Interne notater

STATISTISK SENTRALBYRÅ

83/22 4. august 1983

MACROECONOMIC PLANNING STRATEGIES UNDER UNCERTAINTY OF FUTURE OIL PRICE AND RATES OF RETURN: A DYNAMIC PROGRAMMING APPROACH

by

Iulie Aslaksen and Olav Bjerkholt

CONTENTS

Paper presented to the 11th IFIP Conference on System Modelling an Optimization, Copenhagen, July 25-29, 1983 and to the 10th EARIE Conference, Bergen, August 23-25, 1983.

Abstract. An oil exporting country, paying in oil for a substantial amount of manufactured imports, is vulnerable to changes in the international oil price and should take into account in its national economic planning that oil revenues are highly uncertain. The paper discusses how this uncertainty can be dealt with. A stochastic dynamic approach to national economic planning is introduced in which the uncertainty is connected with the rates of return on various assets. For oil resources the world market price is the source of uncertainty in the rate of return. The optimization framework is based on an assumption of constant absolute risk aversion. The paper discusses how a stochastic optimization framework can be combined with a deterministic multisectoral simulation model used for long-term economic planning in Norway.

1. INTRODUCTION

Norway has become an oil exporting country in recent years. The current level of production of oil and gas amounts to 15 per cent of GDP and 6-8 times the domestic consumption of petroleum. The reserves of oil and gas have been conservatively estimated to be about 100 times the current annual rate of depletion. It is thus within the range of possibilities that Norway will be dependent upon oil and gas production for 20-30 per cent of its GDP for an extended period of time - decades, perhaps even a century.

The Norwegian situation differs from that of most oil-rich countries, because Norway is an industrialized country with a high level of social and economic development. On the other hand the Norwegian position differs from that of the United Kingdom and the Netherlands in the amount of petroleum resources relative to size of economy.

The discovery and exploitation of huge oil and gas reserves have altered the constraints and to some extent also the course of development of the Norwegian economy. In the 1970s the Norwegian economy maintained full employment and accustomed growth rates of domestic demand, while other small open economies were constrained by the sluggish demand for industrial products on the world market. In this period, oil and gas production grew from nil to the current level and oil prices soared. The cost of production of oil and gas amounts to only a small part of the price. The rest is rent.

In the short-run context the rent of oil and gas production is a source of national income. In the long-run perspective the stock of oil and gas in the ground is a part of national wealth - an extraction of an amount of oil and gas represents not income, but only a running down of a large but limited stock. The real source of income connected with petroleum resources is the increase in value of these resources. The rate of return on the stock of petroleum in the ground is the increase in the net price.

Apart from the stock of petroleum and other natural resources, national wealth consists of net foreign reserves (possibly negative) and domestic capital. Petroleum resources differ from most other assets by the higher uncertainty of the rate of return as well as of the amount of resources. This is of great importance in national economic planning for a country in which the oil sector is of substantial magnitude and raises issues such as the proper attitude to risk in economic planning, hedging considerations, etc.

In Norwegian medium and long term economic planning a four-year plan is drawn up every fourth year and at the same time a less detailed "perspective plan" for the following 15-20 years is outlined. (Planning Secretariat, 1981). Integration of the management of petroleum resources in national planning ought to imply an increased emphasis on the long range perspectives, even 20 years is a short horizon in this context, and attention paid to the wealth aspect of petroleum revenues as well as to the uncertainty connected with it.

In this paper we suggest a framework for overall long term economic planning based on optimal management of national wealth under uncertainty of rates of return. First the formal optimization framework is presented, and the model is solved under somewhat simplified assumptions. Secondly, the properties of the solution are discussed, and, finally, we suggest how this framework can be applied in national economic planning. In Aslaksen and Bjerkholt (1983) the same problem is analysed within a continuous time framework while this paper applies a dynamic programming discrete time model.

2. A MODEL FOR OPTIMAL MANAGEMENT OF NATIONAL WEALTH

We assume that the national wealth is distributed over a number of assets - physical and financial assets as well as natural resources. Assets are measured in terms of the purchasing power of consumption goods. The planning horizon is divided into periods of equal length. At the beginning of each period the returns on the various assets are added up and distributed between consumption and accumulation in the same assets. For the decisions to be taken at the beginning of period we have the following budget equation

$$(1) \qquad R_t = C_t + \sum_{i=0}^{n} I_{it}$$

where I_{it} is the investment in asset no. i and C_t is the rate of consumption in period t. All income is assumed to be capital income, accruing from investment undertaken one period earlier, hence

$$(2) \qquad R_t = \sum_{i=0}^{n} r_{it} W_{it-1}$$

where W_{it-1} is the amount of asset no. i invested at the beginning of period t-1 and r_{it} its rate of return. In asset terms the budget equation can be written

$$(3a) \qquad \sum_{i=0}^{n} W_{it-1} + R_t = C_t + \sum_{i=0}^{n} W_{it}$$

or

$$(3b) \qquad G_{t-1} = C_t + \sum_{i=0}^{n} W_{it}$$

where $G_{t-1} = \sum_{i=0}^{n} W_{it-1} + R_t = W_{t-1} + R_t$

Total wealth G_{t-1} at the beginning of period t hence consists of stocks of assets inherited from the past as well as capital income. The rates of return are stochastic variables. We assume that when decisions are to be made at the beginning of period t the outcome of the stochastic rates of return dated t is known with certainty whereas the uncertainty regarding future periods has to be taken into account.

Oil reserves still in the ground can be considered as one type of assets although they are not usually counted as part of national wealth. The value of the oil reserves can be measured as the product of the amount of reserves S_t and the price net of marginal extraction costs, $q_t = p_t - b_t$, where p_t is the current oil price and b_t is marginal extraction cost. We assume that marginal cost is constant with respect to the rate of extraction but is a hyperbolic function of the reserve level. The rate of return on the oil reserves is equal to the rate of growth of the net oil price. Introducing oil as an additional asset in (3) hence gives

$$(4) \qquad W_{t-1} + R_t + q_{t-1}S_{t-1} + \left(\frac{q_t}{q_{t-1}} - 1\right)q_{t-1}S_{t-1} = C_t + W_t + q_tS_t$$

Oil extraction in period t is given by

$$(5) \qquad X_t = S_{t-1} - S_t$$

where the initial level of oil reserves S_o is assumed known with certainty. By netting out oil terms, (4) can be stated as

$$(6) \qquad W_{t-1} + R_t + q_tX_t = C_t + W_t$$

Total wealth G_t and total stock of assets W_t are now redefined to include the oil reserves. The budget equation at the beginning of period t is thus

$$(7) \qquad G_{t-1} = C_t + W_t$$

where $G_{t-1} = \sum_{i=0}^{n} W_{it-1} + R_t + q_tS_{t-1}$

and $W_t = \sum_{i=0}^{n} W_{it} + q_tS_t$

For a small oil exporting country like Norway the oil price is exogenous, independent of domestic reserves and rate of extraction. It may not be so obvious whether the rates of return are independent of the stocks of the respective assets, and whether the stochastic rates of return on assets other than oil are time independent as assumed above.

The planning problem is to maximize the sum of discounted expected utility from consumption over a planning horizon of length T, taking into consideration the discounted utility of terminal wealth. The decision problem at the beginning of each period is deciding on the reinvestment of total wealth and the rate of consumption to be maintained in the period. The results of earlier decisions are represented in period t through the stock of assets inherited from the previous periods. We assume that total wealth unrestricted can be reallocated between assets, i.e. that investment rates may be negative. The decisions to be taken in the following periods up to T have to be taken into account when deciding on consumption and investment at the beginning of period t. Decisions in all periods should reflect an appropriate trade-off between consumption and investment, as well as between consumption in the planning period and terminal wealth.

The objective function at the beginning of period t is

$$(8) \qquad \sum_{\tau=t}^{T} U(C_\tau)(1+\delta)^{t-\tau} + V(G_T)(1+\delta)^{t-T} \qquad t=0,1,\ldots,T$$

where U and V are the utility functions for instantaneous consumption and terminal wealth respectively, and δ is the rate of time preference. The optimization problem can be solved by the method of stochastic dynamic programming, here applied in discrete form. Our approach follows Samuelson (1969) and Chow (1975). Applying the method of dynamic programming, the optimization problem is solved by beginning at the end of the planning horizon and solving the decision problem for each period recursively. At beginning of period T the optimal W_{iT}, S_T and C_T are determined, given the initial condition G_{T-1} and S_{T-1}. Having found the optimal solution for the last period contingent on any initial condition G_{T-1} and S_{T-1}, we solve the two-period problem for the last two periods by choosing the optimal W_{iT-1}, S_{T-1} and C_{T-1}, contingent on the initial condition G_{T-2} and S_{T-2}, and so on. In the last stage the optimal W_{i1}, S_1 and C_1, are determined, given the initial values G_0 and S_0 available at the beginning of period 1. A crucial assumption for the optimality of this procedure is stochastic independence between rates of return, including the oil price, in different periods.

Denote the maximum expected value of (8), contingent on G_{t-1}, by $J_t(G_{t-1})$. The decision problem at the beginning of period t can now be more precisely stated as

$$(9) \qquad J_t(G_{t-1}) = \text{MaxE}\{U(C_t) + \rho J_{t+1}(G_t)\}$$

where the maximization is with respect to W_{it} and S_t and subject to (7). To simplify notation, $\rho = 1/1+\delta$ has been inserted. Before proceeding to the solution procedure, the stochastic assumptions and the specification of the utility function will be introduced.

The stochastic assumptions concerning future oil prices and rates of return are of considerable importance for the optimal solution. We assume that the rates of return are multinormally distributed with expected values r_i^* and variances and covariances σ_{ij}, $i,j=0,\ldots,n$. The oil price is assumed to be normally distributed with expected value p_t^* and variance τ^2. Covariances between the oil price and the rates of return on non-oil assets are given by τ_i, $i=0,\ldots,n$.

For the instantaneous utility function we use the expotential function

$$(10) \qquad U(C_t) = -B\exp(-\beta C_t) \qquad B,\beta>0$$

which implies constant absolute risk aversion. The absolute risk aversion coefficient is given by $-U''/U' = \beta$. For terminal wealth we likewise assume constant absolute risk aversion, i.e

$$(11) \qquad V(G_T) = -G\exp(-\gamma G_T) \qquad G,\gamma>0$$

By the method of dynamic programming we start by solving the maximization problem given by (9) for t=T, i.e.

$$(12) \qquad J_T(G_{T-1}) = \text{MaxE}\{U(C_T) + \rho J_{T+1}(G_T)\}$$

where the maximization is with respect to W_{iT} and S_T and subject to (7). The expectation is contingent on the initial conditions G_{T-1} and S_{T-1} at the beginning of period T. The expected value operator is applied only on the second term since current consumption C_T is known once we have made our decision.

Inserting the objective function given by (8) for t=T in (12) gives

(13) $$J_T(G_{T-1}) \quad = \quad \max\{U(C_T) + EV(G_T)\}$$

The maximal utility of terminal wealth at the end of the planning horizon can thus be interpreted as the maximal expected utility from period T onwards of future consumption and terminal wealth.

We now apply a well-known certainty equivalence result, according to Johansen (1978, 1980). If x is normally distributed and f(x) is an exponential function, $f(x) = -\exp(-\alpha x)$, representing constant absolute risk aversion, then

(14) $$Ef(x) \quad = \quad f(\tilde{x})$$

where $\tilde{x}$, the certainty equivalent of x, is given by

(15) $$\tilde{x} \quad = \quad Ex - \frac{1}{2}\alpha \text{ var } x$$

Applying this certainty equivalence result to (13) gives

(16) $$J_T(G_{T-1}) \quad = \quad \max\{U(C_T) + V(\tilde{G}_T)\}$$

where

$$\tilde{G}_T \quad = \quad EG_T - \frac{1}{2}\gamma \text{ var } G_T$$

$$EG_T \quad = \quad \sum_{i=0}^{n} W_{iT}(1+r_i^*) + q_{T+1}^* S_T$$

$$\text{var } G_T \quad = \quad \sum_{i=0}^{n} \sum_{j=0}^{u} \sigma_{ij} W_{iT} W_{jT} + \tau^2 S_T^2 + 2 \sum_{j=0}^{n} \tau_j W_{jT} S_T$$

Evaluating the terminal value of the oil reserves should take into account future oil price uncertainty beyond the planning horizon. The approach of measuring the terminal value by certainty equivalent net price at the beginning of period T does not capture this future uncertainty. However, the marginal value of the terminal oil reserves is equal to the certainty equivalent net oil price, provided that the terminal level of oil reserves is optimally weighed against consumption throughout the planning period and terminal stocks of non-oil assets.

The first order conditions for the solution of (16) are

$$(17)\quad \begin{cases} \beta U(C_T) = \gamma V(\tilde{G}_T)(1+\tilde{r}_i) & \text{for the non-oil assets} \\ \beta U(C_T) = \gamma V(\tilde{G}_T)\, \tilde{q}_{T+1}/q_T & \text{for the oil asset} \end{cases}$$

where

$$\tilde{r}_i = \frac{\partial \tilde{G}_T}{\partial W_{iT}} = r_i^* - \gamma \sum_{j=0}^{n} \sigma_{ij} W_{jT} - \gamma \tau_i S_T \qquad i=0,\ldots,n$$

and
$$\tilde{q}_{T+1} = \frac{\partial \tilde{G}_T}{\partial S_T} = q_{T+1}^* - \gamma \tau^2 S_T - \gamma \sum_{j=0}^{n} \tau_j W_{jT} - b'(S_T) S_T$$

$\tilde{r}_i$ is the certainty equivalent rate of return on asset no. i, i.e. the marginal increase in certainty equivalent wealth by a marginal increase in asset no. i.

$\tilde{q}_{T+1}$ is the certainty equivalent net oil price. The difference between the certainty equivalent net oil price and the expected net oil price consists of the correction terms due to the uncertainty as well a term due to the dependence of marginal cost on the reserve level. With a hyperbolic marginal cost function, $b(S_T) = m/S_t$, cost function terms in $\tilde{q}_{T+1}$ cancel out, and $\tilde{q}_{T+1}$ appears as

$$\tilde{q}_{T+1} = p_{T+1}^* - \gamma \tau^2 S_T - \gamma \sum_{j=0}^{n} \tau_j W_{jT}$$

To obtain an explicit solution for the optimal portfolio and consumption we make the crucial assumption that asset no. 0 is risk-free, yielding a certain rate of return r_0. Hence, $\tilde{r}_0 = r_0$ and from (17) we get

$$(18)\quad \begin{cases} \tilde{r}_i = r_0 & i=1,\ldots,n \\ \tilde{q}_{T+1}/q_T - 1 = r_0 & \end{cases}$$

Optimal accumulation in the uncertain assets is determined by the condition that certainty equivalent rate of return should be equalized for all assets. Oil extraction is determined by a modified Hotelling rule; certainty equivalent net oil price should grow at a rate of return equal to the certain rate of return.

Substituting (17) and (18) into (16) gives the maximal expected utility at the beginning of period T

$$(19) \qquad J_T(G_{T-1}) \quad = \quad U(C_T^*)(1+\beta/\gamma(1+r_0)) \quad = \quad U(C_T^*)\xi_1$$

where C_T^* is optimal consumption in period T and $\xi_1 = 1 + \beta/\gamma(1+r_0)$

From (19) it is seen that optimal consumption C_T^* can be expressed as a function of total wealth G_{T-1} at the beginning of period T. The explicit solution for optimal consumption C_t^* will be derived from the general solution for $J_t(G_{t-1})$. To realize the recursive nature of the solution, it is elucidating to consider the decision problem for t = T-1 and then derive the general solution for $J_t(G_{t-1})$ by induction. The decision problem at the beginning of period T-1 is

$$(20) \qquad J_{T-1}(G_{T-2}) \quad = \quad \max E\{U(C_{T-1}) + \rho J_T(G_{T-1})\}$$

where the maximization is with respect to W_{iT-1} and S_{T-1} and subject to (7). Observing that J_T is an exponential, we apply the certainty equivalent result to (20)

$$(21) \qquad J_{T-1}(G_{T-2}) \quad = \quad \mathrm{Max}\{U(C_{T-1}) + \rho J_T(\tilde{G}_{T-1})\}$$

However, the appropriate risk aversion coefficient in the certainty equivalent procedure for G_{T-1} is not γ. J_t is an exponential function with time dependent absolute risk aversion coefficient. Differentiating (16) with respect to G_{T-1} and applying the first order condition (17) gives

$$(22) \qquad \frac{dJ_T(G_{T-1})}{dG_{T-1}} \quad = \quad U'(C_T^*)$$

Combining (22) and the solution for J_T given by (19) yields

(23) $$J_T'(G_{T-1}) = U'(C_T^*) = -\beta U(C_T^*) = -\beta/(1+\beta/\gamma(1+r_0)) \cdot J_T(G_{T-1})$$

Hence,

(24) $$\frac{J_T'(G_{T-1})}{J_T(G_{T-1})} = -\beta/(1+\beta/\gamma(1+r_0)) = -\beta/\xi_1$$

The appropriate risk aversion coefficient for $\tilde{G}_{T-1}$ is thus $-\beta/\xi_1$ as given by (24) and we get

$$\tilde{G}_{T-1} = EG_{T-1} - \frac{1}{2}\beta/\xi_1 \text{ var } G_{T-1}$$

The first order conditions for the solution of (21) can hence be stated as

(25) $$\begin{cases} \beta U(C_{T-1}) = \rho\beta/\xi_1 J_T(\tilde{G}_{T-1})(1+r_0) \\ \beta U(C_{T-1}) = \rho\beta/\xi_1 J_T(\tilde{G}_{T-1})\tilde{q}_T/q_{T-1} \end{cases}$$

The solution for J_{T-1} is found by substituting (25) into (21)

(26) $$J_{T-1}(G_{T-2}) = U(C_{T-1}^*)(1+(1+\beta/\xi_1)/(1+r_0) = U(C_{T-1}^*)\xi_2$$

Comparing the solutions for J_T and J_{T-1} the recursiveness of the solution for J_t appears through the coefficient ξ_{T-t}, which is recursively determined by the difference equation.

(27) $$\xi_{T-t} = 1 + \frac{\xi_{T-t-1}}{1+r_0}$$

The solution for ξ_{T-t} is given by

(28) $$\xi_{T-t} = \left(\frac{1}{1+r_o}\right)^{T-t}\left(\beta/\gamma - \frac{1+r_0}{r_0}\right) + \frac{1+r_0}{r_0}$$

with $\xi_0 = \beta/\gamma$

By induction it can be shown that the generalizations of (19), (22) and (24) are

$$(29) \qquad J_t(G_{t-1}) = U(C_t^*)\xi_{T-t+1}$$

$$(30) \qquad J_t'(G_{t-1}) = U'(C_t^*)$$

$$(31) \qquad \frac{J_t'(G_{t-1})}{J_t(G_{t-1})} = -\beta/\xi_{T-t+1}$$

From (31) it is seen that J_t is an exponential

$$(32) \qquad J_t(G_{t-1}) = -H_{T-t+1}\exp(-\beta/\xi_{T-t+1}G_{t-1})$$

The coefficient H_{T-t+1} is determined by inserting (29) in (9) and solving the resulting difference equation for H_{T-t} which yields

$$(33) \qquad H_{T-t} = \xi_{T-t}\exp(\lambda_{T-t})$$

where

$$\lambda_{T-t} = \frac{r_0-\delta^*}{r_0} + \frac{1}{\xi_{T-t}}\left(\frac{1}{1+r_0}\right)^{T-t}\left\{\frac{\beta}{\gamma}\ln\frac{\gamma}{\beta} - \frac{\beta}{\gamma}\frac{r_0-\delta^*}{r_0} + (T-t)(r_0-\delta^*)\left(\frac{\beta}{\gamma} - \frac{1+r_0}{r_0}\right)\right\}$$

and

$$\delta^* = \delta + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(r_i^*-r_0)\hat{\sigma}_{ij} + \left(\frac{q_{t+1}^*}{q_t} - 1 - r_0\right)\sum_{j=1}^{n}(r_j^*-r_0)\hat{\tau}_j + \frac{1}{2}\left(\frac{q_{t+1}^*}{q_t} - 1 - r_0\right)^2\hat{\tau}^2$$

$\hat{\sigma}_{ij}$, $\hat{\tau}_j$ and $\hat{\tau}^2$ are the elements of the inverse of the variance - covariance matrix of σ_{ij}, τ_j and τ^2. As an implication of the certainty equivalence procedure, the stochastic parameters appear only in the risk-adjusted time preference rate δ^*.

We have used the approximation $\ln(\frac{1+r_0}{1+\delta}) \approx r_0 - \delta$, and the coefficients B and G in (10) and (11) are assumed equal to one.

The explicit function relating optimal consumption C_t^* to wealth at the beginning of period t, G_{t-1}, can be derived from the solution for $J_t(G_{t-1})$. Combining (29) and (32) and taking logarithms on both sides yield

$$(34) \qquad c_t = \frac{G_{t-1}}{\xi_{T-t+1}} - \frac{\lambda_{T-t+1}}{\beta}$$

The marginal propensity to consume out of current wealth is the reciprocal of the recursion coefficient ξ_{T-t}. By rewriting ξ_{T-t} given by (28) it is easily seen that

$$(35) \qquad \xi_{T-t} = \beta\left[\frac{1/(1+r_0)^{T-t}}{\gamma} + \frac{1-1/(1+r_0)^{T-t}}{\beta \frac{r_0}{1+r_0}}\right]$$

β/ξ_{T-t} can be interpreted as a weighted harmonic average of the terminal wealth risk aversion coefficient γ and the risk aversion coefficient β adjusted by the term $r_0/1+r_0$. As the time interval from the present date until the planning horizon is increasing, the effect of γ on current consumption is diminishing. In the limiting case where $T-t \rightarrow \infty$, ξ is a constant given by

$$(36) \qquad \xi = \frac{1+r_0}{r_0}$$

In this case the marginal propensity to consume is independent of γ as well as β . However, γ and β appear in the constant term of the consumption function.

When the optimization problen has been solved step by step, optimal consumption is implemented by recording actual development and inserting, period by period, the outcome of the stochastic rates of return, i.e. G_{t-1}, in the consumption function (34). The optimal solution can thus be interpreted as a strategy; decision rules for optimal consumption are calculated initially, whereas actual consumption decisions are postponed until current wealth is known with certainty.

This consumption strategy is consistent with a long-term consumption path given by

$$(37) \qquad C_t = \frac{r_0-\delta}{\beta} t + C_0$$

where C_0 is initial consumption.

The first order conditions for the optimization problem given by (9) combined with (30) gives a relation between marginal utility of consumption in two successive periods,

$$(38) \qquad U'(C_t) = \frac{1+r_0}{1+\delta} U'(C_{t+1})$$

whence the optimal C_t is derived by taking logarithms on both sides and solving the resulting difference equation for C_t.

Given the optimal consumption, the accumulation in the uncertain assets is determined as a one-period portfolio problem.

$$(39) \qquad W_{it} = \frac{\xi_{T-t+1}}{\beta}\{\sum_{j=1}^{n}(r_j^*-r_0)\hat{\sigma}_{ij} + \hat{\tau}_i(p_{t+1}^* - (1+r_0)q_t)\}$$

$$(40) \qquad S_t = \frac{\xi_{T-t+1}}{\beta}\{\sum_{j=1}^{n}(\tau_j^*-r_0)\hat{\tau}_j + \hat{\tau}^2(p_{t+1}^* - (1+r_0)q_t)\}$$

Hence, optimal oil extraction in period t is given by

$$(41) \qquad X_t = S_{t-1} - S_t$$

where S_t is determined by (40) and S_{t-1} is given from the previous period.

Due to the strong assumptions regarding the utility function and the stochastic parameters as well as the production structure and the cost function for oil extraction we have thus obtained explicit solutions with intuitive interpretations.

3. SOME IMPLICATIONS OF THE MODEL

The effect on oil extraction, consumption and accumulation in uncertain non-oil assets of shifts in the parameters is summarized in Table 1. To simplify the discussion of the optimal solution, we have considered the case of time-independent ξ. The restriction

$$(1) \qquad \xi = \frac{1+r_0}{r_0} = \frac{\beta}{\gamma}$$

may either be interpreted as an a priori restriction on the risk aversion coefficients β and γ, or as the limit for ξ_{T-t} when $T-t \to \infty$, i.e. when the planning horizon is a long time ahead. Imposing the restriction (1) simplifies the consumption function considerably, from (34) in section 2 we get

$$(2) \qquad C_t = \frac{r_0}{1+r_0} G_{t-1} - \frac{r_0-\delta^*}{\beta r_0} - \left(\frac{1}{1+r_0}\right)^{T-t} \left\{\frac{1}{\beta} \ln \frac{\gamma}{\beta} - \frac{r_0-\delta^*}{\beta r_0}\right\}$$

or

$$(3) \qquad C_t = \frac{r_0}{1+r_0} G_{t-1} - \frac{r_0-\delta^*}{\beta r_0} + \left(\frac{1}{1+r_0}\right)^{T-t} \frac{r_0-(\delta^*+r_0\ln\frac{\gamma}{\beta})}{\beta r_0}$$

Given the restriction (1), the marginal propensity to consume is constant even when the planning horizon is approached.

Shifts in the stochastic parameters appear through shifts in the risk-adjusted time preference rate δ^*.

Table 1. Effect of Shifts in the Parameters.

	Oil extraction	Accumulation in uncertain asset W_{it}	Consumption $(T-t \to \infty)$
β	+	-	+
γ	+	-	- (0)
δ	0	0	+
r_0	+	-	?
r_j^*	- (+)	+ (-)	+
p_t	+	- (+)	+
p_{t+1}^*	-	+ (-)	+
τ^2	+	?	-

An increase in β or γ reduces the certainty equivalent rate of return on all uncertain assets, including the oil reserves, thereby making accumulation in uncertain assets less attractive. There is an incentive to convert uncertain assets into more safe assets, and oil extraction is increased. However, shifts in β and γ have opposite effect on current consumption. For large T-t an increase in β implies that a higher level of consumption will be aimed at. When future income is uncertain, the assumption of constant absolute risk aversion β implies an incentive to increase current consumption.

The positive effect of β on consumption is diminished as the planning horizon is approached, since the relative importance of γ compared to β is increased.

An increase in γ implies a lower level of consumption throughout the planning period, as the trade-off between consumption and terminal wealth is shifted in favour of terminal wealth. The effect of γ on C_t is less the larger T-t is. In the limiting case of $T-t \to \infty$, γ has no effect on current consumption.

A higher time preference rate δ implies higher consumption throughout the planning period. Oil extraction and accumulation in uncertain assets are independent of δ, since, once optimal consumption is determined, accumulation in the uncertain assets is determined as a one-period portfolio problem.

A higher certain rate of return r_0 reduces the expected gain of holding uncertain assets as compared to accumulation in the certain asset and accumulation will shift in favour of the certain asset. Thus oil extraction will increase. The effect of r_0 on current consumption is ambiguous.

An increase in current oil price p_t provides an incentive to increase oil extraction, whereas an expected increase in future oil price, represented by p^*_{t+1}, gives an incentive to reduce oil extraction and earn a future capital gain on the remaining oil reserves. Whether accumulation in the uncertain non-oil assets will increase or decrease due to a shift in the actual or expected oil price, depends on the covariance structure of the model,

$$\frac{\partial W_{it}}{\partial p^*_{t+1}} \gtrless 0 \quad \text{where } \hat{\tau}_i \lessgtr 0$$

$$\frac{\partial W_{it}}{\partial p_t} \gtrless 0 \quad \text{where } \hat{\tau}_i \gtrless 0$$

An expected increase in future oil price will increase (decrease) accumulation in the uncertain asset W_{it} if the $\hat{\tau}_i$ is negative (positive). This effect illustrates the scope for hedging; future expected declines in the oil price can to a certain extent be safeguarded against by increased accumulation in assets with rates of return negatively correlated with the oil price. The hedging aspect is however taken into account in the optimal solution for W_{it} and X_t, where the trade-off between expected return and safety is given by the assumption of constant risk aversion.

An expected increase in one of the uncertain rates of return r^*_j affects accumulation in uncertain assets W_{it} ($i \neq j$) in the same way as a shift in p^*_{t+1}, i.e. depending on the $\hat{\sigma}_{ij}$'s whereas the effect on oil extraction depends on the $\hat{\tau}_j$ as indicated above.

Due to the wealth effect, a positive shift in any expected rate of return, including future oil price, as well as current oil price, has a positive effect on current consumption.

A larger variance τ^2 of future oil price implies increased oil extraction. As the uncertainty regarding future oil price is larger, there is an incentive to convert the oil reserves into more certain assets. The effect on consumption of a shift i τ^2 is however negative; increased uncertainty affects future wealth and consumption in the same way as a decrease in expected rate of return. However, the assumption of constant τ^2 is very restrictive.

4. INTERPRETATION AND APPLICATION OF THE MODEL FOR USE IN NATIONAL ECONOMIC PLANNING

Uncertainty about the development of exogenous variables is not a new feature in long-term planning situations, but has not been dealt with in a very systematic way. The projected importance of oil and gas revenues is of quite recent origin in the Norwegian economy. The economy has never before been so exposed to variations in one single international price. The need for an explicit consideration of uncertainty issues in the long-term economic planning is thus more strongly felt today than earlier.

The recent turmoil in the international oil market has added emphasis to the uncertainty issue. The latest official long-term programme for the Norwegian economy (Planning Secretariat, 1981) assumed a continued increase in the oil price in the 1981-85 period that soon turned out to be unrealistic. A recent report on the long-range perspectives of the Norwegian economy (Ministry of Finance, 1983) focuses more on uncertainty issues by studying alternative developments with regard to oil price, OECD growth, competitiveness of Norwegian export and import competing industries, etc. A special commission appointed to study the macroeconomic impact of the oil and gas activity in Norway, in general, and to provide guidelines for determining the rate of oil and gas extraction, in particular, has recently submitted its report. Some of the issues discussed by the commission, such as hedging policies or the effect of increased price uncertainty on the rate of extraction, when risk aversion is taken into account could perhaps be more adequately discussed within the framework outlined in this paper when suitably specified and calibrated to provide at least a stylized description of main features of the Norwegian economy. Many important aspects, e.g. the allocation of labour power, are not dealt with at all in the optimization framework outlined above, while others are unrealistically simplified; for instance the instantaneous adjustment of oil extraction to variations in net price or the frictionless conversion from one non-oil asset to another.

The intended application of this framework for use in macroeconomic planning is mainly as a means for evaluating and corroborating the results from the model used in the long-term planning of the Norwegian economy which is a disaggregate (30 industries), deterministic simulation model of the neo-classical equilibrium type called the MSG (Multi-Sectoral Growth) model[1]. The model is supply oriented with exogenously given growth in labour force, capital stock and trends of technological progress. The main

1) The model was originally constructed by Leif Johansen (1960) and has recently been reconstructed and reestimated as MSG-4 (Bjerkholt and others, 1983).

strength of the model is its ability to trace out the long-term equilibrium growth paths of the economy, especially the distribution over industries, the changes in the household consumption pattern, and the development of the equilibrium prices. This model har turned out to be a most useful and valuable tool for providing long-term economic projections and for discussing issues like structural change within a coherent framework. Through relations for producer and consumer behaviour the model takes into consideration important aspects of efficiency and optimality in the economy. Overall optimality, e.g. in the choice of consumption path and accumulation, is left to the planners' discretion and intuitive assessment.

Although stochastic elements are not included in the MSG model, the model is a valuable means for illustrating the wide range of possible long term projections due to alternative oil price assumptions. Model calculations can be performed with alternative oil price scenarios and exogenously stipulated oil and gas production profiles. The choice between domestic use of oil revenues and accumulating foreign reserves is left to the planners' discretion; i.e. the management of the current account surplus is a result of policy decisions. The consequences of alternative oil revenue scenarios are traced out by model calculations. These long-term projections illustrate the considerable impact on sectoral development and accumulated foreign reserves due to alternative oil price assumptions. A consistent evaluation of these long-term equilibrium growth paths in the view of uncertainty and the attitude towards risk requires a stochastic optimization framework as outlined in this paper.

To apply this framework empirically it is necessary to make an assessment of the risk aversion coefficients β and γ as well as of the first and second order moments of the stochastic distributions of uncertain exogenous variables. The risk aversion coefficients are a matter of judgement for the planners and the assumption of constant absolute risk aversion may, of course, not be found acceptable.

In recent MSG-calculations, targets have been stipulated for net foreign reserves at the end of the planning horizon. One aspect of the trade-off between domestic use and net foreign reserves is concerned with the issue of structural change in Norwegian economy (Bjerkholt and others (1981)). Another aspect is the uncertainty of future oil price. Safeguarding against potential oil price declines beyond the planning horizon might imply the conversion of oil reserves into more certain assets. In the present context this aspect is represented by a larger value of the risk aversion coefficient γ. With regard to exogenous variables, such as future international rates of return and oil prices, formal estimation procedures may not be directly applicable and may have to be supplemented by subjective assessments. The first stage of empirical application of this stochastic optimization framework will comprise simulations based on initial values consistent with a MSG reference scenario.

Interpretation of the optimization framework in the context of the MSG model is fairly straightforward. The assets represented in MSG are real capital in each industry and foreign reserves. Industries can be divided into two categories: sheltered industries, mainly services, government administration and some manufacturing sectors, and exposed industries, comprising export-oriented and import-competing industries. Real capital can likewise be divided into sheltered and exposed capital. The capital invested in sheltered industries is our risk-free asset. This may not be so for individual investors, but is not altogether unreasonable from the central economic authority's point of view. The rate of return on the risk-free asset may, of course, not be constant over time as assumed above. Perhaps a better assumption would be a secular downward trend or, with more economic sense, a decreasing function of the amount of sheltered capital. The non-oil assets comprise exposed capital, foreign financial reserves, and foreign investments. Both exposed capital and foreign investments may be subdivided to account for e.g. hedging strategies.

Such model simulations will provide guidelines for optimal oil extraction, accumulation in non-oil assets and consumption under alternative assumptions regarding future oil price. Comparing these optimal planning strategies with calculated MSG long-term growth paths gives an indication as to which extent actual depletion policies and use of oil revenues are optimal in the view of uncertainty and the proper attitude towards risk. Such a comparison will, inter alia, illustrate as to what extent accumulation in non-oil assets is consistent with the equalization of certainty equivalent rates of return, and to what extent oil production is determined by a risk-adjusted Hotelling rule.

Calculations based on the optimization framework can furthermore provide sensitivity analysis as far as shifts in the parameters, like the risk aversion coefficients and the moments of the probability distributions are concerned. Applications of the stochastic optimization framework can thus give guidelines for policy changes due to shifts in expected oil price, degree of uncertainty and risk aversion.

A major shortcoming of the present version of the stochastic optimization framework is the instantaneous adjustment of oil extraction to fluctuations in the projected oil price. Substantial time lags, 5-10 years are involved in off-shore oil field development. This consideration is to some extent taken into account in the MSG model by exogenously stipulating investment in the oil sector as well as the oil extraction profile.

Apart from price fluctuations there are other aspects of uncertainty in connection with oil extraction. The amount of the resource is highly uncertain, and the level of the future extraction costs is dependent upon the amount of the resource as well as the rate of extraction. The latter problem in combination with stochastic oil price is studied by Pindyck (1981). For longer range policy issues it is required to have resource and cost uncertainty included in the model.

Due to the long term character of the MSG-model, policy instruments are not specified in great detail, in contrast to the elaborate specification of the production structure. Hence, the model calculations can be interpreted as strategies; long-term projections calculated initially represent future choices, whereas actual policy decisions in subsequent periods are implemented as to steer the economy towards the projected growth path.

The dynamic programming approach to stochastic optimization as outlined above suggests long-range planning in terms of strategies. Optimal strategies are calculated initially, and policy decisions are implemented at a later stage when the outcome of uncertain variables is known. The optimality of this strategy is contingent on our expectations regarding future oil price and rates of return, i.e. in view of shifts in expected values, we should be prepared to carry out new calculations and elaborate a new optimal strategy. Thus the approach of dynamic programming provides a rationale for rolling planning, which implies revisions of plans, always seeing the first year in the context of a new long-term plan starting that year. In Norwegian medium and long-term economic planning, new calculations are carried out every fourth year based on present estimates for expected values for the important uncertain variables. Even with very simple stochastic assumptions, uncertainty can thus be satisfactorily dealt with in a dynamic programming context.

REFERENCES

Aslaksen, I and O. Bjerkholt (1983). A framework for evaluating planning strategies for an oil exporting country under uncertainty of future oil prices and rates of return. Proceedings of the 4th IFAC/IFORS Conference on the modelling and control of national economies.

Bjerkholt, O., L. Lorentsen and S. Strøm (1981). Using the oil and gas revenues: The Norwegian case. In T. Barker and V. Brailovsky (Eds.), Oil or Industry? Academic press, London. Chap 7, pp. 171-184.

Bjerkholt,o., S. Longva, Ø. Olsen and S. Strøm (Eds.) (1983). Analysis of Supply and Demand of Electricity in the Norwegian Economy. Central Bureau of Statistics, Oslo.

Chow, G.C. (1975). Analysis and Control of Dynamic Economic Systems. John Wiley & Sons, New York. Chap. 8. pp. 197-201.

Chow, G.C. (1979) Optimum control of stochastic differential equation systems. Journal of Economic Dynamics and Control, 143-175.

Johansen, L. (1960). A Multi-Sectoral Study of Economic Growth. North-Holland Publishing Company, Amsterdam.

Johansen, L. (1978). Lectures on Macroeconomic Planning, Part 2. North-Holland Publishing Company, Amsterdam. Chap. 8.

Johansen, L. (1980). Parametric certainty equivalence procedures in decision-making under uncertainty. Zeitschrift für Nationalökonomie, 40, 257-279.

Merton, R.C. (1969). Lifetime portfolio selection under uncertainty: The continuous-time case. Review of Economics and Statistics, LI, 247-257.

Merton, R.C. (1971). Optimum consumption and portfolio rules in a continuous-time model. Journal of Economic Theory, 3, 373-413.

Ministry of Finance (1983). Perspective for the Norwegian economy. Report from a committee (In Norwegian).

NOU (1983). The Future Extent of Petroleum Activities on the Norwegian Continental Shelf. Summary and conclusions of the Report from Commission appointed by the Royal Decree of 5 March 1982.

Pindyck, R.S. (1981). The optimal production of an exhaustible resource when price is exogenous and stochastic. Scandinavian Journal of Economics, 277-288.

Planning Secretariat (1981). Norwegian Long-Term Programme 1982-1985. Report no. 79 to the Storting (1980-81).

Samuelson, P.A. (1969). Lifetime portfolio selection by dynamic stochastic programming. Review of Economics and Statistics, LI, 239-246.