current income relative to current expenditures, and in consequence greater income than expenditure inequality. As a matter of fact, the distribution of expenditure has repeatedly been demonstrated to be less unequal than the distribution of income.<sup>5</sup> Indeed, consumer expenditure analyses show that individuals that are typically defined as poor based on income data for a single year, as the main rule, spend more than their income, which may be interpreted as evidence for borrowing and saving behaviour (Slesnick, 1993, Mayer and Jencks, 1989).<sup>6</sup> Moreover, survey information suggests that a large proportion of individuals with low annual income perceive that they are able to borrow (Mayer and Jencks, 1989). One reason is that numerous behavioural and institutional responses seem to fill in the holes left by market imperfections in the insurance and credit market facing households with temporarily low income (Murdoch, 1995). In particular, individuals appear to utilise "internal capital markets" to smooth consumption by adjusting the purchase of durables to match economic fluctuations.<sup>7</sup> Arguably, methods used to analyse chronic poverty should reflect that income can be and regularly is used for consumption not only within the year that it is obtained, but also between years.

As a reasonable way of reducing the measurement problem of fluctuating annual income, and to obtain a reliable estimate of the economic resources available for consumption and saving (i.e. future consumption), the accounting period of income could be extended from one to several years. When extending the accounting period of income, and consequently incorporating that individuals subject to their budget constraints are capable of undertaking inter-period income transfers, it is for the sake of aggregation necessary to make incomes from different years comparable. To this end, it would seem

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<sup>3</sup> See Fjærli and Aaberge (2000) and Saez and Chetty (2003) who provide empirical evidence of tax-dependent income-reporting behaviour.

<sup>4</sup> See Borsch-Supan (2003) for a cross-country study of life-cycle savings.

<sup>5</sup> Studies from OECD-countries that have found income inequality to be greater than expenditure based inequality include Cutler and Katz (1992) and Johnson and Shipp (1997) for the United States, and Barret, Crossley and Worswick (2000) for Australia.

<sup>6</sup> Alternatively, this may be because low-income households systematically underreport their incomes relative to high-income households.

<sup>7</sup> In fact, empirical analyses of the expenditure pattern on durables and non-durables suggest that unemployed and others that are likely to be liquidity constrained use the purchase of durables as a tool to smooth marginal utility (Gruber and Dynarski, 1997, Browning and Crossley, 1999). This finding breaks the standard relationship between expenditure smoothing and marginal utility smoothing.

attractive to draw on the economic theory of intertemporal choice. For expositional reasons, we will firstly introduce a method based on the assumption of a perfect credit market and with a rather general form of the intertemporal utility function.

## 2.1 Equally-allocated equivalent income

A method for measuring chronic poverty on the basis of income requires normative judgements concerning the measurability and comparability of income streams across heterogeneous individuals. The common practice of using the average real income as a measure of permanent income implicitly relies on rather restrictive conditions for the intertemporal preference structure and the credit market. Thus, it appears attractive to introduce an alternative measure of permanent income which reflects the basic structure of intertemporal choice theory. To achieve interpersonal comparability a common intertemporal utility function will form the basis of computing the permanent income for every individual. The justification for this choice is that the definition and measurement of permanent income will be considered as an integral part of the social choice framework required for analysing poverty. Rather than claiming that the introduced model of intertemporal choice is a descriptively accurate representation of the behaviour of heterogeneous individuals, we justify it primarily as a normative standard for social evaluation determined by the social planner. Specifically, the definition of permanent income that we have in mind is defined as

*the minimum annual income the individual would need in order to achieve the same welfare level as he could by undertaking inter-period income transfers subject to the budget constraint.*

To provide a formal counterpart of this definition, we rely on the conventional discounted utility model, where preferences are assumed to be intertemporally separable and additive.<sup>8</sup> The instantaneous common utility function  $u$  is assumed to be stationary, increasing, concave, and differentiable. Furthermore, we assume that the rate of time preference  $\delta$  is non-negative and constant over time. Let  $(C_1, C_2, \dots, C_T)$  and  $(y_1, y_2, \dots, y_T)$  be the vectors of consumption levels and exogenous real disposable incomes net of interests of the individual for periods  $t = 1, 2, \dots, T$  (after accounting for scale

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<sup>8</sup> See Koopmans (1960) for an attempt to axiomatically justify the discounted utility model in general, and Kahneman et al. (1997) for an axiomatic rationalisation of the assumption of additive separability in instantaneous utility. Over the last decades, important underlying assumptions of the discounted utility model have been questioned and found to be descriptively invalid in at least some situations (for a survey see Frederick et al., 2002). In particular, it is repeatedly claimed that the choice of exponential discounting resulting in a constant rate of time preferences lacks empirical support. Although it would complicate the calculations, it is possible to modify the method for measuring chronic poverty proposed in this paper by permitting, for example, declining discount rates or hyperbolic discounting. Moreover, the discounted utility model can straightforwardly be extended to allow for uncertainty (see e.g. Ljungquist and Sargent, 2004).

economies in consumption by the use of an equivalence scale). Under the assumption of a perfect credit market, the real interest rates on savings and borrowing are equal and invariant across the population, although they may vary over time. Let  $(r_2, r_3, \dots, r_T)$  be the vector of real interest rates for periods  $t = 2, \dots, T$ . The individual's optimal consumption profile  $(C_1^*, C_2^*, \dots, C_T^*)$  is defined as the solution of

$$(2.1) \quad \max_{C_1, \dots, C_T} \sum_{t=1}^T u(C_t)(1 + \delta)^{1-t}$$

subject to the budget constraint

$$(2.2) \quad \sum_{t=1}^{T-1} C_t \prod_{j=1+t}^T (1 + r_j) + C_T = Y,$$

where  $Y$  is defined by<sup>9</sup>

$$(2.3) \quad Y = \sum_{t=1}^{T-1} y_t \prod_{j=1+t}^T (1 + r_j) + y_T.$$

As is well known, the optimal consumption level  $C_t^*$  in period  $t$  can be expressed as a function of the optimal consumption level in period 1

$$(2.4) \quad u'(C_t^*) = \frac{(1 + \delta)^{t-1}}{\prod_{j=2}^t (1 + r_j)} u'(C_1^*), \quad t = 2, 3, \dots, T,$$

where  $u'$  is the derivative of  $u$ .

From (2.4) and (2.2),  $C_t^*$  can be expressed as a function, say  $f_t$ , of  $Y, \delta$  and  $r_2, r_3, \dots, r_T$ , i.e.

$$(2.5) \quad C_t^* = f_t(Y, \delta, r_2, r_3, \dots, r_T) \text{ for all } t = 1, 2, \dots, T.$$

By inserting for (2.5) in (2.1) the maximum utility level ( $\hat{U}$ ) is given by

$$(2.6) \quad \hat{U} \equiv \sum_{t=1}^T u(C_t^*)(1 + \delta)^{1-t}.$$

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<sup>9</sup> It is straightforward to extend the budget constraint to account for wealth, e.g. by assuming that the income in the first period  $y_1$  in (2.3) includes the initial stock of wealth.



The individual-specific optimal consumption profiles derived from a common utility function subject to the budget constraints may form the basis for defining and measuring interpersonal comparable permanent income. Accordingly, the minimum annual income ( $Z$ ) that is required to obtain the maximum utility level  $\hat{U}$  emerges as an appropriate measure of interpersonal comparable permanent income. Thus, replacing the optimal  $C_t^*$  with  $Z$  for every  $t$  in the right-hand side of equation (2.6) yields

$$(2.7) \quad Z = u^{-1}(\Delta^{-1}\hat{U}),$$

where  $u^{-1}(t) = \inf \{x : u(x) \geq t\}$  is the left inverse of  $u$  and  $\Delta$  is defined by

$$(2.8) \quad \Delta = \sum_{t=1}^T (1+\delta)^{1-t} = \frac{1+\delta}{\delta} (1 - (1+\delta)^{-T}).$$

The minimum annual income  $Z$  will be denoted the equally-allocated equivalent income (EAEI). Since the individual-specific  $Z$ s can be considered comparable measures of individual welfare levels that represent individual-specific vectors of incomes over a sequence of years, the distribution of the  $Z$ s constitutes the basis of the definition and measurement of chronic poverty. Thus, the EAEI may be viewed as an attempt to integrate the theory of intertemporal choice into the social choice framework required for analysing poverty.

Note that the notion of EAEI can be considered analogous to the certainty equivalent in the theory of choice under uncertainty and the equally-distributed equivalent income in analyses of income inequality.<sup>10</sup> However, while the equally-distributed equivalent income represents a money-metric measure of the social welfare for a given distribution of income across individuals, the EAEI represents a money-metric measure of the welfare level associated with the income stream for a given individual. Thus, the social planner considers the income vector  $(y_1^1, y_2^1, \dots, y_T^1)$  to be preferable to the income vector  $(y_1^2, y_2^2, \dots, y_T^2)$  if and only if  $Z^1$  exceeds  $Z^2$ .

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<sup>10</sup> See Atkinson (1970).

### 3. Evaluation of alternative methods for measuring chronic poverty

Below, we show the usefulness of the EAEI method for evaluating the theoretical basis of the standard approaches for measuring chronic poverty. In fact, the method introduced in this paper proves to encompass alternative methods for measuring chronic poverty based on an extended accounting period of income.

#### 3.1. Measuring chronic poverty on the basis of the maximum sustainable consumption

Motivated by the idea of letting the individuals' permanent incomes form the basis for analysis of chronic poverty, Rodgers and Rodgers (1993) introduced the maximum sustainable consumption of an income stream as a device for making incomes added up over several years comparable across individuals. Specifically, their focal variable for the measurement of chronic poverty is the annual income that is equal to the maximum sustainable annual consumption level ( $A$ ) an individual can achieve for a given real income stream. The behavioural counterpart of this method is that each individual carries out intertemporal income transfer at the prevailing interest rates to achieve a constant consumption level over time. For simplicity, suppose that the real interest rates on saving and borrowing in each year are equal. Then, the permanent income measure introduced by Rodgers and Rodgers (1993) coincides with the annuity equivalent ( $A$ ) of the individual's actual income stream over the given period, which is derived by replacing the variables representing the consumption levels of the various periods in the intertemporal budget constraint (2.2) by  $A$ . Provided that year  $T$  is the basis for the annuity calculations, the annual income level that is equal to the maximum sustainable annual consumption level is, as proposed by Rodgers and Rodgers (1993), given by

$$(3.1) \quad A = \frac{Y}{1 + \sum_{t=1}^{T-1} \prod_{j=t+1}^T (1+r_j)},$$

where  $Y$  is defined by (2.3).

The MSC approach can, as stated in Remark 3.1, be considered a special case of the more general EAEI method introduced in Section 2. The proof of the result in Remark 3.1 follows by straightforward calculation from equations (2.4) and (2.2).

**Remark 3.1.** Let  $(y_1, y_2, \dots, y_t)$  and  $(C_1^*, C_2^*, \dots, C_T^*)$  be the vectors of real incomes and optimal consumption levels for periods  $1, 2, \dots, T$  where  $\delta$  is rate of time preferences and  $r_2, r_3, \dots, r_T$  are the real interest rates, and let  $Z$  and  $A$  be defined by (2.7) and (3.1), respectively.

Then

$$Z = A = C_1^* = C_2^* = \dots = C_T^*$$

if and only if

$$\delta = r_2 = r_3 = \dots = r_T.$$

It follows from Remark 3.1 that the permanent income measure stemming from the EAEI method is equal to that of the MSC method if, and only if, the rate of time preferences and the real interest rate coincide in each period.<sup>11</sup>

As stated in Remark 3.2 below, the MSC method can be considered as an extension of a frequently applied approach using average income over a sequence of years after adjusting for yearly inflation as a measure of permanent income.<sup>12</sup> This method implicitly assumes that the rates of time preferences as well as the real interest rates are equal to zero.

**Remark 3.2.** Let  $(y_1, y_2, \dots, y_T)$  and  $(C_1^*, C_2^*, \dots, C_T^*)$  be the vectors of real incomes and optimal consumption levels for periods  $1, 2, \dots, T$  where  $\delta$  is the rate of time preferences and  $r_2, r_3, \dots, r_T$  are the real interest rates,  $\bar{y}$  the mean of the real income vector, and let  $Z$  and  $A$  be defined by (2.7) and (3.1), respectively.

Then

$$Z = A = C_1^* = C_2^* = \dots = C_T^* = \bar{y}$$

if and only if

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<sup>11</sup> Note that the condition  $\delta=r_t$  for all  $t$  can be abandoned in the case where the intertemporal utility function  $u$  coincides with the maximin criterion, i.e. when the utility function is of Leontief type where the periods are viewed as perfect complements. This implies that the utility is equal to the minimum consumption level over the sequence of years. An underlying assumption for such an utility function is that each individual is assumed to prefer to make intertemporal income transfer independent of the price he has to pay in terms of interest rate payments, as long as such transfer do not reduce his maximum sustainable annual consumption level.

<sup>12</sup> See Duncan and Rodgers (1991), Aaberge et al. (2000), Hill and Jenkins (2001), OECD (2001) and Valletta (2006) for analyses of poverty based on average real income over a sequence of years.

$$\delta = r_2 = r_3 = \dots = r_T = 0.$$

Thus, by using average income as a measure of permanent income one attempts to make the incomes added up over time comparable by accounting for the average change in prices for a representative basket of consumer goods. A person may then be defined as chronically poor if he has an average real income level below the average poverty line over the T periods.

Making incomes from different periods comparable, however, is not merely a question of accounting for changes in the *price of goods*, but also for changes in the *price of consumption*. Should not the price of consumption today reflect its opportunity cost? The opportunity cost of consumption today has to do with the alternative use of one's income, which depends on the real interest rates determining how much future consumption we must give up in exchange for being able to consume more today. Hence, it is the real interest rates that should be used to make incomes from different periods comparable. The price of goods coincides with the price of consumption as long as it is costless to carry out intertemporal transfers of income (i.e. the real interest rates are equal to zero). In fact, relying solely on the consumer price index to make incomes from different periods comparable implies that one only accounts for part of the costs and benefits of receiving income at different times. According to the average real income method, the real incomes for two periods given by, say, 1 in the first period and 100 in the second period is viewed as equally good as the real incomes given by 100 in the first period and 1 in the second period. However, the latter income profile can obviously produce higher consumption and welfare level in each period compared to the former income profile simply by utilizing the opportunity to save at a positive real interest rate. Thus, we may infer that the average real income method fails to provide an adequate description of the constraints in the intertemporal optimisation problem of the individuals. Furthermore, the fact that the exact choice of the rate of time preference can be controversial is, of course, not an argument for setting the rate of time preferences equal to zero, which is implicitly done in the average real income approach.

### **3.2. On the use of the MSC approach when optimal consumption levels differ between time periods**

The discussion in Section 3.1 demonstrates that the theoretical justification of the MSC approach introduced by Rodgers and Rodgers (1993) relies on rather restrictive intertemporal preferences for consumption. At first glance, it may appear that the MSC approach based on A lacks the strict theoretical basis of intertemporal optimisation that it claims to have. While the theory of intertemporal choice assumes that individuals can, *if they prefer*, make consumption-equalising income transfers, the

MSC approach insists that they *strictly prefer* to make such transfers. Indeed, within the intertemporal choice framework income transfers are made in order to achieve constant marginal utility of consumption over time, which means that consumption levels normally will differ between time periods. Thus, an interesting question is whether  $A$  defined by (3.1) can be considered as a valid measure of comparable permanent income even in cases where the assumption of equal consumption levels over time is abandoned. To discuss this question we will consider the case where the optimal consumption profile is separable in the sense that the optimal consumption for any period can be expressed as a product of a function of the real interest rates and the real income vector as well as a time-dependent function of preference parameters and interest rates

$$(3.3) \quad C_t^* = q_t C_1^*, \quad t = 1, 2, 3, \dots, T,$$

where  $q_t$  is defined implicitly by

$$(3.4) \quad g(q_t) = \frac{(1 + \delta)^{t-1}}{\prod_{j=2}^t (1 + r_j)}, \quad t = 2, 3, \dots, T,$$

$g(x) = u'(x)$  and  $q_1 = 1$ . Thus, the ratio between the optimal consumption levels for two arbitrarily chosen periods depends on the instantaneous utility function  $u$ , the rate of time preference  $\delta$ , and the real interest rates  $r_2, r_3, \dots, r_T$ , but not on the present value of the real income  $Y$ . As demonstrated by Theorem 3.1 below the consumption profile (3.3) is optimal if and only if the utility function is of the Box-Cox type, which is by far the most popular specification of the instantaneous utility function in intertemporal choice theory (Davies and Shorrocks, 2000).

**Theorem 3.1.** *Let  $(C_1^*, C_2^*, \dots, C_T^*)$  be the vector of optimal consumption levels for periods  $1, 2, \dots, T$  defined by (2.4) where  $u'$  is the derivative of the instantaneous utility function  $u$ ,  $\delta$  is the rate of time preferences and  $r_2, r_3, \dots, r_T$  are the real interest rates, and let  $q_t$  be defined (3.4). Then*

$$(i) \quad C_t^* = q_t C_1^* \quad \text{for } t = 1, 2, 3, \dots, T$$

*if and only if the instantaneous utility function  $u$  has the following form*

$$(ii) \quad u(x) = \begin{cases} \frac{1}{1 - \varepsilon} (x^{1 - \varepsilon} - 1) & \text{if } \varepsilon \neq 1 \\ \log x & \text{if } \varepsilon = 1, \end{cases}$$

*where  $\varepsilon^{-1}$  is the intertemporal elasticity of substitution.*

**Proof.** Assume that  $C_t^* = q_t C_1^*$  where  $q_t$  is defined by (3.4). Now, inserting for (3.3) and (3.4) in (2.4) we obtain the following functional equation

$$g(q_t C_1^*) = g(q_t) g(C_1^*) \quad \text{for all } q_t \text{ and } C_1^*,$$

which has the solution (see Aczél, 1966)  $g \equiv 0$  or  $1$ , or there exists a real number  $\varepsilon^{-1}$  such that

$$u'(x) = g(x) = x^{-\varepsilon}.$$

Hence, (i) implies (ii).

The converse statement follows by inserting (ii) in (2.4). □

**Remark 3.3.** *Although Theorem 3.1 is motivated by the call for a framework for analysing chronic poverty, the result of Theorem 3.1 also proves useful in the context of intertemporal consumption behaviour of individuals. In this context Theorem 3.1 demonstrates that individuals will behave in accordance with the separable optimal consumption profile (i) if and only if the instantaneous utility function of the intertemporal utility function (2.1) is of the Box-Cox type (ii).*

As will be demonstrated below, the result of Theorem 3.1 proves useful for identifying the relationship between  $A$  defined by (3.1) and  $Z$  defined by (2.7). To this end, it is convenient to introduce the notation  $a_t$  defined by

$$(3.5) \quad a_t = \frac{1 + \sum_{i=1}^{T-1} \prod_{j=i+1}^T (1+r_j)}{\sum_{i=1}^{T-1} q_i \prod_{j=i+1}^T (1+r_j) + q_T} q_t, \quad t = 1, 2, \dots, T,$$

and  $k(\varepsilon, \delta)$  defined by<sup>13</sup>

$$(3.6) \quad k(\varepsilon, \delta) = \begin{cases} \left[ \frac{\sum_{t=1}^T a_t^{1-\varepsilon} (1+\delta)^{1-t}}{\Delta} \right]^{\frac{1}{1-\varepsilon}} & \text{when } \varepsilon \neq 1 \\ \prod_{t=1}^T \frac{a_t (1+\delta)^{1-t}}{\Delta} & \text{when } \varepsilon = 1, \end{cases}$$

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<sup>13</sup> For convenience the dependence of  $k$  on  $r_2, r_3, \dots, r_T$  is suppressed in the notation for  $k$ .

where  $\hat{u}$  is defined by (2.8).

**Theorem 3.2.** Let  $(C_1^*, C_2^*, \dots, C_T^*)$  be the vector of optimal consumption levels for periods  $1, 2, \dots, T$  defined by (2.4) where  $u'$  is the derivative of the instantaneous utility function  $u$ ,  $\delta$  is the rate of time preferences and  $r_2, r_3, \dots, r_T$  are the real interest rates. Moreover, let  $Z, A, q_t$  and  $k(\varepsilon, \delta)$  be defined by (2.7), (3.1), (3.4) and (3.6). Then

$$(i) \quad C_t^* = q_t C_1^* \quad \text{for } t=1, 2, 3, \dots, T$$

implies

$$(ii) \quad Z = k(\varepsilon, \delta)A.$$

**Proof.** By inserting for  $C_t^* = q_t C_1^*$  in equation (2.2) we get

$$(3.7) \quad C_1^* = \frac{Y}{\sum_{t=1}^{T-1} q_t \prod_{j=t+1}^T (1+r_j) + q_T},$$

where  $Y$  is defined by (2.3).

Next, inserting for (3.1), (3.5) and (3.7) in  $C_t^* = q_t C_1^*$  yields

$$(3.8) \quad C_t^* = a_t A \quad \text{for } t=1, 2, 3, \dots, T.$$

Moreover, it follows from Theorem 3.1 that the instantaneous utility function  $u$  is given by (ii) of Theorem 3.1. By inserting (3.8) and specification (ii) for  $u$  in equation (2.6) we get

$$(3.9) \quad \hat{U} = \begin{cases} \frac{1}{1-\varepsilon} \left[ A^{1-\varepsilon} \sum_{t=1}^T a_t^{1-\varepsilon} (1+\delta)^{1-t} - \Delta \right] & \text{when } \varepsilon \neq 1 \\ \Delta \log A + \sum_{t=1}^T (1+\delta)^{1-t} \log a_t & \text{when } \varepsilon = 1. \end{cases}$$

Now, inserting for (3.9) and the inverse of the Box-Cox utility function  $u$  (defined by (ii) in Theorem 3.1) in (2.7) yields

$$(3.10) \quad Z = k(\varepsilon, \delta)A. \quad \square$$

Since a common intertemporal utility function determined by the social planner forms the basis for computing the permanent incomes, it follows that  $\varepsilon$  and  $\delta$ , and thus  $k(\varepsilon, \delta)$ , is the same for all individuals. Furthermore, because methods for quantifying (relative) poverty are required to be invariant with respect to scale transformations of the chosen income measure (permanent income in this case), Theorems 3.2 and 3.1 demonstrate that  $A$  and  $Z$  will produce identical poverty results for a given set of income data when the utility function  $u$  is of the Box-Cox type. Thus, in contrast to what is claimed by Rodgers and Rodgers (1993) the permanent income measure  $A$  may form the basis of poverty analysis even in cases where the optimal consumption levels differ between time periods.

Furthermore, it follows from Theorems 3.2 and 3.1 that poverty estimates based on the EAEI approach are *independent* of the actual parameter values for  $\varepsilon$  and  $\delta$  provided that the chosen instantaneous utility function is of Box-Cox type.<sup>14</sup> These results obviously generalise to distributional assessments requiring the income measure to be invariant with respect to scale transformations, such as in the standard measurement of income inequality. In this regard, note that under the assumption of a perfect credit market and a Box-Cox instantaneous utility function, the EAEI permanent income measure is equivalent to the concept of utility-equivalent annuity used by Nordhaus (1973) as a measure of lifetime income. Nordhaus (1973) is concerned with the sensitivity of the analysis of lifetime income distributions based on utility-equivalent annuity measures to the arbitrary nature of the parameter values for  $\varepsilon$  and  $\delta$ .<sup>15</sup> As demonstrated by Theorems 3.2 and 3.1, however, inequality estimates as well as poverty estimates based on the Box-Cox family of utility-equivalent annuity measures will in fact be independent of the actual parameter values for  $\varepsilon$  and  $\delta$ .

### 3.3. An illustration of the EAEI, the MSC and the average real income method

In Figure 1 the various measures of permanent incomes are shown graphically in the two-period case. Consider the vectors of real income  $(y_1^1, y_2^1)$  and  $(y_1^2, y_2^2)$  with the same average real income  $\bar{y}$ , which is given by the point where the upward sloping 45 degree line intersects the downward sloping 45 degree line. The maximum sustainable consumption  $A$  for each real income vector is defined by its annuity value and obtained from the point of intersection between the corresponding budget constraint with gradient  $1+r_2$  and the upward sloping 45 degree line. In comparison, the equally-allocated

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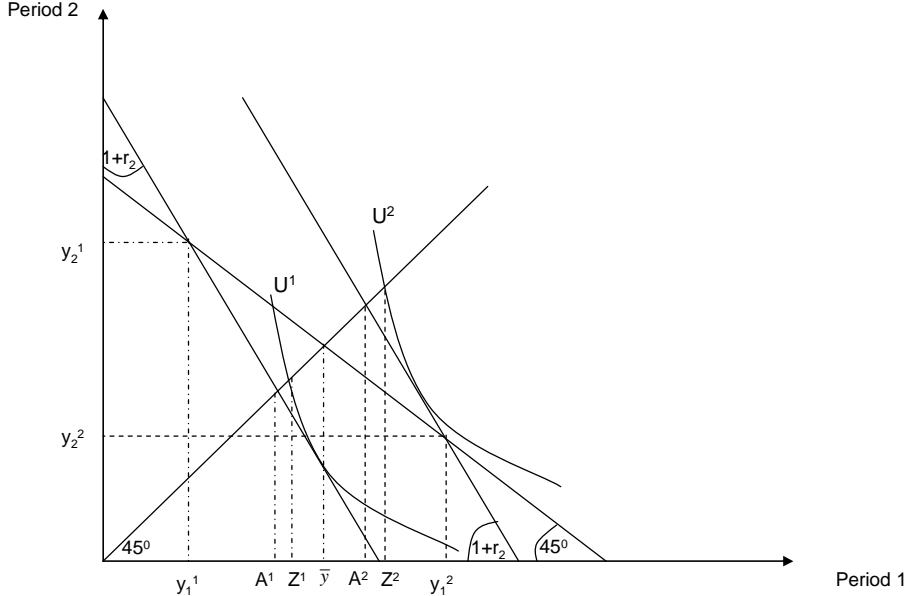
<sup>14</sup> Thus, unlike the cost-benefit literature where results appear to be rather sensitive to the choice of social discount rates, poverty measurement based on the EAEI approach will in fact be independent of the actual parameter values for  $\varepsilon$  and  $\delta$ . In practice, countries have used widely different rate of time preferences for official cost-benefit analysis, ranging from 3 per cent real rate in Germany to 8 per cent real rate in France (Evans and Sezer, 2005)

<sup>15</sup> This concern is also raised by Creedy (1999) in his survey of lifetime income distribution measures.



equivalent income  $Z$  associated with each real income vector is given by the point where the indifference curve crosses the upward sloping 45 degree line.

**Figure 1. Permanent income measures for a set of real income vectors**



The average real income  $\bar{y}$  will differ from  $A$ , and thus from  $Z$ , when the real interest rates on inter-period income transfers are non-zero. As illustrated in Figure 1,  $\bar{y}$  will be larger (less) than  $A$  when the real income vector leads to borrowing (saving). This is due to the fact that the average real income disregards the costs and benefits of inter-period income transfers. The permanent income measure based on the EAEI method will exceed or be equal to that of the MSC method. The reason is that only the method for deriving  $Z$  is based on a preference structure flexible enough to allow individuals to adjust their consumption profiles to take advantage of intertemporal incentives caused by any deviation between the rates of time preferences and the real interest rates. However, as demonstrated by Theorem 3.2, the ratio of  $Z$  and  $A$  will be equal for all real income vectors independent of the chosen values of the preference parameters  $\delta$  and  $\epsilon$  as well as the magnitudes of the real interest rates as long as they are equal for all individuals.

**4. Measurement of permanent income and chronic poverty when the credit market is imperfect**

Under the assumption of a perfect credit market we have demonstrated that analyses based on permanent income measures from the MSC and the EAEI approach will produce identical poverty

results, provided that the underlying instantaneous utility function is a member of the Box-Cox family of utility functions. Below, we abandon the assumption of a perfect credit market and explore whether any of the previously discussed approaches can be extended to account for individual-specific interest rates on savings and borrowing as well as for liquidity constraints.<sup>16</sup> If chronic poverty is essentially about identifying those with the inability to pursue welfare due to lack of economic means, it might be important to take into account that at least certain sub-groups of the population face constraints in the credit market as borrowers. Specifically, individuals can face borrowing constraints since judgements about future labour income involve not only uncertainty but also asymmetric information about skills and propensities to induce effort. This may, in particular, be the case for individuals such as unemployed or disabled who are outside the labour force, and therefore are at risk of seriously falling short of the resources commanded by most of the individuals or households in the society in which they live.

#### 4.1 The EAEI approach

When interest rates on borrowing and savings differ then (2.2) and (2.3) are no longer a valid representation of the budget constraints. Consequently, the optimal consumption levels defined as the solution to (2.1), (2.2) and (2.3) will in this case not form an appropriate basis for defining and measuring the EAEI. Thus, the method outlined in Section 2 for deriving the optimal consumption profiles has to be revised.

In order to make the derivation of optimal consumption profiles under the assumption of imperfect credit markets as transparent as possible, consider first the simple two-period case illustrated in Figure 2 where the real interest rate on borrowing ( $r_b$ ) exceeds that of savings ( $r_s$ ). This leads to a piecewise-linear and concave budget constraint with two segments. The individual concerned has three options: save (segment 1), borrow (segment 2), or consume his entire income in each period (kink).

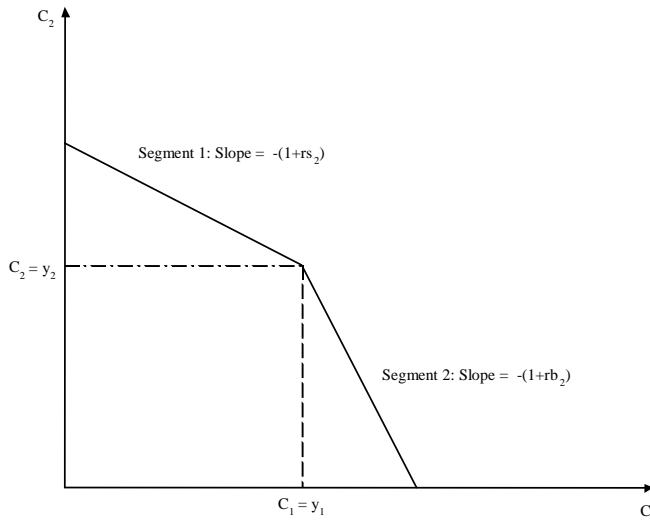
Prohibiting borrowing corresponds to segment 2 being a vertical line that intersects the horizontal axis at  $C_1 = y_1$ . To determine the optimal consumption profile, we first derive the conditional consumption profile for each of the linear segments; the term conditional is used because it describes the choice of consumption conditional on having chosen to save or to borrow. By computing the utilities of the conditional consumption profiles, we can determine whether the individual is considered to be a saver, a borrower or neither a saver nor a borrower and, hence, his optimal consumption profile. Notice that

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<sup>16</sup> The empirical studies carried out by Flavin (1981) and Blinder and Deaton (1985) reject the permanent income hypothesis with perfect credit markets. There are also strong theoretical reasons why the credit market does not clear and liquidity constraints arises, e.g. such that borrowers may default on their loans (Stiglitz and Weiss, 1981). See Hayashi (1987) for a review of other possible mechanisms that result in liquidity constraints.

the income and substitution effects of changes in real interest rates and incomes will depend on whether the individual remains at the segments or the kink. Small changes can, therefore, affect the consumption profile through adjustments within the given segment and/or by changing the choice of segment or kink.

**Figure 2. Piecewise-linear and concave intertemporal budget constraint with two segments**



Formally, we can apply the Kuhn-Tucker method to derive the conditional and optimal consumption profiles in the case of imperfect markets.<sup>17</sup> For simplicity, assume that each individual is faced with a single borrowing interest rate and a single savings interest rate (but different individuals may face different borrowing and/or savings rates). If there are no liquidity constraints, the optimal consumption profile  $(C_1^*, C_2^*, \dots, C_T^*)$  is defined as the solution of (2.1) subject to the budget constraints

$$(4.2) \quad \begin{aligned} S_0 &= 0 \\ S_t &= (1 + r\gamma_t)S_{t-1} + y_t - C_t, \quad t = 1, 2, \dots, T-1 \\ S_T &= (1 + r\gamma_T)S_{T-1} + y_T - C_T = 0 \end{aligned}$$

where  $S_t$  represents the assets at the end of period  $t$  earning an interest rate  $r\gamma_{t+1}$ , and

$$(4.3) \quad r\gamma_t = \begin{cases} rs_t & \text{if } S_{t-1} \geq 0 \\ rb_t & \text{if } S_{t-1} < 0 \end{cases}, \quad -1 < r\gamma_t < \infty, \quad t = 2, 3, \dots, T.$$

<sup>17</sup> See Ljungquist and Sargent (2004) for an in-depth discussion of the discounted utility model under various forms of credit market imperfections.

Solving this maximization problem requires comparison of  $3^{T-1}$  conditional consumption profiles. The consumption profiles are distinctive in terms of whether the individual in the various periods is considered to be a saver, a borrower or located at the kink and thereby consuming all his assets. Each of these conditional consumption profiles is a candidate for the individual's optimal consumption profile provided that the budget constraints are satisfied for the given values of  $y_t$  and  $r\gamma_t$ . The optimal consumption profile is determined as the utility maximising choice among the conditional consumption profiles satisfying their respective budget constraints. By inserting the consumption levels of the optimal consumption profiles into (2.6), the corresponding EAEI ( $Z$ ) is obtained from (2.7).

The presence of liquidity constraints reduces the number of available conditional consumption profiles that have to be compared. For example, the case where borrowing in each period is prohibited corresponds to reducing the number of conditional consumption profiles to those satisfying  $S_t \geq 0$  for all  $t$ . Thus, deriving EAEI subject to liquidity constraints is straightforward and can be considered a special case of the method outlined above.<sup>18</sup>

## 4.2. The MSC approach

When the real interest rate on savings differs from that of borrowing, the maximum sustainable annual consumption level defined by (3.1) is, as acknowledged by Rodgers and Rodgers (1993), not valid as a measure of permanent income. This is due to the fact that we do not know in advance whether the individual in question will be a saver or a borrower in the various periods. For this reason, Rodgers and Rodgers (1993) introduced an iterative procedure based on mean annual income as a first approximation of  $A$ ; this approximation is then adjusted according to the corresponding savings/borrowing pattern until assets at the final year is acceptably close to zero. In end effect, they obtain an estimate of  $A$  defined as annual income plus savings or minus borrowing required to achieve a constant consumption level over time.

By contrast, the framework outlined in Section 4.1 does not require application of an iterative procedure for computing the annual income that corresponds to the maximum sustainable annual consumption level of a given income stream. Specifically, replacing  $C_t$  with  $A$  for each  $t$  in (4.2) yields  $T$  equations for determining  $A$  and  $S_t$ . Since  $S_t$  can be larger, less, or equal to zero in each of the  $T-1$  periods, there are  $3^{T-1}$  candidates for  $A$ . Note that these candidates share the common feature of

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<sup>18</sup> A program for computing the permanent income  $Z$  with or without credit market imperfections is available from the authors upon request.

constant consumption levels over time, but are distinctive in terms of the values of  $S_t$  and thus  $r\gamma_t$ . Specifically,  $A$  is determined as the highest consumption level among the candidates satisfying their respective budget constraints.

The method outlined above is not only simpler and computationally less burdensome compared to the iterative procedure proposed by Rodgers and Rodgers (1993), but also provides an exact measurement of  $A$ . But more importantly, the above method serves as a framework for identifying the underlying assumptions inherent in the iterative procedure. In fact, it is easy to demonstrate that a necessary condition for the optimal consumption profile  $(C_1^*, C_2^*, \dots, C_T^*)$  defined as the solution to the maximisation of (2.1) subject to (4.2) under the constraint  $C_1^* = C_2^* = \dots = C_T^*$ , is that  $\delta = rb_t$  for any  $S_{t-1} < 0$  and  $\delta = rs_t$  for any  $S_{t-1} > 0$ .<sup>19</sup>

This result has some troublesome implications for the MSC approach. On the one hand, interpersonal comparability of income streams requires a common intertemporal utility function. On the other hand, introducing an imperfect credit market, where some individuals are borrowers and others savers, implies interpersonal variation in real interest rates. However, allowing interpersonal variation in the real interest rates and insisting on identical rate of time preferences across individuals is incompatible with the MSC requirement that the rate of time preferences has to be equal to the real interest rate for each individual. Hence, comparability of incomes in the MSC approach is incompatible with an imperfect credit market. By contrast, the EAEI approach justifies interpersonal comparability of permanent incomes even when there is interpersonal variation in the real interest rates, since it does not require constant consumption levels over time. Furthermore, even if one ignores the comparability problem, the MSC method is not able to handle liquidity constraints in a sound manner, since constant consumption levels may not be attainable when borrowing is restricted. Consider, for example, an individual restricted from borrowing with a two-period vector of real incomes given by 1 in the first period and 100 in the second period. In this case, the annual income equal to the maximum sustainable annual consumption level obtained from the iterative procedure is equal to the relatively low income in the first period and is in fact unaffected by the magnitude of the income in the second period. By contrast, the EAEI of the individual will not be determined solely by the income level in the first period; although he is not as well off as he would be if there were no borrowing constraint, he will still receive some utility from the high income in the second period. This illustrates that the MSC method

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<sup>19</sup> As long as there is an interior optimum, this condition is necessary and sufficient for the optimal consumption profile to consist of consumption levels that are constant over time. Suppose there is a boundary optimum, i.e.  $S_t = 0$  for at least one  $t=1, 2, \dots, T-1$ . In this case, it is also required that  $C_{t-1}^* = C_t^* = C_{t+1}^*$  for any  $S_t = 0$ , which in general will not be the case.

is not flexible enough to incorporate liquidity constraints. In fact, in the presence of liquidity constraints the MSC method suffers from the same weakness as cross-section snapshots of poverty, since neither recognizes that a bad year does not render an otherwise well-off person poor.

### 4.3. The spell approach

"Any model of income estimated with longitudinal data implicitly or explicitly provides a model of intertemporal dynamics" (Bane and Ellwood, 1986, p2).

A widely used method for measuring chronic poverty is based on the time span individuals have endured low annual income. After Bane and Ellwood (1986), it has become common to use hazard regression models and/or first-order Markov models to estimate the lengths of poverty spells which are defined as continuous periods during which income falls below the year-specific income threshold.<sup>20</sup> Chronic poverty is then defined by relatively long continuous spells or somewhat shorter but recurrent spells. Essentially, the novelty of Bane and Ellwood (1986) was to apply methods designed for analysis of duration events to analyse changes in annual income over time.<sup>21</sup> At first glance, this methodological framework appears promising since concern is changed from the question of 'who has low income at a given point in time' to the question of 'who has persistently low income'. Moreover, this approach enables researchers to study the association between transitions into and out of a low annual income status and demographic and macroeconomic variables. However, acknowledging that income is a flow variable, it becomes apparent that methods developed to analyse the dynamics of stock variables, such as unemployment, may not be directly applicable to studies of poverty. In particular, it is not clear whether the model of intertemporal dynamics underlying the spell-based approach produces a reasonable relationship between the observed income pattern and the associated welfare level.

Arguably, the intertemporal model of dynamics underlying the spell-based approach to chronic poverty suffers from an unsatisfactory specification of the budget constraints and a controversial intertemporal preference structure. The spell-based approach assumes, as pointed out by Rodgers and Rodgers (1993), that income is perfectly transferable within the period that it is earned, typically a

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<sup>20</sup> Prior to Bane and Ellwood (1986), it was common to simply tabulate the proportion of the population with annual income below the year-specific income threshold in  $t$  out of  $T$  years, where  $0 \leq t \leq T$ . In this case, the chronically poor are defined as those who had annual income below the year-specific income threshold in most or all years. Essentially, the distinguishing feature of spell studies compared to tabulation studies is that they sample from the flow rather than the stock to avoid selection problems. See for example Levy (1977), Coe (1978), Rainwater (1981), Hill (1981), and Duncan et al. (1984) for tabulation studies, and Stevens (1994, 1999) and Cappellari and Jenkins (2004) for spell studies.

<sup>21</sup> See e.g. Lancaster (1989) for a review.

year, but not transferable between periods. As previously addressed, this assumption is in conflict with substantial empirical evidence, which shows that most households undertake inter-period income transfers. A model of intertemporal dynamics should reflect that incomes could be used for consumption not only within the year that it is obtained, but also between years. In fact, fluctuating income is, as the main rule, neither the same as fluctuating consumption or welfare level. Thus, the recent trend of studying the entries and exits associated with poverty based on changes in annual income risks to be misleading.<sup>22</sup> Indeed, Duncan et al. (1995) and other applications of the spell-based approach implicitly acknowledges the problem of excess sensitivity to fluctuations in incomes as they restrict poverty transitions to those that involve an income change of, say, at least 20 per cent. Even if one takes the stand that certain individuals are constrained from borrowing they may be capable of saving. Others can undertake inter-period income transfers by borrowing as well as by saving. By treating income as a stock variable and thus assuming away any kind of inter-period income transfers for the entire population, the spell-approach fails to account for interpersonal variation in real interest rates in general and liquidity constraints in particular; not only incomes but also the opportunities to smooth consumption by borrowing may vary across the population.

Even if one for some reason finds it reasonable that each individual is incapable of saving and borrowing, the spell-based approach implicitly assumes that the intertemporal utility function is exclusively determined by the length of time in which a person happens to have income below a specific annual income threshold. Thus, the method pays excessively attention to the time-patterns of income, but is rather insensitive to the magnitude of the actual income streams. For example, a person that on a single occasion experiences annual income above the year-specific income threshold, and for that reason is not defined as chronically poor, may have an income stream with less consumption potential in each period compared to the income stream of a chronically poor person experiencing a complete poverty spell. Moreover, since the spell approach only utilises parts of the available income information, it is impossible to consider distributional issues such as the poverty gap and the inequality among the poor using this method.

#### **4.4. Illustrating the sensitivity of poverty estimates to the choice between the EAEI method and the spell approach**

Below, we illustrate the extent to which chronic poverty estimates depend upon the choice of method for measurement in the case of an imperfect credit market. Specifically, Table 1 contrasts the EAEI

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<sup>22</sup> See for example Stevens (1994, 1999) for studies of transition into and out of poverty based on information about changes in annual incomes.

approach with the frequently applied spell-approach in the three-period case. The reason for excluding the MSC approach is partly because it fails to account for liquidity constraints, but also due to lack of interpersonal comparability of permanent incomes when the real interest rates on savings and borrowing differ across individuals.

**Table 1. Chronic poverty status for a set of real income vectors according to the EAEI method and the spell-approach\***

$(y_1, y_2, y_3)$	Spell-approach (Complete poverty spell)	Spell-approach (2-period poverty spell)	EAEI ( $rs_t = 0.02,$ $rb_t = 0.05,$ $t=2,3$ )	EAEI ( $rs_t = 0.02,$ $rb_t \rightarrow \infty, t=2,3$ )	EAEI ( $rs_t = -1,$ $rb_t \rightarrow \infty, t=2,3$ )
(90, 90, 90)	Chronic poor	Chronic poor	Chronic poor	Chronic poor	Chronic poor
(10, 110, 10)	Not chronic poor	Chronic poor	Chronic poor	Chronic poor	Chronic poor
(500, 90, 90)	Not chronic poor	Chronic poor	Not chronic poor	Not chronic poor	Not chronic poor
(90, 90, 130)	Not chronic poor	Chronic poor	Not chronic poor	Chronic poor	Chronic poor
Poverty threshold	100	100	100	100	100

\* The poverty thresholds are assumed to be fixed and equal to 100 for both methods and under all specifications of the credit markets. The instantaneous utility function is under the EAEI method assumed to take the Box-Cox functional form with  $\delta = 0.03$  and  $\varepsilon = 2$ .

Let us first consider the chronic poverty status following from the spell-approach when the chronically poor are defined as those experiencing a complete poverty spell over the three years. In this case, the first individual is the only one who is chronically poor, despite his relatively high income stream compared to the second individual. This illustrates that studies relying on the spell-approach risks that a non-poor individual has an income stream that is well below the income stream of a person defined as chronically poor. The reason is that the second individual happens to have, in a single year, annual income above the year-specific poverty threshold, which excludes him from the poor population regardless of the relative size of his actual income stream. Classifying the first individual, but not the second individual, as chronically poor seems flawed, since the first individual could, by saving, sustain a higher consumption level than the second individual in each year.

At first glance, this problem appears to be solvable by adjusting the time span necessary for individuals to have endured low annual income to be defined as chronically poor. Suppose that the chronically poor are defined as those experiencing a poverty spell of at least two years in length out of the three years. In this case, all four individuals presented in Table 1 are classified as chronically poor.



However, the classification of the third individual as chronically poor is clearly unreasonable, since he by saving could achieve an annual consumption level in each year that is well above the poverty threshold. This demonstrates that the weaknesses of the spell-approach cannot simply be solved by ad-hoc adjustments of the length of poverty spells it takes to define chronic poverty. On the contrary, one is forced to deal explicitly with the unsatisfactory specification of the budget constraints and the controversial intertemporal preference structure inherent in the spell-approach.

Consider the chronic poverty status in Table 1 following from the EAEI method under the various specifications of the imperfections in the credit market. In the case of no liquidity constraints but real interest rates that differ between borrowing and saving, the first two individuals are chronically poor whilst the last two individuals are non-poor. This means that we do not have the problem as in the spell-approach where the first individual, unlike the second, was classified as chronically poor. In the case where borrowing is prohibited, individual four is also classified as chronically poor. Although individual four receives some benefits from having a relatively high income in the last year, he is not as well off as he was when he could undertake intertemporal income transfers to smooth consumption. This leads to a relatively low permanent income  $Z$  for individual four when he is faced with a binding borrowing constraint, since this reduces the welfare level associated with his real income. Finally, consider the special case of the EAEI method where, just as in the spell-based approach, it is assumed that individuals cannot borrow or save. However, due to a less restrictive intertemporal utility function the EAEI method does not suffer from the excess sensitivity of poverty estimates to the time-pattern of annual incomes. In particular, the second individual with high income in a single year but low overall consumption potential is classified as chronically poor according to the EAEI method with liquidity constraints. By contrast, according to the spell-based approach the high income in a single year is sufficient to exclude individual two from the population of chronically poor, independent of the size of his incomes in the other periods. This demonstrates that arguments of the kind that individuals are incapable of carrying out income transfers between years are not only empirically false, but even if they were true they do not imply that one should apply the spell-based approach. Instead, one should account explicitly for liquidity constraints as in the EAEI method.

## **5. Concluding remarks**

As an alternative to the conventional methods for measuring chronic poverty, this paper proposes an interpersonal comparable measure of permanent income as a basis for defining and measuring chronic poverty. The introduced measure of permanent income is compatible with a general preference structure of intertemporal consumer behaviour and, moreover, can be considered as a money-metric

measure of the welfare level associated with the income stream for a given individual. Acknowledging that not only incomes but also the opportunities to smooth consumption by borrowing may vary across individuals, the proposed method is flexible enough to allow for individual-specific interest rates on borrowing and saving as well as for the presence of liquidity constraints. Although this paper has focused on the definition and measurement of chronic poverty, the results generalise to any distributional assessments where the income measure is required to be invariant with respect to scale transformations, such as in the standard measurement of income inequality.

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