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THE EFFECT OF UNCERTAINTY AND LEARNING ON RESOURCE DEPLETION

by

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Abstract

The paper analyzes the effects of uncertainty and learning on investment decisions when the information gain is a by-product of planned economic activity. We consider two resources, α and β, one of which must be extracted before the other. During the production of α, information relevant to the development and production of β is obtained. The aim of the paper is to study the impact of this information on the optimal production time of α. The analysis shows that several different classes of optimal solutions are possible: which class the optimal decision belongs to depends, among other factors, on the rate of information gain relative to the discount rate.

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1. INTRODUCTION

The economic literature on the extraction of exhaustible resources has grown appreciably over the last few years (see the reference list for a partial list of some relevant works). One of the main themes in these investigations has been how to incorporate the effect of uncertainty and learning into the theoretical framework in a 'sensible' way. The concept of option value, or quasi option value [Henry, 1974a,b; Arrow and Fisher, 1974] has come to the forefront in these discussions. As Conrad [1980], among others, has shown, (quasi)-option value is essentially the expected value of information gained by delaying irreversible decisions. One of the main conclusions which emerges from these studies is that the prospect of getting fuller information works against irreversible decisions, i.e. the prospect of fuller information combined with the irreversibility of the planned investment, brings forth a positive option value in favour of preservation [Henry, 1974a]. However, more often than not, useful information is not gained by just postponing decisions. Some sort of economic action, with associated costs, is usually required. Furthermore, the amount and type of information gained is seldom independent of the outcome of previous decisions. Thus option value is not a one-sided argument for slower, or less, development and greater preservation [Freeman, 1984; Miller and Lad, 1984].

The present note is an attempt to study the problem of individual decision-making under uncertainty in the context of natural resource extraction when learning is possible. More specifically, we adress the problem of when the prospect of more information justifies the added costs necessary to obtain this information. We shall do this within the framework of a simple model describing the optimal extraction of <u>two</u> non-renewable resources, here denoted α and β respectively. We will be thinking of the resources as oil and gas, but other types of deposits of resources will surely also fit the model. Because of technical considerations, e.g. a rapid fall in pressure as gas is produced, one of the resources (α or oil) will have to be produced <u>before</u> the main extraction of the other resource (β or gas) starts, if it is to be produced at all. Similar situations may arise during exploitation of other resources, e.g. mining of minerals. The model is a generalization of the model analyzed in Hoel [1978].

We remark that the model may also apply to more general situations if α and β are viewed as two activities, one which by necessity must be performed before the other and where the first activity produces information which is valueable for the next activity. Examples of such situations are marked research done before a new product is released, and wildcat drilling for oil and gas.

During the production of resource α (oil), information will be gained concerning the reservoir characteristics. Estimates of future costs associated with the production of resource β will also be affected, and some of the uncertainty associated with this production will probably diminish. Future prices obtainable for the second resource will also generally be less uncertain. We shall restrict ourselves to the situation where the information gain is a function of the production time of resource α alone. This may be reasonable in situations where the production rate of α is constant, or alternatively, where the activity level of process α is fixed. The problem we shall consider is the determination of the optimal production time T, taking the above information gain into account. Formally, we want to optimize the total net discounted profit W(T) with respect to T:

$$W(T) = F(T) + \exp(-rT)EG(\underline{X}) \qquad (1)$$

The profit function is composed of two parts:

$F(T)$	: The net discounted profit of developing and producing resource α from time zero to T. F(T) may have a local maximum at time $T=T^*$.
$\exp(-rT)EG(\underline{X})$	: Discounted (to time zero) expected net profit obtained from the development and production of resource β.

r being the discount rate, assumed constant for simplicity, and $EG(\underline{X})$ is the expected net profit from the extraction of resource β discounted to time T. $\underline{X}$ is a stochastic vector with components consisting of the variables determining the profit of resource β through the functional expression G. We assume that the joint probability density of $\underline{X}$, $\varphi(\underline{X};T)$, is known. The learning process is represented by the time dependence of the probability distribution, i.e. it depends on the length of the interval over which resource α is produced. In order not to complicate the mathematical expressions unduly, we have supressed T as an argument in the function EG.

We shall allow the (co-)variances (denoted by $\underline{\sigma}^2$) to vary with the production time T of resource α, while the mean values of $\underline{X}$ (denoted by $\underline{\mu}$) is kept constant. This is a generalization of the approach taken by Hoel [1978], where no change in the probability distribution is made before resource α is completly depleted. At this point in time Hoel assumes that the decision makers obtain perfect knowledge of stock size and extraction costs of deposit β (the two variables considered by Hoel).

The optimal production time of resource α is formally found from the following set of equations:

First order condition:

$$W'(T) = F'(T) - \exp(-rT)\,[rEG(\underline{X}) - \partial(EG(\underline{X}))/\partial T] = 0 \qquad (2)$$

Second order condition:

$$W''(T) = F''(T) - \partial\{\exp(-rT)\,[rEG(\underline{X}) - \partial(EG(\underline{X}))/\partial T]\}/\partial T < 0 \qquad (3)$$

Note that the solution of (2) and (3) should be compared to the situation where resource α is not produced, i.e. with the profit $W_\beta = EG(\underline{X})$, and with the profit of producing α alone, $W_\alpha = F(T^*)$, where T^* is the optimal production time of resource α. In this note we shall assume that W_β is larger than W_α, thus indicating that resource α is in some sense an inferior resource compared to β. The profit of extracting α alone may even be negative. This is typically the case when 'production of α' is a pure information gathering process.

Before proceeding with the study of the effects of the learning process and the solution of the above equations, let us say a few words about the net discounted profit, F(T), of resource α, and discuss some possible functional forms of the profit function of resource β.

1.1 The profit function of resource α.

Our aim in this section is to find a reasonable description of the general shape of the utility function for resource α, F, considered as a function of the production time T alone. We do this by noting that a reasonable expression for the utility function associated with resource α is the discounted net profit:

$$F(T) = \int_{0}^{T} \{q(t)z(t) - c(z(t), Z(t), \underline{Y}(t))\}\exp(-rt)dt \qquad (4)$$

where q(t) is the price, z(t) the production rate, and Z(t) the accumulated production of resource α. The vector $\underline{Y}(t)$ in eq.(4) indicates that the cost may depend on other variables than those directly associated with the production volume.

By making reasonable assumptions about the the various terms entering (4) it is possible to say something about the general form of the profit function F(T).

First of all we assume a smooth price trend. This is a prerequisite in order to be able to say anything meaningful about the global structure of F(T). In the following analysis the price is, for simplicity, set equal to a constant (figure 1a).

Secondly, the production rate z(t) is assumed to be single peaked with zero as a limit as $t \to \infty$ (figure 1b).

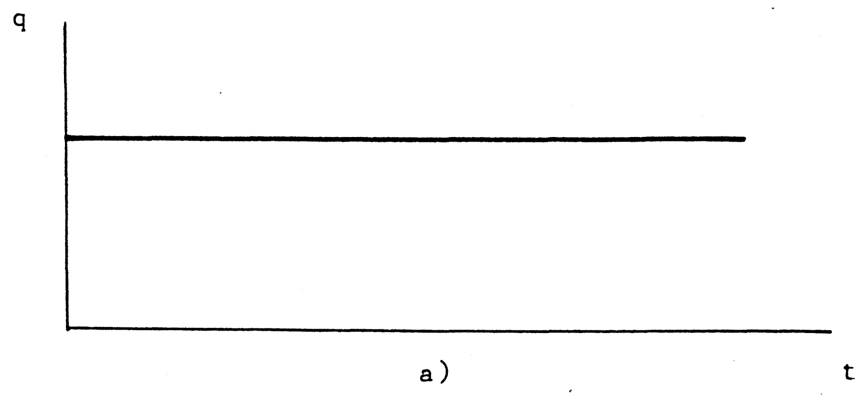
Finally, the cost term of the profit function is assumed to be composed of three parts:

i) Investment or capital costs c_1 pr. unit time. This contributes to the total costs at the initial stage of the project.

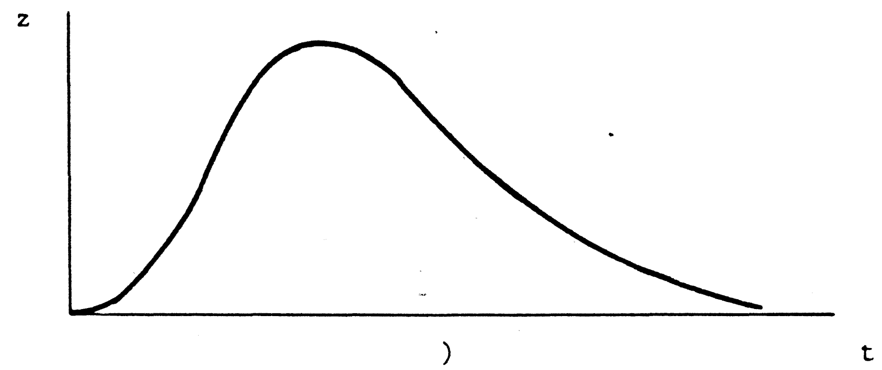
ii) Production costs pr. unit time, c_2, which follows the production rate z(t) closely.

iii) Depletion costs pr. unit time, c_3, which takes account of the increased production costs as the resource is depleted. The last barrel of oil is more expensive to produce than an earlier one. Also dismantling of the production equipment has a cost.

The shape of the total cost function, $c = c_1 + c_2 + c_3$, as a function of time, is shown in figure 1c. Discounting the instant profit produce a profit profile similar to the one shown in figure 1d.



a)



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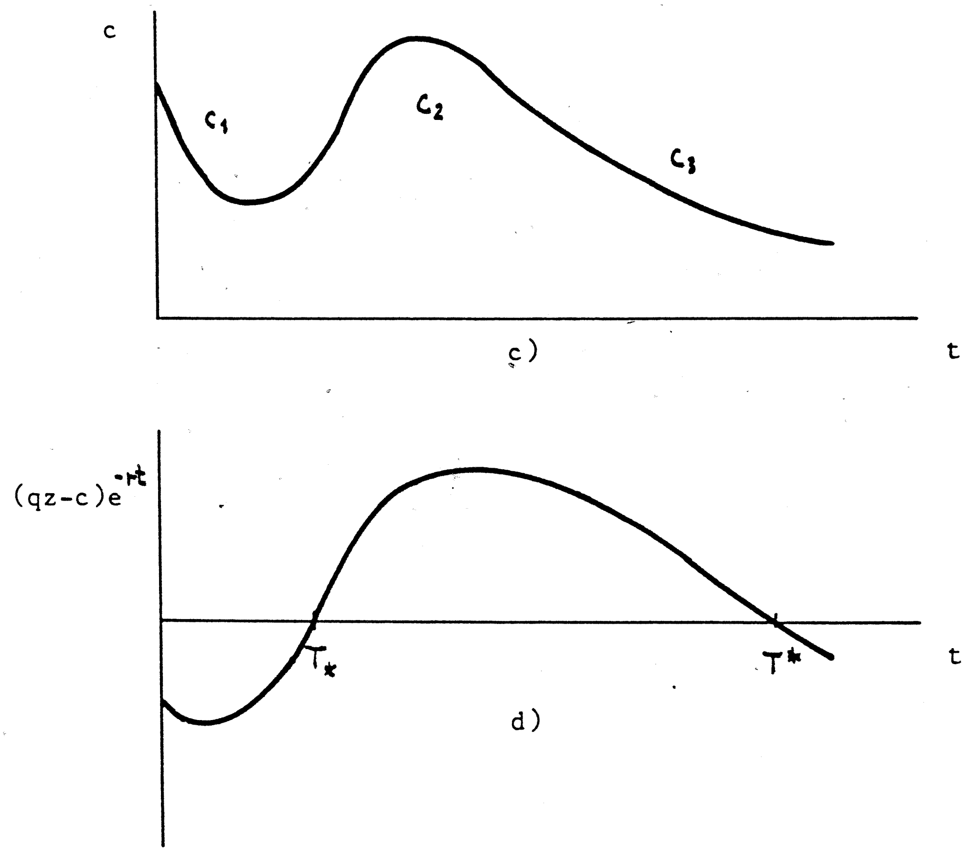

Figure 1. Components of the profit function for resource α. a) The price profile. b) The production profile. c) The cost profile. d) Discounted profit rate.

Finally, integrating the discounted profit rate over time from $t = 0$ to $t = T$ we obtain a function of the form shown in figure 2.

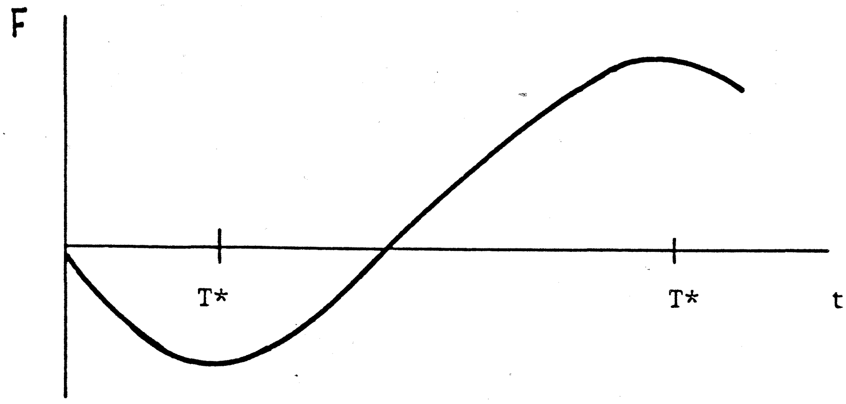

Figure 2. A profit function for resource α.

Taken by itself, i.e. neglecting the possible impacts on resource β, the net discounted profit of resource α is maximized by producing up to time $T = T^*$. Note that the maximized profit, $F(T^*)$, may be positive or negative. A negative profit may be acceptable if it is needed to obtain a bigger positive profit in the utilization of resource β.

We note in passing, that the shape of the profit function F(T) is very similar to what is sometimes assumed to be the shape of the 'net value of information' as a function of the amount of information [Dasgupta, 1982].

Occasionally, when we want to illustrate some of our results with numerical calculations, we will have the need for a specific discounted profit function for resource α. A simple example of such a function is:

$$F(T) = -S\,[T^3/3 - (T^*+T_*)T^2/2 + T^*T_*T] \qquad (5)$$

This function has the general shape shown in figure 2, where also the locations of the parameters T_* and T^* are shown. At $T=T_*$ the profit from α stops declining, while T^* is the optimal production time. S is a scaling factor. We have assumed that the discount rate r is constant, and have neglected it in (5). This is reasonable if current prices and costs increase with a rate equal to the discount rate. We shall return to this model when we illustrate some of our results below.

1.2 Some possible profit functions for resource β.

We have represented the physical and economic variables associated with resource β by the stochastic vector $\underline{X}$. The components of $\underline{X}$ may be reservoir size (volume, tonnes of oil, cubicmeters gas etc.), total (integrated) development and production costs, price obtainable for the produced stock etc. At this point we shall not specify $\underline{X}$ further, only assume that it has a joint probability density $\varphi(\underline{X};T)$ which also depends on the total production time of resource α. Below we shall study in more detail the case where X is a scalar variable. This is done for illustrative purposes, since in this case the complicating correlations between the elements of $\underline{X}$ disappear.

The expected profit of resource β enters in the expression (1) for the total discounted net profit. It is possible to relate the expected value of $G(\underline{X})$ to known properties of the distribution of $\underline{X}$ in three situations: when the function G is quadratic in its argument, when G is of an exponential type, and when the distribution function φ is sharply peaked.

1.2.1 Quadratic profit function.

A quadratic profit function for resource β has the form:

$$G(\underline{X}) = a + \sum_i b_i X_i + \frac{1}{2} \sum_{ij}\sum X_i c_{ij} X_j$$

$$= a + \underline{bX} + \frac{1}{2} \underline{X}' c \underline{X} \qquad (6)$$

where a, $\underline{b}$, and $\underline{c}$ are a constant scalar, a vector, and a n x n matrix, respectively. The expected value of $G(\underline{X})$ is given by

$$EG(\underline{X}) = G(\underline{\mu}) + \frac{1}{2} \sum_{ij}\sum c_{ij}\sigma^2_{ij} \qquad (7)$$

where

$$\underline{\mu} = \underline{\mu}(T) = \int \underline{X}\ \varphi(\underline{X};T)d\underline{X} \qquad (8a)$$

$$\sigma^2_{ij} = \sigma^2_{ij}(T) = \int [X_i-\mu_i(T)][X_j-\mu_j(T)]\varphi(X;T)dX \qquad (8b)$$

Thus, $\underline{\mu}(T)$ is the expected value of $\underline{X}$ and $\underline{\sigma}^2(T)$ is the covariance matrix of $\underline{X}$. T is the life time of resource α.

Assuming, for a scalar X, b>0 and c<0 the form of G(X) is shown in figure 3. Marginal utility is decreasing and the absolute risk aversion

$$R_A = -G''(X)/G'(X) = - c/(b + cX) \qquad (9)$$

is increasing with X and tend towards infinity at the point of maximal profit. Although a quadratic function is easy to handle analytically the two last implications can be serious drawbacks.

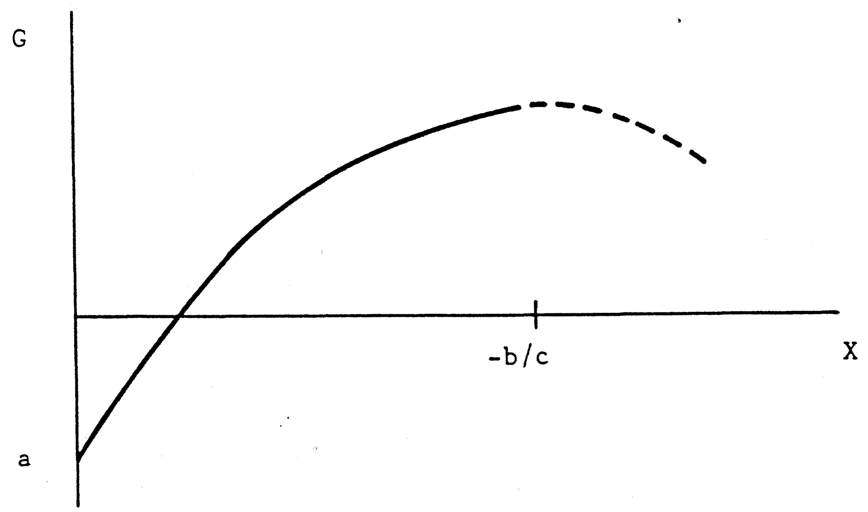

Figure 3. A quadratic profit function.

Note that in the simple case with a scalar X and c<0, an increased uncertainty (variance) leads to a decreased expectation value of G. This is a simple consequence of the concavity of G

(Jensen's inequality).

We also remark that for sufficiently sharply peaked density functions $\varphi(\underline{X};T)$, the expected value of a general profit function $G(\underline{X})$ can, to a good approximation, be written as in eq. (7), with $c_{ij} = \partial^2 G/\partial X_i \partial X_j$.

1.2.2 Exponential profit function.

A profit function which has a constant absolute risk aversion, (equal to $-\underline{a}$), is

$$G(\underline{X}) = b + c \exp(\sum_i a_i X_i) \tag{10}$$

If the X's are assumed to be normally distributed (but not necessarily independent) with mean $\underline{\mu}$ and covariance σ^2, then

$$EG(\underline{X}) = b + c \exp(\sum_i a_i \mu_i) \exp(\frac{1}{2} \sum_{ij}\sum a_i \sigma_{ij}^2 a_j) \tag{11}$$

If X is a scalar and c and a are negative constants, (see figure 4 below), G is concave and an increased uncertainty implies a decrease in the expected value of G.

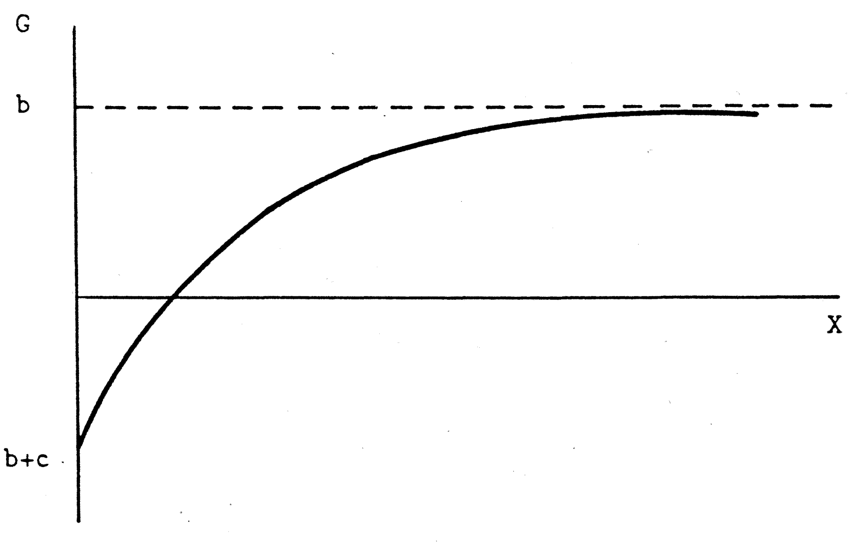

Figure 4. An exponential profit function.

2. FIRST ORDER CONDITIONS.

Returning to the analysis of the first order condition, (2), we rewrite it to yield

$$H(T;r) = EG(\underline{X}) - (1/r)\partial EG(\underline{X})/\partial T \qquad (12)$$

where the function H(T;r) is defined by

$$H(T;r) = (1/r)F'(T)\exp(rT) \qquad (13)$$

We have indicated that the discount rate r enters as a parameter in the defining equation.

Eq.(12) says that the optimal production time of resource α is obtained when the marginal profit of continued production of α, divided by the discount rate, equals the discounted expected utility of β adjusted for learning. The optimal production time is thus determined by balancing the cost of postponing the production of β against the value of information obtained by extending the production time of α.

Not much can be said about the effects of uncertainty and learning without specifying the functional form of G further. The results will be sensitive to the formulation (properties) of the function G and the covariance structure of the stochastic vector $\underline{X}$. For this reason we restrict ourselves to a situation where the net profit obtainable from resource β depends on a scalar variable X and the profit function G(X) is concave. X may, for instance, be the reservoir size of an oil field (cfr. Hoel [1978]). Illustrations of our results will be based on examples using one of the functional forms discussed in the last section. Both of these are of the above type.

In order to isolate the effects of uncertainty and learning on the optimal production time of resource α, it is useful to have as reference case the situation where uncertainty, and hence learning, is absent. We now proceed with the description of such a situation.

2.1 The certain case.

Without uncertainty the first order condition (12) takes the form

$$H(T;r) = EG \qquad (14)$$

where now $EG = G(\mu)$, independent of T, and $\mu = X$.

From the generic form of the profit function F(T) for resource α (figure 2), we deduce that H(T;r) have a shape similar to the curve shown in figure 5. It is negative for $T < T_*$ and $T > T^*$, positive when $T_* < T < T^*$.

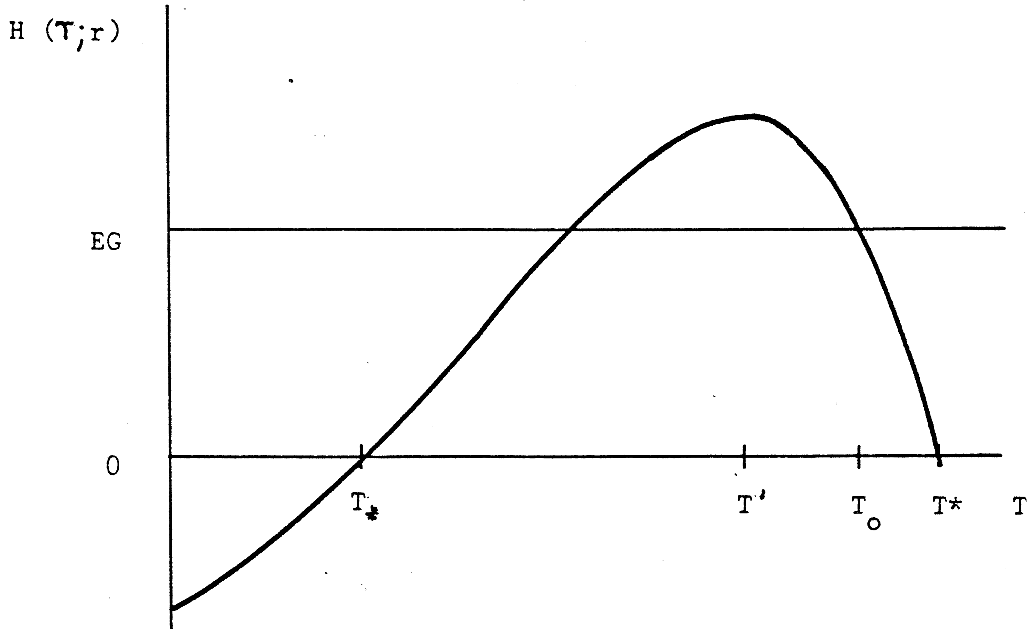

Figure 5. Illustration of the function H(T;r) and the first order condition in cases where learning effects are absent.

The (constant) value EG is represented as a straight horizontal line in the figure. If this line is above the maximum of H, we conclude that the production of β should start right away without any production of resource α. Total net profit is W = EG in this case.

β

If the EG-line crosses the H-curve, however, production of resource α for a time T_0 could be optimal. From the second order condition it follows that it is the rightmost solution, i.e. the solution where the H-curve approach the EG curve from above, that gives a (local) maximum of the total net discounted profit. If we denote the time where $H(T;r)$ is maximal by T', we conclude that $T' < T_0 < T^*$. Whether $T=0$ or $T=T_0$ is the global optimum of the total net discounted profit $W(T)$, depends, among other things, on the absolute level of the profit function $F(T)$ of resource α.

2.2 <u>The effects of a fixed uncertainty and changes in the discount rate.</u>

As pointed out previously, EG is a decreasing function of σ^2. Introducing a T-independent uncertainty alters the expected value of G to a new constant $EG_0 < G(\mu)$. We shall assume EG to be independent of the discount rate r. This, of course, is unrealistic, but is done in order to simplify the following discussion.

The possible solutions of the problem when a T-independent uncertainty is included are as in the certain case, only now there is an increased likelihood for production of both resource α and β (since the horizontal line in figure 5 is lowered). In other words, <u>we may have situations where without uncertainty no production of α should be undertaken, while inclusion of uncertainty makes such production profitable</u>.

If we denote the solution of the first order condition with uncertainty by T_1, it can be shown that $T_0 < T_1 < T^*$. This is done by proving, from the first and second order conditions (2) and (3), that T_1 is an increasing function of the variance. It is also easy to deduce from figure 5. Thus, in our model <u>uncertainty tends to extend the production period of resource α, compared to the certain situation</u>.

It is sometimes assumed that the effect of uncertainty can be captured by increasing the discount rate. Next we shall show that this is not always the case. Increasing the discount rate changes the H-curve in a rather complicated manner. Since

$$\partial H(T;r)/\partial r = H(T;r)(T - 1/r) \tag{15}$$

H will increase where $T > 1/r$ and $F'(T) > 0$, and where $T < 1/r$ and $F'(T) < 0$. (Remember that we have assumed F(T) to be independet of r). A typical result of increasing r is shown as the dotted curve in figure 6.

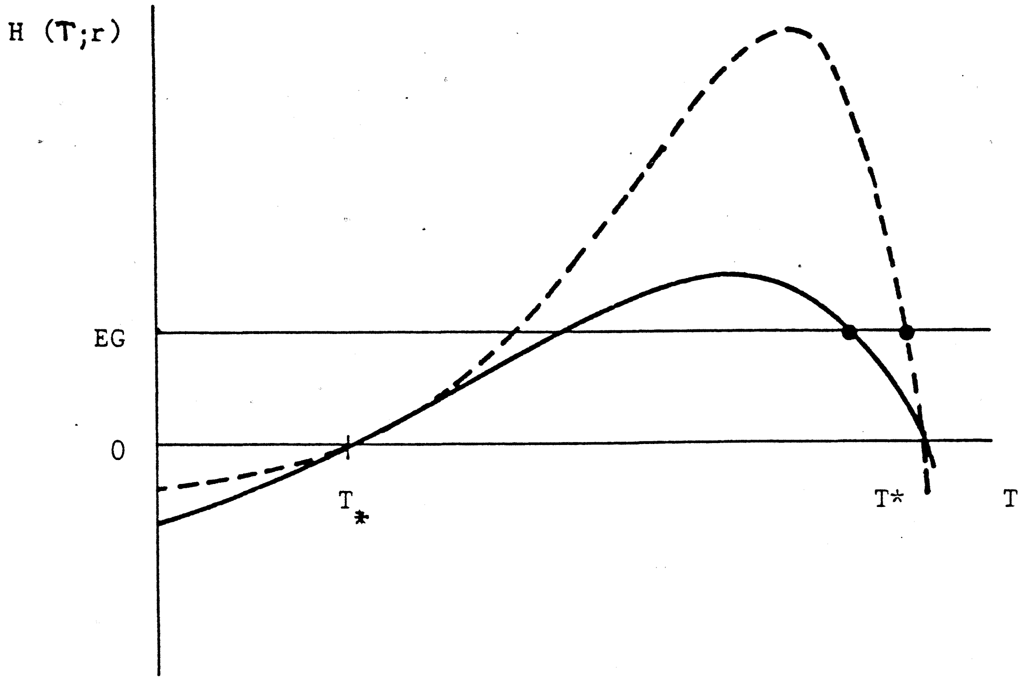

Figure 6. The effect of a changed discount rate on the first order condition.

If $1/r < T_0$, we conclude that an increased discount rate will increase the production time T_0. This is an effect similar to an increase in the uncertainty. However, the opposite may take place when $1/r > T_0$. Hence, only for large enough discount rates is it in principle possible to represent the effect of uncertainty by an increased discount rate, in the sense that they affect the outcome in the same direction. The actual increase in r needed to represent a given uncertainty depends in a complicated manner on the discount rate itself, the profit functions F and G, and the uncertainty.

2.3 The effect of learning.

Allowing a T-dependent variance in the distribution of X, introduces a new term in addition to those already considered in the first order condition (cfr. eq. 12). Since learning implies that $\partial\sigma^2/\partial T < 0$, and EG is a decreasing function of the variance σ^2, we conclude that $\partial EG/\partial T > 0$, i.e. the marginal expected profit from resource β due to an increase in the production time of α is positive. The effect of learning on the first order condition, eq. (12), is, therefore, to increase the first term on the right hand side, EG, but at the same time subtract a positive term, $(1/r)\partial EG/\partial T$. Apriori it is difficult to predict the net outcome.

We note that for 'small' discount rates, $r < r_0$, where

$$r_0 = (\partial EG/\partial T)/EG = \partial(\ln EG)/\partial T \tag{16}$$

the right hand side of eq.(12) becomes negative. r_0 could be termed the <u>learning rate</u>. Without learning r_0 is zero. Generally, r_0 will depend on the production time T. When we below discuss the types of solutions we get when r_0 is in certain intervals, we are referring to the value of r_0 at the (local) optimal production time.

When $r < r_0$, corresponding to a high learning rate, we can only have interior solutions with either $0 < T < T_*$ or $T > T^*$. The first interval contains solutions where the value of learning obtained from the production of resource α outweighs the cost of producing α for a short time. These solutions have no counterpart in the case without learning. Thus, no production of α would take place if learning effects were disregarded. We observe that if the production of resource α shows a negative marginal profit for all T, i.e. T_* is effectively infinite, solutions of this type are the only ones possible with $T > 0$. Examples of such situations are wildcat and appraisal drilling for oil and gas, and mineral exploration. <u>In order for learning to have an effect on the production time in this case, the learning rate r_0 must exceed the discount rate r.</u>

Solutions in the second interval ($T > T^*$) are of a different nature in that they extend the production time of resource α beyond the optimal single-resource time T^*. The small discount rate (or high learning rate) assumed ($r < r_0$) makes the costs associated with the delay in the production of resource β lower than the value of added information obtained from the extended production period of α.

When $r > r_0$, corresponding to a low learning rate, the first order condition (12) gives solutions located in the interval between T_* and T^*. In this case the costs of delaying the production of β beyond T^* are larger than the benefit of added information. This is reasonable since a small learning rate implies a relatively high uncertainty and, hence, a relatively low expected profit from resource β. By increasing the learning rate (but still $r > r_0$), the expected profit increases and together with it the cost of delaying the production start of β. The production time of α will, therefore, decrease with increasing learning rate as long as $r > r_0$. This is perhaps opposite to what one intuitively would think the effect of learning should be.

If we interpret the right hand side of the first order condition (12) as expected net profit from resource β adjusted for learning effects, we observe that for small T it will be lower than the expected profit when learning is disregarded. Only after a time TL, where TL is defined implicitly as the solution of the equation

$$EG(T) - (1/r)\partial EG(T)/\partial T = EG(T=0) \qquad (17)$$

is the expected adjusted profit equal to the unadjusted profit. (Here, the argument T signifies that the expected value is to be calculated with the probability density at time T). Thus, only after a time TL is the learning process able to reduce the effects on uncertainty below the analogous T-independent level. We shall return to this when we illustrate some of our results with a specified profit function G below.

So far, we have only assumed a discounted profit function for resource α of the general form shown in figure 2, and a single variable, concave profit function for resource β. In order to illustrate our results, we now turn to a study of some concrete examples.

3. EXAMPLES.

We shall now evaluate the effect of learning in the cases where the utility of resource β is represented by a quadratic and exponential function of X, respectively. We assume the profit function for resource α to be of the form given in eq. (5).

We specify the T-dependence of the variance of the variable X by

$$\sigma^2(T) = \sigma_0^2 \exp(-\gamma T) \tag{18}$$

A high decay rate γ indicates that the variance, i.e., the uncertainty, decrease fast with increasing production time T. In other words, much is learned in a relatively short time under the production of resource α.

3.1 Quadratic profit function.

The quadratic profit function for resource β was given in eq.(6) as

$$G(X) = a + bX + \frac{1}{2} cX^2 \tag{19}$$

where now a, b, and c are scalar constants. The expected profit from resource β after having produced α for a time T is

$$EG = G(\mu) + \frac{1}{2} c\sigma^2(T) \tag{20}$$

From eqs.(12) and (20) we obtain the first order condition for a maximal total net discounted profit in the following form:

$$H(T;r) = G(\mu) + \frac{1}{2} c\sigma_0^2 (1 + \gamma/r)\exp(-\gamma T) \tag{21}$$

With a T-independent uncertainty, i.e. no learning (γ=0), the first order condition takes the form:

$$H(T;r) = G(\mu) + \frac{1}{2} c\sigma_0^2 \tag{22}$$

Thus, provided the marginal profit of resource β is a decreasing function of X ($c<0$), the T-independent uncertainty decreases the expected profit. By including a learning effect this lowering of the expected profit is enhanced by a factor $(1 + \gamma/r)$, but at the same time reduced by a factor $\exp(-\gamma T)$. The final outcome is that only for production times T greater than a time TL, where TL is defined by

$$TL = (1/\gamma)\ln(1 + \gamma/r) \tag{23}$$

is the learning process able to reduce the effect of uncertainty below the analogous T-independent level. TL is plotted in figure 7 for various values of the discount rate r.

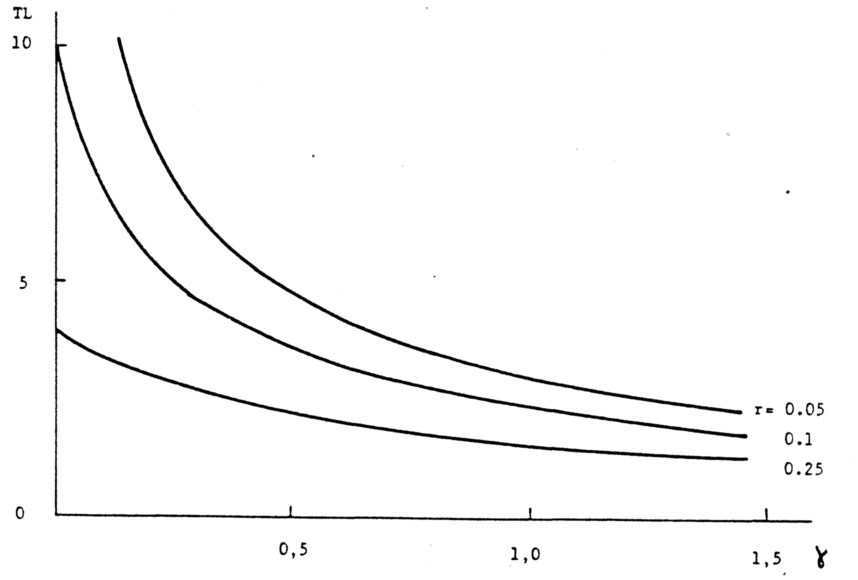

Figure 7. The 'learning time' TL as function of the decay rate γ for various values of r.

We observe that TL is a decreasing function of both r and γ. Hence, the 'learning time' TL is decreased by either an increase in the speed of learning (γ) or an increase in the discount rate (r).

The solution of the first order condition (21) is illustrated in figure 8 for two values of γ, keeping r fixed.

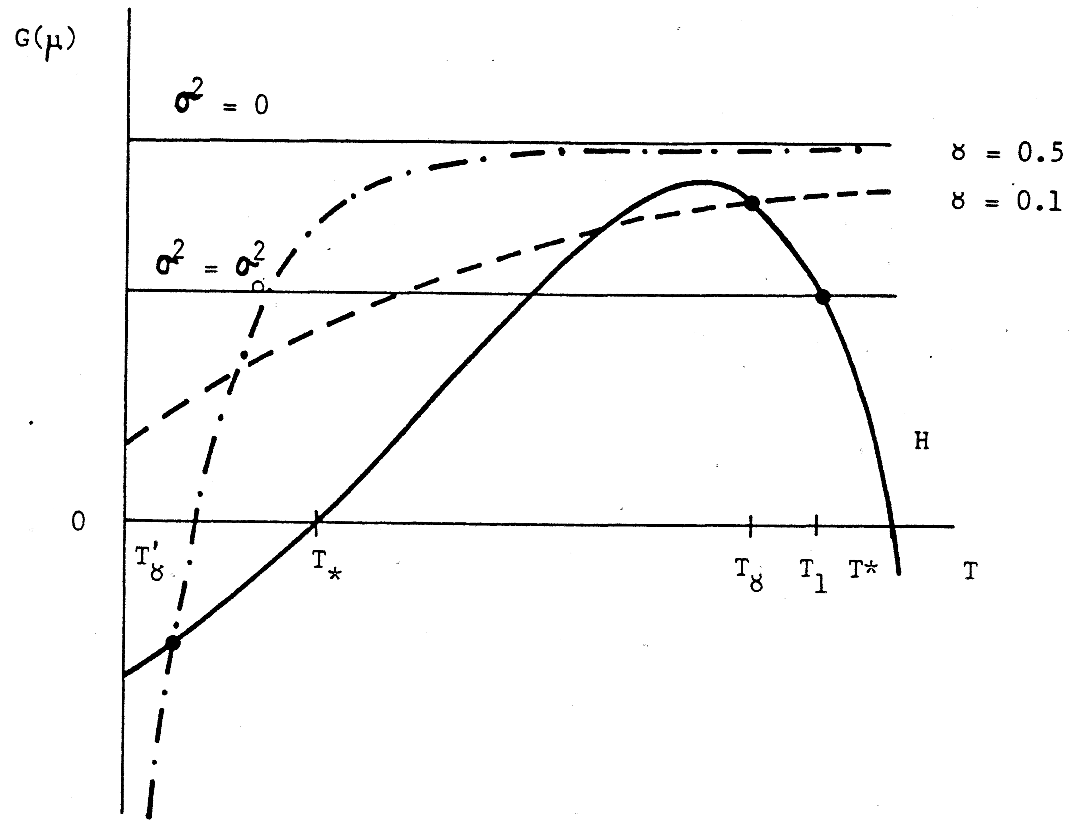

Figure 8. The first order condition when learning effects are included.

In this figure we have used a profit function for resource α of the form given in (5) with scale factor S = 0.0002, $T_* = 5$, $T^* = 20$, and a quadratic profit function for resource β (eq. 19) with parameters a = -0.497, b = 0.547, and c = -0.1. We note that the maximum net profit from α alone is rather low: $F(T^*) = 0.07$. The mean value of X was equal to 2.3, while the initial variance was $\sigma_0^2 = 4$. The discount rate r was 0.10 throughout the calculations.

As shown in the figure, G(μ) is above the maximum of H(T;r) (upper horizontal line), indicating that development and production of resource α should not take place in a situation without uncertainty. The optimal net profit in the example is G(μ) ≈ 0.50 in this case.

Including a T-independent uncertainty, σ_0^2, make possible an interior (T > 0) solution of the first order condition at $T = T_1 \approx$

18.20 (lower horizontal line in figure 8). In our example, the total net discounted profit from producing α for a time T_1 followed by the production of β, is $W(T_1) \approx 0.11$. The net expected profit from a production of resource β alone is $W_\beta = W(0) \approx 0.30$. This last solution is, of course, preferable to the interior (or $\alpha + \beta$) solution.

Finally, including learning by letting the decay rate of the variance of X, γ, be equal to 0.1, results in a right hand side of the first order condition (eq.21) as shown by the broken curve in figure 8. The interior solution is now at $T = T\gamma \approx 16.50 < T_1$, with associated profit $W(T\gamma) \approx 0.14$. This is still below the profit obtainable from the production of resource β alone.

Increasing the decay rate γ to 0.5 changes the situation rather dramatically, as illustrated by the dash-dotted curve in figure 8. The new interior solution at $T\gamma' \approx 1.20$ is now in a region where the production of resource α considered by itself gives a net loss. Nevertheless, the total net discounted profit from the production of both α and β is approximatly $W(T\gamma') \approx 0.32$, i.e. above the maximum profit obtainable from resource β alone. Qualitative changes in decisions may thus be brought about when the possibility of gaining information during economic activities is recognized.

3.2 Exponential profit function.

The results described above remains largely unchanged when we assume an exponential profit function for resource β;

$$G(X) = b + c\exp(aX) \qquad (24)$$

In our example we have used the following parameters: $a = \ln(0.5)$, $b= 1.0$, and $c = -2.0$. The mean value and variance of X and the discout rate r were as in the previous example. The first order condition for optimal production time T is now

$$H(T) = b + c[1 + \gamma a^2\sigma_0^2\exp(-\gamma T)/(2r)]\exp[a\mu + \frac{1}{2}a^2\sigma^2(T)] \qquad (25)$$

In comparison, a T-independent uncertainty gives as first order condition:

$$H(T) = b + c\exp(a\mu + \frac{1}{2} a^2\sigma_0^2) \qquad (26)$$

Again, we observe a double effect of a T-dependent variance. The constant c is enhanced by a factor $[1 + (\gamma/r)\exp(-\gamma T)]$, while the variance in the exponential part of (25) decrease with increasing T. It is, thus, possible to define a 'learning time' TL analogous to the case with the quadratic profit function. This time, however, TL is only defined implicitly by the equation.

$$\frac{1}{2} a^2\sigma_0^2[\exp(-\gamma TL)-1] = -\ln[1+(\gamma a^2\sigma_0^2/2r)\exp(-\gamma TL)] \qquad (27)$$

From the graphical representation of H(T;r) (figure 5) it is again possible to obtain the optimal production time for resource α. The picture that emerges is very similar to what was presented in figure 8 for the quadratic profit function and is, for this reason, omitted.

This similarity is hardly surprising, since the exponential and the quadratic profit functions are quite similar in global behaviour. The main difference that emerges is that the profit from β when uncertainty, but not learning, is included, is lower in the exponential case than in the quadratic example (-0.06 and 0.30, respectively). This behaviour, which do not alter the optimal solution described in the last section, is due to the difference in absolute risk aversion associated with the two functions.

4. SUMMARY AND CONCLUSIONS

We have in this paper considered the extraction of two resources, α and β, one of which must be produced before the other. During the production of the first resource, α, information relevant to the development and production of the second resource, β, is obtained. Our main aim was to study the impact of this information gain on the determination of the optimal production time T of resource α.

Conditions determining the optimal production time has been given in the form of first and second order conditions. Not much could be said from these conditions, however, in the general case where the net profit of resource β was determined by a multi-component stochastic vector $\underline{X}$. For this reason we explored some of the consequences of the learning process when a scalar variable (X) could be assumed to determine the profit obtainable from resource β. A decreasing marginal utility of β with respect to X was also assumed. Under these conditions we showed that several qualitatively different types of solutions were possible, depending on

- the costs (due to the discounting of future profit) associated with a delay in the production start of resource β,

- the benefit of information obtained during the production of resource α, i.e. the benefit of a decreased uncertainty,

- the profit potential of resource α and β, considered by themselves.

We found that production of α only should take place when, loosely speaking, the profit from β was below a certain threshold value. Uncertainty lowered the expected profit from the production of β, thus making a solution with production of both α and β more probable. By including learning effects, however, the uncertainty was decreased. This made the delay in the production of β, due to the production of α, more expensive. At the same time the production of α was necessary in order to get the information needed to lower this uncertainty. Balancing these effects lead to basically three different classes of solutions.

One class implied a production of resource β only. This was found to be optimal when the learning rate was small compared to the discount rate and the expected profit from β was in some sense 'large'. Any delay in the production of β in this situation would be too expensive to justify the relative small increase in information one could obtain from a production of α.

The second class of solutions was characterized by small modifications of a scheme where α was produced for a time not very different from the optimal production time of α, T^*, when considered as a single resource, followed by production of β. The modified optimal production time could be longer or shorter than the single-resource time T^*, depending on the value of the learning rate relative to the discount rate. A high learning rate will extend the production period of α while a low learning rate will shorten it.

Finally, we found a third class of solutions, possible only when the learning rate was high compared to the discount rate. These solutions implies production of α for a <u>short</u> time, before the production of β starts. The benefit of reduced uncertainty outweighs the costs of a short production time of α in this case. Most of the information gathering taking place in real life, like marked research, wildcat drilling and other exploratory activities, presumably represent solutions of this type. In the resource context described above we find that the information gain could make it profitable to develop and produce a resource that otherwise would not have been exploited.

Most of the above conclusions are based on the assumption of a concave profit function for resource β. If the profit function instead is assumed to be convex most of the conclusions would have to be modified. However, the interesting problem is rather which conclusions could be made when we have a 'realistic' profit function, e.g. one that is composed of different parts depending on multicomponent variables in different manners. The answer is that few general statements could be made in this case. Rather, the results will to a large degree depend on the covariances between the different parts of the multi-component variables and the functional dependencies on these variables. It is, of course, very difficult to predict the effects of uncertainty and learning in such situations. Nevertheless, as we have shown in this paper, such effects are potentially very important and should not be disregarded when planning investments strategies.

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6. SOME COMMENTS ON THE LITERATURE

Below follows a few short comments and incomplete summaries of some of the papers listed in the reference list which has most direct relevance for the discussion in our paper.

HOEL, MICHAEL, Resource extraction, uncertainty, and learning. Bell Journal of Economics, 9, 642-645, 1978

Hoel considers two resources; A and B. A (=oil) must be extracted before B (=gas) can be explored. Production rate of A determined by extraction period T. T is to be decided.

Utility of A: F(T).

Utility of B: U(T;R,b)=g(R,b)exp(-rT)
where

- r: constant rate
- R: size of reservoir (or other reservoir characteristics)
- b: cost of production

R and b assumed independently distributed stochastic variables.

$$\underset{0<T}{\text{Maximize:}} \left[F(T)+EU(T;R,b) \right]$$

Conclusion: Since g depends on R and b in different manners, the uncertainties in R and b affect T in different ways.

Note: There is no difference between the fixed (no-learning) and the flexible (sequencial, with learning) strategy in this case. Furthermore, there are no irreversibility effects, since A is to be utilized in any case. Therefore, the result is rather difficult to compare with 'classical' works such as Arrow & Fisher (1974).

HENRY, CLAUDE, Option values in the economics of irreplacable assets.
Review of Economic Studies, , 89-104, 1974

Henry considers two periods, j=1,2, each with an utility function of the form U(Y,D). D = development, Y = profit from development.

Y is a concave function of D with negative derivative: Y=f(D), $f'<0$, $f''<0$. U is assumed to be strictly quasi-concave.

Irreversibility is included by requiering $0 \leq D2 \leq D1$ ($\leq$ Dmax).

Period j=1 has utility function U1(D1), while the utility function for period 2 depends on the 'state-of-the-world' through the stochastic variable w; U2(D2;w). The probability distribution of w is considered as fixed.

Problem: Determine the effect of two different decision strategies (fixed and flexible) on the initial period D1 to be chosen.

Strategy 1: Fixed (without learning):

$$\max_{D1,D2} [U1(D1) + EU2(D2;w)]$$
$$\text{s.t. } 0 \leq D2 \leq D1 \leq D_{max}$$

Denote solution by D^*1 and D^*2.

Strategy 2: Flexible (with learning):
This is done in two steps:

(1) $\max_{0 \leq D2 \leq D1} [U2(D2;w)]$ for given D1 and w.

Denote solution by $D_*2(d1;w)$.

(2) $\max_{0 \leq D1 \leq D_{max}} [U1(D1) + EU2(D_*2(D1;w);w)]$

Solution: D_*1.

Henry shows that $D_{*}1 \geq D^{*}1$.

Remark: It may be possible to interpret D as oil production, Y as gas production.

Note: The paper also includes a discussion of the concept 'option value'.

HENRY, CLAUDE, Investments decisions under uncertaint: The 'irreversibility effect'.
The American Economic Review, 64, 1006-1012, 1974.

Main results:
'Equivalent certainty case' not applicable when irreversibility is taken into account. This refutes the arguments of Herbert A. Simon (Econometrica 24,74-81,1956), Henri Theil (Econometrica,25,346-349,1957), and Edmond Malinvaud (Econometrica,37,706-718,1969).

Risk-neutral decision maker facing binary decision is led to adopt irreversible decisions more often than he should if he neglect irreversibility effects.

If fixed strategy indicates no irreversible decision, then so does a flexible strategy.

Finally, Henry shows in a concrete example that the irreversibility effect may be fairly important.

Interpretation: Gas production precludes oil production and hence is an irreversible act.

ARROW, KENNETH J. and ANTHONY C. FISHER,
Enviromental preservation,
uncertaint, and irreversability.
Quartely Journal of Economy, 88, 312-319, 1974.

A 'classical' paper based on the problem of land development over two time periods. Main conclusions:

i) Uncertainty lower expected benefits.
ii) The effect of irreversibility is like risk aversion.

HANEMANN, W. MICHAEL, On reconciling different concepts of option value.
Preprint, 1984.

The paper is mainly concerned with the interpretation of different use of the concept 'option value'. The discussion of the Arrow-Fisher-Henry option value is close to Henry(1974a).

MILLER, JON R. and FRANK LAD, Flexibility, Learning, and irreversibility: A Bayesian approach.
Journal of enviromental economics and management, 11, 161-172, 1984.

Miller and Lad points out that a flexible strategy may imply larger costs than a fixed strategy, and that the learning is dependent on decisions in period 1. Thus a conditional probability distribution should be used when calculating the expected utility in period 2.

Concludes that a flexible strategy not neccesarily implies a more conservative decision; i.e. irreversible decisions may under certain circumstances be taken more often by use of flexible strategies than by use of a fixed strategy.

FREEMAN III, A. MYRICK,
The quasi-option value if irreversible development. Journal of Enviromental Economics and Management, 11,292-295,1984.

Points out that option value is not a one-sided argument for slower development and greater preservation. Option value is related to information gain in uncertain situations, and only occationally is such information obtained by waiting alone. See also Miller and Lad for similar arguments.

7. FIGURE CAPTIONS

Figure 1. Components of the profit function for resource α. a) The price profile. b) The production profile. c) The cost profile. d) Discounted profit rate.

Figure 1. Components of the profit function for resource α.

Figure 2. A profit function for resource α.

Figure 3. A quadratic profit function.

Figure 4. An exponential profit function.

Figure 5. Illustration of the function H(T;r) and the first order condition in cases where learning effects are absent.

Figure 6. The effect of a changed discount rate on the first order condition.

Figure 7. The 'learning time' TL as function of the decay rate γ for various valuer of r.

Figure 8. The first order condition when learning effects are included.

I N N H O L D S F O R T E G N E L S E