WORKING PAPERS FROM DEPARTMENT FOR STATISTICS ON INDIVIDUALS AND HOUSEHOLDS

ARBEIDSNOTAT FRA AVDELING FOR PERSONSTATISTIKK

POPULATION AND LIVING CONDITIONS

BEFOLKNING OG LEVEKÅR

7/1992

Report from Multidisciplinary Research Conference on Poverty and Distribution
Oslo, November 16–17, 1992

Part 3
Parallel Session 2
Income and Consumption
Distribution and Poverty

CENTRAL BUREAU OF STATISTICS OF NORWAY

Statistisk sentralbyrå

FORORD

I denne serien samles notater innen feltet befolkning og levekår som har krav på en viss allmenn interesse, men som ikke presenterer avsluttede arbeider. Det som presenteres vil ofte være mellomprodukter på vei fram mot en endelig artikkel eller publikasjon, eller andre arbeider som forfatteren eller avdelingen er interessert i en viss spredning av og å få kommentert. Når de er ferdig bearbeidet, vil noen av arbeidene bli publisert i andre sammenhenger.

Synspunktene som presenteres er forfatternes egne, og er ikke nødvendigvis uttrykk for for SSBs oppfatning.

PREFACE

This series contains papers within the field of population and living conditions. The papers are expected to be of some general interest, and presents work in progress, or other notes worth a limited distribution.

The views expressed in this paper are those of the author(s) and do not necessarily reflect the policies of the Central Bureau of Statistics of Norway.

Statistisk sentralbyrå

CENTRAL BUREAU OF STATISTICS OF NORWAY

OSLO
PB 8131 DEP
N-0033 OSLO
TELEFON (02) 86 45 00
FAX (02) 86 49 88
TELEX 11 202 SSB-N
BESØKSADRESSE
SKIPPERGATEN 15

KONGSVINGER
POSTUTTAK
N-2201 KONGSVINGER
TELEFON (066) 85 000
FAX (066) 85 289
BESØKSADRESSE
OTERVEIEN 23

Report

from

Multidisciplinary Research Conference

on

Poverty and distribution

Oslo, November 16-17, 1992

Parallel session 2
Income and consumtion. Distribution and poverty

November 16th and 17th 1992 the Central Bureau of Statistics, Norway arranged a multidisciplinary research conference on poverty and distribution in Oslo.

The aim of the conference was

* to present and discuss various approaches and methods in the study of poverty and distribution,

* to present and discuss results of Norwegian and foreign investigations of the scope of poverty, its distribution and development, its causes and remedies, and

* to identify relevant areas for research on poverty in Norway and other countries.

Researchers from more than twenty countries participated. The conference partly consisted of plenary lectures and discussions, and partly of parallel sessions where individual participants had the opportunity to present and discuss their own papers.

The conference report includes the lectures of the main speakers and the papers presented at the the conference, and consists of seven issues of Working papers from Department for Statistics on Individuals and Households. The first one includes the lectures given in the plenary sessions, while the others includes the papers from each of the parallel sessions:

1 Plenary lectures
2 Paralell session 1. Approaches to the study of poverty. Subjective and objective indicators of poverty.
3 Parallel session 2. Income and consumption. Distribution and poverty.
4 Parallel session 3. Who are the poor? Comparisons between groups and countries.
5 Parallel session 4. Poverty - development and duration.
6 Parallel session 5. The welfare state, distribution policy and poverty.
7 Parallel session 6. Less developed countries: Who are the poor, where are they located and why are they poor?

Report

from

Multidisciplinary Research Conference

on

Poverty and distribution

Oslo, November 16-17, 1992

Parallel session 2
Income and consumtion. Distribution and poverty

November 16th and 17th 1992 the Central Bureau of Statistics, Norway arranged a multidisciplinary research conference on poverty and distribution in Oslo.

The <u>aim</u> of the conference was

* to present and discuss various approaches and methods in the study of poverty and distribution,

* to present and discuss results of Norwegian and foreign investigations of the scope of poverty, its distribution and development, its causes and remedies, and

* to identify relevant areas for research on poverty in Norway and other countries.

Researchers from more than twenty countries participated. The conference partly consisted of plenary lectures and discussions, and partly of parallel sessions where individual participants had the opportunity to present and discuss their own papers.

The conference report includes the lectures of the main speakers and the papers presented at the the conference, and consists of seven issues of Working papers from Department for Statistics on Individuals and Households. The first one includes the lectures given in the plenary sessions, while the others includes the papers from each of the parallel sessions:

1 Plenary lectures
2 Paralell session 1. Approaches to the study of poverty. Subjective and objective indicators of poverty.
3 Parallel session 2. Income and consumption. Distribution and poverty.
4 Parallel session 3. Who are the poor? Comparisons between groups and countries.
5 Parallel session 4. Poverty - development and duration.
6 Parallel session 5. The welfare state, distribution policy and poverty.
7 Parallel session 6. Less developed countries: Who are the poor, where are they located and why are they poor?

Multidisciplinary Research Conference on Poverty and Distribution
Soria Moria Conference Center, Oslo

Programme

November 16th:

10.30 - 10.45	Opening
10.45 - 11.45	Prof. Jonathan Bradshaw, University of York, Britain: Why and how do we study poverty in industrialized western countries. Various approaches to the study of poverty. Lecture and plenary discussion.
11.45 - 12.45	Lunch
12.45 - 13.45	Prof. Bernard M.S. van Praag, Erasmus University, Netherlands: How poor are the poor? Relative and absolute poverty. Subjective and objective indicators of poverty.
13.45 - 14.00	Pause
14.00 - 15.00	Prof.Lee Rainwater, Harvard University USA: Who are the poor? The distribution of poverty. Comparisons between various groups and various countries.
15.00 - 15.15	Pause/coffee
15.15 - 17.15	Parallel sessions with presentations and discussions of contributed papers.
17.15 - 18.15	Prof.Greg Duncan, Ann Arbor, USA: Poverty's development and duration. Panel studies.
19.30	Get-together
20.00	Festive dinner

November 17th:

08.45 - 11.00	Parallel sessions with presentations and discussions of contributed papers.
11.00 - 11.15	Pause/coffee
11.15 - 12.15	Prof.Stein Ringen, University of Oxford, Britain: The welfare state, distribution policies, and poverty. Analyses of measures and policies to combat poverty.
12.15 - 13.15	Lunch
13.15 - 14.30	Presentation of International Research and statistical Programmes on Poverty.
14.30 - 14.45	Pause
14.45 - 15.45	Panel discussion: Challenges and possibilities facing poverty research focusing on data requrements.
15.45 - 16.00	Conclusion and closing led by a representative of the Central Bureau of Statistics.

22. september 1992

RESEARCH CONFERENCE ON POVERTY AND DISTRIBUTION
OSLO, NOVEMBER 16-17, 1992

Parallel session 1
Approaches to the study of poverty. Subjective and objective indicators of poverty.

Session leader: Dr. philos Lars Gulbrandsen, INAS, Norway

Mr. Karel Van den Bosch, UFSIA, Belgium: Poverty and Social Security in Seven Countries and Regions of the E.C.

Prof. John Veit-Wilson, Dept. of Applied Social Science, England: Confusions between Goals and Methods in the Construction & Use of Poverty Lines.

Mr. Arne S. Andersen and mr. Jan Lyngstad, Central Bureau of Statistics, Norway: Payment problems or poverty? Norwegian households 1987 - 1991.

RESEARCH CONFERENCE ON POVERTY AND DISTRIBUTION
OSLO, NOVEMBER 16-17 1992

Parallel session 2.
Income and consumption. Distribution and poverty.

Session leader: Mr. Ib Thomsen, Central Bureau of Statistics, Norway.

Mr. Thor Olav Thoresen, Central Bureau of Statistics, Norway: Child Care Subsidies and Effect on Distribution.

Ms. Hilde Bojer, Department of Economics, University of Oslo, Norway: Gender, occupational status and income inequality in Norway.

Prof. Leif Nordberg and Rec.ass. Markus Jäntti, Åbo Akademi University, Finland: Statistical inference and the measurement of poverty.

Dr. Jolanda van Leeuwen, Erasmus University Rotterdam, The Netherlands: The Leyden Poverty Line when Prices are Income-Dependent. Abstract

Dr. Jørgen Aasness and Ms. Jing Li, Central Bureu of Statistics, Norway: A microsimulation model of consumer behavior for tax analysis. Abstract

Mr. Ib Thomsen and Mr. Dinh Quang Pham, Central Bureau of Statistics, Norway: An application of latent Markov models to estimate response errors from repeated surveys.

RESEARCH CONFERENCE ON POVERTY AND DISTRIBUTION
OSLO, NOVEMBER 16-17 1992

Parallel session 3.
Who are the poor? Comparisons between groups and countries.

Session leader: Ms. Gunvor Iversen, Central Bureau of Statistics, Norway.

Dr. A. Jan Kutylowski, Poland: Distribution of subjective income deprivation in Poland 1981 -1990.

Ms. Iulie Aslaksen, Central Bureau of Statistics, Norway and ms. Charlotte Koren, INAS, Norway: A women's perspective on poverty: Time use, income distribution and social welfare.

Dr. Björn Gustafsson, Göteborg University, Sweden and Dr. Ludmilla Nivorzhkina, Rostov University, Russia: Relative Poverty in two egalitarian societies. A comparison between Taganrog, Russia during the Soviet era and Sweden.

Mr. Lars B. Kristoffersen, NIBR, Norway: Social Indicators of Child Poverty.

Ms. Randi Kjeldstad, Central Bureau of Statistics, Norway: Pre valence and Change in Low Income among Male and Female Singles and Lone Parents in Norway through the Nineteen Eighties.

Mr. Børge Strand, Central Bureau of Statistics, Norway: Regional location of Poverty in Norway.

Dr. Hans de Kruijk, Erasmus University, The Netherlands: Location of poverty in Pakistan.

RESEARCH CONFERENCE ON POVERTY AND DISTRIBUTION
OSLO, NOVEMBER 16-17 1992

Parallel session 4.
Poverty - development and duration.

Session leader: Dr. Kari Skrede, INAS, Norway.

Dr. R. Muffels, Tilburg University, The Netherlands: The Evolution of poverty according to objective and subjective standards.

Mr. Kjell Jansson, Statistiska Centralbyrån, Ørebro, Sweden: Low income per year is not enough to measure poverty.

Prof. Dr. Bea Cantillon, UFSIA, Belgium: The "zero-sum crisis": the stability in the distribution of income and welfare in a period of economic crisis.

Mr. Jon Epland and Mr. Leif Korbøl, Central Bureau of Statistics, Norway: Duration of Poverty in Norway in the 1980s. Some longitudinal results from the Norwegian socio-economic panel (NSP)

RESEARCH CONFERENCE ON POVERTY AND DISTRIBUTION
OSLO, NOVEMBER 16-17 1992

Parallel session 5.
The welfare state, distribution policy and poverty.

Session leader: Mr. Knut Halvorsen, NKSH, Norway.

Dr. Ivar Lødemel, FAFO, Norway: European Poverty Regimes.

Dr. Jørgen Elm Larsen, The Danish Equal Status Council, Denmark: Poverty debate and poverty research in Denmark.

Mr. Tapio Salonen, Sosialhögskolan, Sweden: Social assistance in a longitudinal perspective.

Mr. Sven-Åke Stenberg, Swedish Institute for Social Research, Sweden: Welfare Dependence in the Welfare State: A Cross-Generational Study in Post-War Sweden.

Dr. Lutz Leisering and Dr. Wolfgang Voges, Bremen University, Germany: Poverty produced by the welfare state. An application of longitudinal analysis.

Mr. Peter Whitesford, University of York, United Kingdom: Assessing the Impact of Anti-Poverty Policies: - the Australian Experience

RESEARCH CONFERENCE ON POVERTY AND DISTRIBUTION
OSLO, NOVEMBER 16-17, 1992

Parallel session 6.
Less developed countries: Who are the poor, where are they located and why are they poor ?

Session leader: Mr. Bjørn K. Wold, SSB, Norway

Mr. Mohamed Ould Abba, Ministry of Plan, Mr. Sidna Ould N'Dah, National Statistical Office, Mauretania: Le Profil de la Pauvrete en Mauretanie: Questions Conceptuelles, Instruments et Principaux Resultats.

Mr. William Bender and Mr. Simon Hunt, Ministry of Plan, Luanda, UNICEF, Luanda, Food Studies Group, University of Oxford, Angola & Great Britain: Poverty and Food Insecurity in Luanda.

Mr. Christian Grootaert, World Bank, USA: The evolution of welfare and poverty during structural change and economic recession - the case of Cote d'Ivoire 1985-88.

Mr. Wilson Mazimba and Mr. Emmanuel Silanda, Central Statistical Office, Zambia: Some indicators of poverty in Zambia.

Mr. Sidna Ould N'Dah, National Statistical Office, Mauretania: Enquete Permanente sur les Conditions de Vie des Menages en Mauretanie.

Mr. Jeannot Ngbanza and Mr. Perkyss Mbayndoudjim, ECAM, Bangui, Central African Republic: Mesure de la Pauvrete: Les Travaux en Cours en Republique Centrafricaine.

Child Care Subsidies and Inequality*)

Thor Olav Thoresen
Central Bureau of Statistics, Norway

Abstract

In Norway spaces at child care centers are rationed. This paper examines the impact of child care subsidies on income distribution and inequality in Norway in 1990 based on data from the Income and Wealth Survey in combination with the Survey of Level of Living. Two methods of measuring the income to households from subsidies are applied, which both rest on rather strong assumptions and simplifications. Our results indicate that more child care subsidies are transferred to well-off households. Among households with preschool children the child care subsidies make a substantial contribution to inequality. The results are however preliminary and the approach must be regarded as a first crude step towards a more comprehensive evaluation of the subsidies.

1 Introduction

Studies of welfare bring up difficult problems touching upon fundamental questions in economics. The welfare in a society is assumed to depend on the individual welfare of households and the distribution among households. One approach to the interpersonal comparability issue involves the assumption that individuals with the same observed characteristics, such as income, can be deemed to have the same level of welfare (Atkinson and Stiglitz 1980). Income can be defined in various ways. The welfare for households with children is affected both by money income and the cost and quality of child care.

In many countries the government uses various welfare policy instruments to improve the well-being of

*) Preliminary version. A version of this paper was presented at the Research Conference on Poverty and Distribution in Oslo, November 16-17, 1992. I benefitted from suggestions by Rolf Aaberge and Olav Ljones. Author's address: Research Department, Division for Public Economics, Central Bureau of Statistics, P.O. 8131 Dep., N-0033 Oslo. Telephone: (+47-2)864500

households. A large portion of governmental expenditures is in-kind transfers to individuals and households which justifies the inclusion of in-kind transfers in the definition of income. Several studies have shown a large decline in the number of people living in poverty when in-kind benefits have been included in the definition of income (e.g. Smeeding 1977). When trying to determine the effects of in-kind benefits on the distribution of economic well-being, one faces difficulties. First one has to identify the distribution of benefits across the population. Then there is the valuation-problem, how to determine the value of benefits to recipients (Smeeding and Moon 1980).

In this paper we will estimate some effects of child care subsidies on the distribution of income. Since in Norway there is no established tradition for poverty lines, this approach is chosen instead of measuring the proportion under the poverty line. The study can easily be extended in this direction.

In Norway, approximately 39 per cent of all children aged 0-6 years attended child care centers by the end of 1991 (Central Bureau of Statistics 1991). In spite of governmental programs, for many years aiming at increasing the supply of child care services, there is still an excess demand for child care services. The number of slots available in kindergartens is not adequate to meet the demand for nonparental care at existing subsidized rates. We examine the effect of subsidizing child care centers (kindergartens), but the distinction between the subsidized and the unsubsidized child care market is not clear cut. Recently there has been a growing number of childminders receiving public subsidies. They take care of 3-5 children, their services are approved by the local authorities and they receive pedagogical guidance from a teacher. The term child care center also includes related, subsidized services like care in the homes of regulated public caregivers.

Most child care centers are either owned by the municipality or by private organisations. For all kindergartens the general principle is that expenditures are financed from three sources:

- government transfers (not included in the general transfer to municipalities)
- municipal transfers
- parental payments

Nearly all child care institutions are subsidized by the government. All kindergartens owned by the municipalities will receive support from local governments, and some of the private child care institutions are subsidized by the municipalities too. Others are financed through governmental subsidies and the parental fee only. The governmental transfer is calculated per child and is dependent on age of the child.

Laws and regulations entail higher costs on smaller children. The government's objective is to cover about 40 per cent of the operational expenditures and let parents and local authorities share the remaining 60 per cent equally (Forbruker- og administrasjonsdepartementet 1987).

The households have to apply for access to institutional day care for their children. The criteria for selection varies dependent on ownership and among municipalities. The municipalities usually formulate the criteria for the child care service which they partially finance and will generally attach importance to social matters. The privately owned centers (which are subsidized by governmental transfers) state their own rules for selection.

In Norway the market for child care is rationed and on average the fees cover only a portion of the actual expenses. In addition some of the households' expenses are tax-deductible. The welfare effects of child care centers come from the fact that the parents get access to a pedagogical, reliable service for their preschoolers while they are able to consume more leisure, do more hours of household work or supply more paid work. As a consequence there has been a focus on the unfairness of public support to child care. Large differences between municipalities regarding the probability of having access to a child care institution are observed. There are also substantial differences between municipalities in how much of the expenses that are covered by the payments from parents. On average the child care fees cover about 35 per cent of the actual expenses connected with institutional child care (Ergoplan 1992).

Studies on the distribution of child care in Norway have shown that workers and parents without higher education have lower use of center-based care than more highly educated and wealthy parents (see e.g. Herigstad 1986, Gulbrandsen & Ulstrup Tønnessen 1988, Kristiansen 1989). There may be several explanations of this fact. One reason might be that people have different ability to handle the bureaucratic process of applying for child care and therefore have different search-costs.

To formalize the discussion we introduce a conceptual framework of households' choices, based on a standard theory of labour supply (see e.g. Connelly 1992, Leibowitz, Klerman and Waite 1992, Gustafsson and Stafford 1992). Uncertainty is ignored. The utility function for the decisionmaker (for simplicity assumed to be the mother) is

(1) $$U = U[X,L,C]$$

where X is consumption of a composite commodity, L is leisure and C is total child well-being. The household face a budget constraint

(2) $$Y_h + Y_n + t(w-p) = X$$

where Y_h is husband's income, Y_n is nonlabour income, t is wife's hours of work, p is price of child care per hour and w is wage rate after tax. The price of consumption goods is normalized to unity.

The household's decision-problem is then to determine consumption, the wife's labour supply and child care arrangements. The ratio between the marginal utility of work and the marginal utility of consumption defines the shadow-value of leisure. The wife's shadow-value of leisure, faced with a wage rate net of child care costs, at the point where she is indifferent whether to work or not, defines the "reservation wage".

(3) $$s = s(Y_h,Y_n,C)$$

Expression (3) states that the mother's reservation wage is affected by the children's well-being, husband's income and nonlabour income. In reality the reservation wage is a function of numerous variables.

It follows that access to child care will imply a lower reservation wage if the service is considered to be of higher quality than other modes of care. Labour supply will therefore be increasing in the quality of child care. Due to the rationing of spaces at child care centers, parents might choose to take care of the children themselves when access to the high standard service of child care centers is not available. These households renounce income in order to undertake child care at home. Hence the correlation between income and access to child care centers is caused by the fact that day care generally results in a larger income, because both parents (or single-parent families) are able to do paid work. For the parents who choose to take care of the preschoolers themselves, there are no acceptable substitute-services. This means that other modes of care like grand parents care, childminders, etc. is considered to be inappropriate, or is impossible for other reasons. However, Heckman (1974), in his pioneering study in this field, recognized that a majority of working women with young children used informal methods of child care, like family members or other relatives.

In addition, the mother's labour supply will be increasing in her own net wage. Child care expenses can be regarded as entry cost to the labour market. Free or low cost care, due to large subsidies, will make work more attractive. Analogously, women facing greater child care subsidies through the tax system would more likely work.

It is clear from this model that being at work implies that wages exceed child care costs. One would expect that lower wage women will be greater users of informal child care, like grand parents care, etc. If such low cost care were not available to them, they would not work at all. The relatively high incidence of subsidized child care among well-off households are then caused by other households' decision not to supply labour in the market because of low wages or (and) negative effects of child care costs. We would expect that the attitude to kindergartens versus care for own children will vary among different social and educational groups.

In addition to the questions of inequality, caused by the fact that only some households receive child care subsidies and thereby have the possibility of doing paid work, there are considerable efficiency-aspects related to the question of supply and distribution of child care. Increasing the number of child care centers may have profound effects on women's labour supply. The female labour market participation in Norway has increased during the eighties, especially among women with young children (Kjeldstad 1991), which is in accordance with trends in other countries (see e.g. Hofferth and Phillips 1987). Of course, it can be argued that the real national income is increased by higher female rate of employment, mainly because the production of care within the household does not influence the national income (Waring 1989).

2 Approaches to Valuation of In-kind Subsidies

No single measure of the value of in-kind subsidies is adequate for all applications. This part of the paper will introduce some concepts in studies of measurement and valuation of in-kind subsidies (Smeeding 1984) and examine how child care expenses fit in with the different approaches.

a) Market Value

The market value of an in-kind subsidy is equal to the private market cost or the purchasing power of benefits received by the individual or the household. The market value is intuitively attractive to economists, relatively easy to estimate in many cases, and is the measure most often used in studies of the value and distribution of in-kind transfer benefits.

The market value of child care institutions is difficult to calculate for several reasons. First, there are very few child care institutions operating without public support. Most kindergartens receive subsidies from the government and many get subsidies from the local authorities as well or are owned by them. Due to the current rationing of spaces at child care centers, the market price for access to child care centers is most likely to exceed the private cost (or the marginal utility of child care spaces measured in money exceed the costs). But this market price is only hypothetical. One has to apply for access to this service and spaces in public child care can not be resold. Since the right to trade child care allowances doesn't exist and a completely private market is almost absent, the market value is very difficult to estimate.

Next, child care institutions do not provide a homogeneous kind of service. There are differences according to opening time, how many adults per child, the staff's pedagogical abilities, etc. Heterogeneity in quality is often alleged to be an important factor confronting parents in making arrangements for the care of their young children (Hofferth and Wissoker 1992). Even if there had been a well developed private market for child care institutions, it might not have given any guidance to the value of subsidies, because of differing ambitions in the private and the public sector.

b) Government Cost

Government cost includes the cost of benefits provided, costs covered by subsidies from the state and local authorities plus the associated economic costs of provision and program management. Government cost is normally the proper measure to determine net changes in budget outlays resulting from a given change in program rules and regulations.

Government cost may also be compared to the market value of in-kind subsidies to determine the net efficiency cost or efficiency benefits from the form in which the transfer is provided. Most likely, the significance of efficiency loss or gain in the case of child care centers are rather limited. For instance, the costs in child care centers owned by local authorities and private (but subsidized) child care institutions are basically equal (Ergoplan 1992).

c) Social Benefit Value

When estimating the total welfare effects of a transfer program, the concept of social benefit value offers a plausible approach. The social benefit value includes consumption and production externalities and other efficiency and equity benefits accruing to taxpayers who finance the subsidies, as well as benefits to the program recipient net of recipient charges. Child care subsidies will, for instance, give both parents the opportunity to supply labour. A larger labour force will in turn increase the society's production.

Obviously, social benefit values are difficult to estimate. We will turn to two different concepts closely related to the social benefit value. When introducing the social benefit value, the cash equivalent value and nonrecipient benefits, we are moving towards approaches which include indirect effects of public subsidies.

d) Recipient or Cash Equivalent Value

The recipient value reflects the recipients' own valuation of the benefit. The amount of cash transfer that would leave the recipient at the same level of well-being or utility as the in-kind subsidy is referred to as the "Hicksian equivalent variation". If the beneficiary would have chosen the same consumption pattern with a cash-equivalent transfer as without, the utility approach provides the same estimate as the market value approach. When assessing distributional effects of in-kind subsidies, the cash equivalent value is useful because it translates the market value of goods into cash values conceptually equivalent to the money income to which they are added (Smeeding and Moon 1980). An accurate assessment of the cash equivalent value involves assigning a utility function to the subsidy recipients.

Smeeding (1984) estimates the cash equivalent value, without assigning utility functions, by assuming that the recipient value of an in-kind transfer is equal to the normal expenditure revealed by unsubsidized consumer units. Subsidized units were matched by unsubsidized units with similar characteristics. If the similar nonrecipient normally spent less than the market value of the in-kind benefit, the recipient value was measured by the level of normal expenditures. If normal expenditures exceeded the market value, the recipient value equalled the market value.

Lacking suitable estimates for the market value of center-based care, this approach is difficult to apply in the case of child care. Intuitively, the cash equivalent value of day care subsidies is not very far from the market value. Some parents may want to spend more time with their children at the expense of paid work, but the absence of flexibility in the labour market might prevent such an allocation. Then the main problem is rigidities in the labour market, not the size of the child care subsidy.

e) Nonrecipient Benefits

When the cash equivalent value is less than the market value and the government cost of in-kind subsidies, which is probable in most cases, the term nonrecipient benefits represents the difference. The more "expensive" in-kind subsidies can only be justified by the existence of benefits for nonrecipient taxpayers and policymakers from in-kind subsidies. For instance, subsidizing the child care for a child with drug addicted parents would save the child from suffering and give a better start in life which might prevent the dependence on future social transfers.

Ideally, one would like to distribute the difference between the social benefit value of the in-kind subsidy and the cost of a cash transfer program to nonrecipient beneficiaries. Social benefit value is, as mentioned earlier, not easily estimated, so total benefits are usually assumed to be at least as great as the government cost and nonrecipient benefits are derived by taking the difference between government cost and the lower-cost cash subsidy.

In addition to the "donor benefits" caused by individual welfare interdependency, in-kind subsidies may have efficiency advantages compared to an equal-cost cash transfer program. Murray (1980) has shown that means-tested in-kind transfers lead to a smaller labour supply reduction than equal-cost cash transfers. Subsidizing child care programs may in fact increase labour supply relative to an equal-cost cash transfer program. Smeeding (1984) argues that all taxpayers receive nonrecipient benefits in proportion to federal income tax payments. In this sense nonrecipient benefits can be treated as a public good.

The discussion so far has introduced a number of concepts and methods in analyzing the impact of in-kind subsidies. Obviously, it is of great importance to include external effects in studies of in-kind subsidies. When estimating the impact of subsidized child care, it is highly important to be aware of the substantial influence on female labour supply (see e.g. Gustafsson and Stafford 1992). However, it is beyond the scope of this study to make a comprehensive presentation of consequences on society of the existence of child care institutions. This paper is a first step toward assessing distributional impacts of child care subsidies. The focus will be on the direct effects of subsidizing institutional child care: namely, to see how the subsidies influence the degree of inequality in a distribution, without incorporating any behavioural changes and externalities. This means that the government cost approach is chosen in this study.

3 Measurement of Subsidies

As interpreted above, there are substantial differences between the households' payment and the actual value of the service received. The previous section outlined the conceptual basis for valuing in-kind benefits. By adding the subsidies to the households' disposable income in accordance with the actual expenses, we will examine the impact of child care subsidies on the distribution of income.

Disposable income is defined as gross income minus income taxes and is calculated by the Norwegian microsimulation model LOTTE. The database or the model population in the model is derived from the latest yearly Income and Wealth Survey, at present from 1990. Data is collected from income tax returns, social security registers, other administrative registers and by interviews. The Income and Wealth Survey does not contain any information about the households' consumption of institutional child care. However, a subsample of the Income and Wealth Survey sample is included in the Survey of Level of Living. In the Survey of Level of Living the households are asked about what kind of child care their children attend, for how many hours per week and their expenses on child care per month.

By the end of 1990 about 35 per cent of all children under 7 years were accommodated in child care institutions, not including the children attending educational programs for 6 years old. Only about 29,5 per cent of the preschool children in the Survey of Level of Living attend child care centers. This poor coverage in the sample is due to a partial nonresponse (nonrespondents in this context are persons not responding on questions about child care, but answering other questions in the survey). Withdrawing the nonrespondents in the denominator, the coverage in the sample is raised to almost 33 per cent. Neither do all respondents give satisfactory information about expenses and hours of child care per week. As a consequence one has to be somewhat modest when drawing conclusions from this material.

It is not easy to give precise estimates of the significance of child care subsidies. On average the parents cover about 35 per cent of the child care expenses, but the price differs from one municipality to another and even for a given quality standard there are substantial price differences (Statistisk sentralbyrå 1992). To illustrate, in 1992, for a family with an income of 250 000 Norwegian kroner (Nkr), the monthly parental fee in Oslo is 2 680 Nkr on average, whereas in the county of Finnmark the corresponding fee is 1 774 Nkr. There are no laws or regulations setting a limit to the parental fee (Barne- og familiedepartementet 1992).

In a survey of the parental fee in 109 Norwegian municipalities it was observed that about 50 per cent of the municipalities charge parental fee subject to family-income (Statistisk sentralbyrå 1992). Most local governments give a discount if more than one family member attend child care centers. Hence it follows that calculating subsidies on the basis of the parental fee is problematic. The size of the public subsidy depends inter alia on geographical location, income, number of family members attending child care, the age of the children and ownership of the center (private or public). The parents' payments reflect the size of the subsidy only to a limited extent. A rather small parental fee might indicate that the parents are paying a small part of the expenses themselves because of low income, but might also reflect that the child is attending the service in part-time manner during the week.

Table 1 describes how the expenses on child care are distributed in the official child care statistics (Central Bureau of Statistics 1991) compared to the Survey of Level of Living.

The table shows that there are remarkably many child care spaces which are free (without parental fee) in the Survey of Level of Living and the lack of consistence between the two statistics might lead to substantial weaknesses when calculating the subsidies. As a consequence of the weaknesses in the data set and due to the discussion above, the present study employs two different methods of estimating the subsidies. In one of the methods the point of departure is the monthly child care costs as stated by the households in the Survey of Level of Living. The other method rests on an estimate of the total expenses on child care in Norway in 1990. Based on these estimates the subsidies are allocated according to averages within groups of households. In the Appendix the estimation procedures are outlined in more detail.

It must be emphasised that the methods of estimating the subsidies are insufficient. It has not been possible to include all elements which affect the size of the subsidy. For instance, it is worth noting that it is not possible to distinguish between the children who attend the privately owned child care centers and the children attending centers run by municipalities. In general, day care centers owned by municipalities are receiving larger subsidies than the private centers. There are reasons to believe that more well-off households make use of the private part of the service. If that is the case, the subsidies as calculated by method 1 and 2, tend to overestimate the subsidies received by richer households.

In addition, we are not able to separate child care subsidies to households with small children. Spaces for children younger than 3 years are more expensive than other spaces. However, the overall average subsidy rate includes both services, the work intensive service for small children and the less costly day care for older preschoolers.

4 Household Welfare Comparisons

It is usually assumed that larger households need more income than a small household to reach the same welfare level. One way to take the size of the household into account is to use household income per capita as a welfare indicator. This method ignores economies of scale in producing and consuming household goods and services. Equivalence scales are designed by taking into account the main household characteristics which do affect needs.

In addition to a whole range of different methodologies for setting an equivalence scale, it is also questionable whether one should take household characteristics into account at all when comparing different households. Pollak and Wales (1979) point to the fact that demographic variables also represent values to the household. Is it fair to compensate a couple with one child at the expense of an involuntarily childless couple to give them an equal level of welfare?

In spite of this serious objection and other objections to the use of equivalence scales, we employ equivalence scales to examine how a correction of the household composition may affect the results. The OECD scale is based on a normative approach and equals 1 for the first adult, 0.7 for each additional adult and 0.5 for each child (younger than 14).

The welfare measurement scale, which we also have made use of, is a subjective scale to measure directly the utility associated with particular income levels for families of given characteristics. This method uses survey questions to measure the welfare level of a household and allows for the possibility that having children yield positive utility to the parents (Hagenaars 1991, originally from a study by Van Praag, Hagenaars and Van Weeren (1982) on Dutch data). Table 2 shows the equivalence scales.

5 Measurement and Decomposition of Inequality

In order to evaluate how child care subsidies affect the distribution of income, we will employ the Gini coefficient. Given the inequality in a distribution function measured by the Gini coefficient (G), the next step is to identify the sources that make substantial contribution to the inequality. Assume that the income, *D*, is the sum of different factor components. Among them are the in-kind child care subsidy and all the components which constitute disposable income.

As demonstrated by Rao (1969) the Gini coefficient admits the following decomposition

$$G = \frac{\mu_d}{\mu}\gamma_d + \frac{\mu_s}{\mu}\gamma_s \tag{4}$$

where μ_d is the mean of disposable income, μ_s is the mean of the child care subsidy and μ is the mean of D. γ_d and γ_s are concentration coefficients which can be interpreted as the conditional G-inequality of factor d and s respectively, given the units rank order according to D (Aaberge, Chen, Li and Li 1992).

Notice that γ_d and γ_s are measures of correlation between disposable income and D and the correlation between the subsidy and D, respectively. Assuming that μ_s is positive, then a negative value of γ_s expresses negative correlation between the in-kind child care subsidy and D and means that the subsidy has an equalizing effect on the inequality in the distribution. In this case the poorest part of the population receives an average transfer which is larger than the average transfer to the whole population. To have a neutral effect on the distribution, the households receive an equal amount of the child care subsidy. Then γ_s is equal to 0.

This interpretation of the influence from the different income components is based on a simultaneous examination. Be aware that the marginal effect on G from a small increase of an income factor, depends on the size of the concentration coefficient compared to the overall inequality. The overall inequality will increase (decrease) if and only if the concentration coefficient (concentration coefficients are assumed fixed) of the income factor is larger (smaller) than the overall inequality (Aaberge, Chen, Li and Li 1992).

6 Child Care Subsidies and Inequality

We have estimated the impact of child care subsidies in 1990. Child care subsidies imply a major transfer to the households with children attending child care centers as they receive about 28 000 Nkr (method 1) or 29 500 Nkr (method 2) per year in subsidies on average. This constitutes more than 10 per cent of the average disposable income. But the subsidies are unevenly distributed. A few households are in fact paying more than the actual costs for their children's child care (calculations done by method 1, method 2 gives positive subsidies to all households).

In calculating the in-kind child care subsidies and adding them to disposable income, we introduce two new income concepts, D_1 and D_2. The D_1 income concept consists of disposable income and the child care subsidy as calculated with method 1. D_2 consists of disposable income and the child care subsidy as calculated with method 2 (see Appendix). Table 3 and table 4 give estimates of Gini coefficients for disposable income, D_1 and D_2. The tables also give estimates of inequality when taking the size of the households into account. Table 3 provides Gini coefficients for distributions of disposable income for households with at least one preschool child, in table 4 the population is restricted to households receiving child care subsidies. As explained above (section 3) our methods do not distinguish between children attending public child care centers and preschoolers in privately owned centers. When restricting to the smaller population of households receiving child care subsidies, the results become very sensitive to an possible over-estimation of subsidies transferred to well-off households. The same weaknesses in methods of calculating subsidies also apply to the estimations for households with at least one preschool child, but are of less importance as these calculations also include households not receiving subsidies.

Including child care subsidies in the income concept in table 3 does not make any large alteration in the degree of inequality, which means that child care subsidies are distributed about as (un)evenly as disposable income. When taking uncertainty into account there are no significant differences between the distributions of D_1 and D_2 and disposable income. The two methods of calculating subsidies (method 1 and method 2) give similar results.

The large standard deviations in table 4 are reflecting the small number of observations as bases for this analysis. However, the overall impression from the table is that the new income concepts might imply less inequality.

The results from table 3 and table 4 indicate that the inclusion of child care subsidies in the income concept does not increase inequality, but this does not imply that child care subsidies are not prorich. Dividing with G on both sides of (1) above give us

$$(5) \qquad 1 = \frac{\mu_d}{\mu}\frac{\gamma_d}{G} + \frac{\mu_s}{\mu}\frac{\gamma_s}{G}$$

μ_d/μ is the fraction of total income. The expression (5) states that the fraction of overall inequality is the fraction of total income multiplied by the concentration coefficient and divided by G (the overall inequality).

Table 5 displays the results of the decomposition of the Gini coefficient. As in table 3 and 4 we are employing equivalence scales. Income factor 1 is disposable income, while income factor 2 is child care subsidies in the tables.

The positive concentration coefficients in table 5 demonstrate that both disposable income and child care subsidies have a disequalizing effect on the distributions of income (D_1 and D_2). Table 5 shows that the contribution to inequality from child care subsidies is quite substantial. Calculations performed using equivalence scales do not differ considerably from the other calculations.

Table 6 shows positive concentration coefficients also when restricting to households receiving child care subsidies. We assume that this conclusion is sensitive to improvements in child care data, which makes it possible to correct the methods of calculating subsidies.

Note that this interpretation of the concentration coefficients is based on a simultaneous examination of the influence from the different factor components on the overall inequality (Aaberge, Chen, Li and Li 1992). In this context the child care subsidies contribute to inequality.

Table 7 confirms the results from table 5. In table 7 D_1 and D_2 are decomposed in deciles of income for households with preschool children. Column 2 in the table reflects that the 10 per cent richest households receive more subsidies than the 10 per cent poorest households. Table 7 also documents the accordance between the results from the two methods of calculating subsidies.

To sum up, the results are indicating that child care subsidies on average are transferred to quite well-off households, when studying the whole group of households with preschoolers. Other studies have shown similar effects. Above it was emphasised that the subsidies itself may have given larger income through increased labour supply from the household.

When restricting to households which are receiving child care subsidies, table 6 indicates that child care subsidies still contribute to inequality. One might believe that the lower income households are favoured by the income dependent parental fee and receives larger subsidies due to smaller parental fees. Our results might indicate that children of households in the lower income deciles have a larger probability to attend the service in part-time manner compared to children of well-off households. The subsidization increases with larger quantities of the service consumed. However, it must be stressed that the estimations are based on a small number of observations and the methods of estimation are poor, due to insufficiencies in data.

7 Summary

In this paper we have added child care subsidies to disposable income to examine the impact of in-kind child care subsidies on inequality. By doing this we are aware that our choice of income concept will never represent a true reflection of all income components. Income is not the only welfare measure and there are other in-kind transfers which have a substantial influence on households well-being, for example. It is further questionable if and how one should take household size into account in studies of welfare. In spite of these objections, it is of interest to study and to give a description of how the welfare system contribute to the level of living.

The focus in this paper has been on the impact of child care subsidies on inequality. By presenting a framework of the households' decision to supply labour and by introducing several methods and concepts related to the problem of estimating in-kind subsidies, it becomes clear that the methods in use are only focusing on the direct effects of in-kind subsidies. It is reasonable to believe that child care subsidies have large external effects and behaviourial changes are probably contributing to the inequality which we are observing. This raises several questions: Do child care subsidies contribute to the inequality in disposable income through a positive correlation between availability of institutional child care and labour supply, are the households in lower income deciles restricted by the wage (net of child care costs) or do the allocation mechanism of child care itself act in favour of well-off households. A more comprehensive study of child care must bring these questions into consideration.

Two methods of calculating the direct effects of child care subsidies has been presented, both suffering from weaknesses. However, when calculating the distribution of in-kind subsidies in a population, one will always have to make simplifying assumptions. Finally, substantial deficiencies in the child care data has been documented, and a larger sample would have reduced the uncertainty.

The results in this study indicate that child care subsidies favour relatively well-off households. Among households with preschool children the child care subsidies make a substantial contribution to inequality in the distributions of the new income concepts. It must be stressed that this conclusion rests heavily on the methods employed to estimate the subsidies. Further improvements in data and further developments in approaches and methods to value in-kind subsidies will test the validity of these results.

APPENDIX: Further Description of Methods for Calculating Child Care Subsidies

The child care spaces in the material are subdivided into three groups, according to the degree of utilization of the service:

Category 1. More than 30 hours of child care per week.

Category 2. From 20 to 30 hours of child care (both endpoints included).

Category 3. Less than 20 hours of child care per week.

This categorization is done instead of dividing the material into subgroups in accordance with type of child care. The latter method would group the spaces into full time child care, half time and short time care and care in the homes of regulated public caregivers. The categorization in use here take into account that the household consume different quantities of the service, but the boundaries are set quite arbitrarily.

a) Method 1

This approach rests on the fundamental assumption that the cost for each space within each category is equal. Total costs are calculated as the average parental fee (according to the Survey of Level of Living) with the addition of average public subsidies. For example, within category 1 the average parental fee is 1540 Nkr per month or about 16920 Nkr per year. Parents cover on average about 35 per cent of the expenses. Thus the total estimated cost within this category is about 48340 Nkr and the subsidy is about 31420 Nkr per year on average. The assumption of equal costs implies that the subsidies are distributed among the households dependent on the size of the parental fee. A household which is paying more than 16920 Nkr per year is receiving a subsidy less than 31420 Nkr. A household which is paying less than 16940 per year is given a fairly large in-kind subsidy compared to a household which pay a higher rate. For category 2 and category 3 the average subsidy is 24800 Nkr and 13420 Nkr per year, respectively.

b) Method 2

With this method it is also assumed that the cost within each category is equal. The point of departure is an estimate of the society's total expenses on child care and not the parental fee for each household as stated in the Survey of Level of Living. Total costs is estimated to 5.325 billion Nkr for 1990 (based on a sample survey executed at the Central Bureau of Statistics, not published) and the parents cover 35 per cent. Households with children in child care centers then received subsidies at a total of 3.462 billion Nkr in 1990. The 3750 households in the Survey of Level of Living represent 0.2 per cent of all households. It is assumed that the sample received about 7 mill. Nkr which is distributed to the 285 child care spaces. When distributing the subsidies it is assumed that child care of category 1 is receiving twice as much subsidies as category 3 and category 2 is receiving the average of category 1 and 3. It follows that the households are subsidized by 32600 Nkr, 24450 Nkr and 16300 Nkr per space in category 1, 2 and 3, respectively. This approach is related to the method used in Herigstad (1986).

REFERENCES

Aaberge, Rolf, Xiaojie Chen, Jing Li and Xuezeng Li (1992): "The Structure of Economic Inequality among Households Living in Urban Sichuan and Liaoning, 1990", Discussion Paper No. 70, Central Bureau of Statistics, Oslo, Norway.

Atkinson, Anthony B. and Joseph E. Stiglitz (1980): Public Economics, Maidenhead: McGraw-Hill Book Company, England.

Barne- og familiedepartementet (1992): "Juridiske sider ved fastsettelsen av foreldrebetaling i barnehager", rapport fra en arbeidsgruppe.

Central Bureau of Statistics (1991): "Kindergartens 1990 (NOS B 975)".

Connelly, Rachel (1992): "The Effect of Child Care Costs on Married Women's Labor Force Participation", The Review of Economics and Statistics.

Ergoplan (1992): "Kostnader i barnehager".

Forbruker- og administrasjonsdepartementet (1987): "Barnehager mot år 2000", St.meld. nr. 8 (1987-88).

Gulbrandsen, Lars og Cathrine Ulstrup Tønnessen (1988): "Barnehageutbyggingens fordelingsmessige virkninger", Tidsskrift for samfunnsforskning nr.6 1988.

Gustafsson, Siv and Frank Stafford (1992): "Child Care Subsidies and Labour Supply in Sweden", Journal of Human Resources, Vol. 27.

Hagenaars, A.J.M. (1991): "Family Equivalence Scales", Living Conditions of the Least Privileged in the European Community, Doc. No. Po. 39/91, Erasmus University Rotterdam.

Heckman, James J. (1974): "Effects of Child Care Programs on Women's Work Effort", Journal of Political Economy, Vol. 82, No. 2.

Herigstad, Helge (1986): "Inntekt og offentlige ytingar", Rapportar SSB 86/2.

Hofferth, Sandra and Deborah Phillips (1987): "Child Care in the United States 1970 to 1995", Journal of Marriage and the Family, Vol.49.

Hofferth, Sandra and Douglas Wissoker (1992): "Price, Quality, and Income in Child Care Choice", The Journal of Human Resources, Vol. 27, No. 1.

Kjelstad, Randi (1991): "1980-årene: Småbarnsmødrenes tiår på arbeidsmarkedet", Samfunnsspeilet nr.3-1991.

Kristiansen, Jan-Erik (1989): "Barnehagetilsyn : Først og fremst for de ressurssterke", Samfunnsspeilet nr. 3-1989.

Leibowitz, Arleen, Jacob A. Klerman and Linda J. Waite (1992): "Employment of New Mothers and Child Care Choice", The Journal of Human Resources, Vol. 27.

Murray, Michael P. (1980): "A Reinterpretation of the Traditional Income-leisure Model, with Application to In-kind Subsidy Programs, Journal of Public Economics", Vol. 14.

Pollak, Robert A. and Terence J. Wales (1979): "Welfare Comparisons and Equivalence Scales", American Economic Review, Vol. 69.

Rao, V.M. (1969): "Two Decompositions of Concentration Ratio", Journal of the Royal Statistical Society, Vol. 132.

Smeeding, Timothy (1977): "The antipoverty effectiveness of in-kind transfers", The Journal of Human Resources, Vol. 12, No. 3.

Smeeding, Timothy and Marilyn Moon (1980): "Valuing Government Expenditures: The Case of Medical Care Transfers and Poverty," Review of Income and Wealth, Vol. 26.

Smeeding, Timothy (1984): "Approaches to Measuring and Valuing In-Kind Subsidies and the Distribution of Their Benefits" in Marilyn Moon (ed): Economic Transfers in the United States, Chicago: University of Chicago Press.

Statistisk Sentralbyrå (1992): "Foreldrebetalingsundersøkelsen 1992".

Van Praag, B.M.S., A.J.M. Hagenaars and J. van Weeren (1982): "Poverty in Europe", Review of Income and Wealth, Vol. 28.

Waring, Marilyn (1989): If Women Counted, London: Macmillan.

Table 1. Frequencies of expenses on child care in the official child care statistics (CBS 1991) and in the Survey of Level of Living 1991.									
Parental fee (N kr)	0	1-399	400 -799	800 -1199	1200 -1599	1600 -1999	2000 -2499	2500 -2999	3000-
Kindergartens 1991	5.7	11.5	19.7	14.7	11.1	15.1	15.2	5.3	1.8
Survey of Level of Living 1991	20.4	5.3	15.1	11.9	9.8	17.5	14.7	3.9	1.4

Table 2. Equivalence scales		
Household size	OECD scale*)	Welfare measurement scale
1	1.0	1.00
2	1.7	1.12
3	2.2	1.21
4	2.7	1.27
5	3.2	1.32
6	3.7	1.37

*) Each additional household member after household size 2 is a child in the table

Table 3. Estimates of the Gini coefficient[*] in the distribution of disposable income and in the distribution of disposable income pluss child care subsidies for households with preschool children (697 obs.)									
Income concept	Disposable income			D_1: Disposable income + child care subsidies (method 1)			D_2: Disposable income + child care subsidies (method 2)		
Type of equivalence scale	Unadjusted disposable income	OECD scale	Welfare measure-ment scale	Unadjusted D_1	OECD scale	Welfare measure-ment scale	Unadjusted D_2	OECD scale	Welfare measure-ment scale
Gini coeffient	0.218 (0.013)	0.204 (0.015)	0.211 (0.014)	0.217 (0.013)	0.203 (0.014)	0.209 (0.013)	0.217 (0.013)	0.204 (0.014)	0.210 (0.013)

[*] Standard deviations are given in parentheses.

Table 4. Estimates of the Gini coefficient[*] in the distribution of disposable income and in the distribution of disposable income pluss child care subsidies for households receiving child care subsidies (239 obs).									
Income concept	Disposable income			D_1: Disposable income + child care subsidies (method 1)			D_2: Disposable income + child care subsidies (method 2)		
Type of equivalence scale	Unadjusted disposable income	OECD scale	Welfare measure-ment scale	Unadjusted D_1	OECD scale	Welfare measure-ment scale	Unadjusted D_2	OECD scale	Welfare measure-ment scale
Gini coefficient	0.202 (0.013)	0.180 (0.011)	0.194 (0.012)	0.185 (0.011)	0.162 (0.010)	0.176 (0.011)	0,186 (0,011)	0,165 (0,010)	0,177 (0,011)

[*] Standard deviations are given in parentheses

Table 5. Decomposition of the Gini coefficient in distributions of D_1 and D_2 with respect to disposable income (1) and child care subsidies (2) for households with preschool children (697 obs.)

Income concept	Type of equivalence scale	Level of inequality	Income factor	Fraction of overall inequality (per cent)	Fraction of total income (per cent)	Concentration coefficient
D_1	Unadjusted D_1	0.217	1	95.4	96.3	0.215
			2	4.6	3.8	0.264
	OECD scale	0.203	1	95.0	96.1	0.200
			2	5.0	3.9	0.261
	Welfare measurement scale	0.209	1	95.5	96.2	0.208
			2	4.5	3.8	0.250
D_2	Unadjusted D_2	0.217	1	95.1	96.1	0.215
			2	4.9	3.9	0.271
	OECD scale	0.204	1	94.5	95.9	0.159
			2	5.5	4.1	0.216
	Welfare measurement scale	0.210	1	95.1	96.1	0.208
			2	4.9	4.0	0.259

Table 6. Decomposition of the Gini coefficient in distributions of D_1 and D_2 with respect to disposable income (1) and child care subsidies (2) for households receving child care subsidies (239 obs.)

Income concept	Type of equivalence scale	Level of inequality	Income factor	Fraction of overall inequality (per cent)	Fraction of total income (per cent)	Concentration coefficient
D_1	Unadjusted D_1	0.185	1	96.9	90.4	0.198
			2	3.2	9.6	0.061
	OECD scale	0.162	1	97.5	90.0	0.176
			2	2.5	10.0	0.040
	Welfare measurement scale	0.176	1	97.6	90.3	0.190
			2	2.4	9.7	0.044
D_2	Unadjusted D_2	0.186	1	96.6	90.0	0.200
			2	3.4	10.1	0.057
	OECD scale	0.165	1	96.4	89.5	0.178
			2	3.7	10.5	0.057
	Welfare measurement scale	0.177	1	97.2	89.9	0.192
			2	2.8	10.1	0.050

Table 7. Mean D_1 and mean D_2 for households with preschool children*) with respect to disposable income (1) and child care subsidies (2), (697 obs).

Decile	Mean D_1	Decile spesific mean factor income		Mean D_2	Decile specific mean factor incomes	
		1	2		1	2
1	94828	91013	3816	94757	89179	5578
2	159873	150514	9359	160029	151378	8652
3	199261	191827	7434	199305	192499	6806
4	223930	218734	5196	224452	219144	5308
5	247145	244187	2958	247080	242763	4317
6	267188	261405	5783	266631	259255	7377
7	286679	274053	12626	287480	276730	10749
8	313263	301221	12041	314773	302218	12556
9	348918	329784	19133	349816	328383	21433
10	477389	457600	19788	479278	458791	20486

*) D_1 and D_2 are unadjusted.

Gender, occupational status and income inequality in Norway

Hilde Bojer
Department of Economics
University of Oslo[1]

[1]Most of the data are taken from Surveys of Income and Property 1970— 1989, carried out by the Norwegian Central Bureau of Statistics. NSD (Norwegian Social Science Data Services) put them at my disposal, after clearing them of all means of identifying individuals. Neither CBS nor NSD are in any way responsible for my analysis and interpretation of the data. I wish especially to thank Helene Roshauw of NSD for her generous help. Thanks are also due to Kari Skrede of INAS for giving me access to her cohort data, and to Grete Zimmer for letting me use results from her masters thesis. Helpful comments from Frances Wooley, Ingunn Lønning and Steinar Strøm are gratefully acknowledged.

Abstract

The inequality of individual incomes is analysed with particular attention paid to the structural differences between men and women. In the nineteen seventies, Norway experienced an explosive increase in the labour force participation of women. The inclusion of zero incomes in the data makes it possible to assess the impact on individual income distribution of this important structural change. Previous analyses of changes in Norwegian income distribution have not been able to include this effect.

Inequality is computed for men and women separately, and for self-employed, employees, pensioners and others.

The inequality measure used is called the generalised entropy measure.

The internal inequality of men is more or less unchanged in the period. Women's internal inequality, and therefore also the income inequality for the whole adult population, has decreased strongly. The inequality between men and women is decreasing, but still substantial.

Women overall have greater internal inequality than men, but women's inequality is smaller than men's within each category of occupational status.

Cohort data show, as expected, large transition rates in and out of the labour market for women of all ages and for young and old men, while men between 30 and 50 are fairly stable.

1 Introduction

Previous analyses of individual income distribution in Norway show a surprising stability, both overall and within groups. In particular, my own computations of Gini-coefficients for men and women from 1970 up to 1984 show no systematic change.[1] Also, women's mean income relative to the mean income of men was more or less unchanged during all these years.

Now, the seventies and eighties were years of dramatic change in the labour force participation of Norwegian women. The proportion of married women in paid work more than doubled. Moreover, women's employment increased mainly in part-time work and service work, particularly in the public sector. Women's new work patterns are markedly different from those of men, and from those of women previously. I was puzzled that such developments in no way affected inequality measures and women's relative mean income.

The solution is very simple. Previous to 1982, Norwegian income statistics comprised so called income-earners only: that is, persons with non-zero income. The great change in the economic status of women during the seventies was the transition from zero income to a small, but positive income. The size of the population covered by income statistics increased, but the population size in itself does not influence ordinary measures of inequality.

When I recomputed inequality and relative mean income for the whole adult population, including non income-earners, I obtained very different results indeed, as shown in section 8.

2 The population

Theoretical as well as empirical analyses of income distribution tend to assume income to be strictly positive for every unit of analysis. Indeed, widely used distribution functions such as Pareto's law and the log-normal distribution are undefined for zero and negative values of the variable. Theorems about e. g. inequality measures also assume a strictly positive resource variable for the sake of simplicity. I have never seen this assumption seriously queried. It is meaningful to the extent that it is meaningful to view income earners and non-income earners as two distinct, not overlapping, groups of

[1] Bojer (1987)

the population, with transition between the groups to be analysed separately from the analysis of distribution of income itself.

I think the picture of reality underlying much traditional analysis of income distribution can fairly be described as follows: Income earners are adult males and unmarried women working full time. Children, youths and married women are economically supported by the income earners. Transition between the two states are made twice in a lifetime: at the beginning and end of a man's working life.

This picture is no longer entirely correct even for men, and is completely misleading for youngsters, pensioners and women. Women typically move in and out of the labour force several times in a lifetime, working full time, long part time, short part time, maybe taking odd jobs or seasonal work as personal finances demand or child care and husband care allow. Students as well as old age pensioners of either sex often do odd jobs and part time work to supplement their income.

Since almost every adult person now spends at least some of their time as an income-earner and some time as a non-income earner, with no definite once-and-for-all transition between these two states, I feel meaningful analysis of the income distribution must comprise the whole adult population.

3 The definition of income

Analysing positive incomes only is, of course, also meaningful if observations of zero income in reality represent measuring errors. Pareto was explicit on this point, and is worth quoting:[2]

> We should observe that when researching into the distribution of income, we are not concerned with the sources of income. Even the poorest man must be regarded as having sufficient income to keep him alive. It doesn't matter whether this sum comes from the fruit of his work, or whether it comes to him from charity, or indeed whatever source, legal or illegal.[3]

In my view, Pareto here confuses income and consumption. Every surviving person does indeed consume, but not necessarily out of income. As every

[2] For Pareto, of course, it goes without saying that the person to be studied is a man.
[3] Quoted from Brown (1976)

woman knows, transfers in kind, or even in cash when received as charity, are something very different from earning your own cash income. Economic emancipation for women is the right to own and control economic property, including the right to paid work (not to work in itself, which few women have lacked). Income in cash instead of in kind is important, as implying greater choice, but the crucial point is not the cash but the right. Income from paid work is a legally enforceable entitlement, giving the right to control economic resources.

In more recent literature, the authoritative definition of income is usually taken to be that of Simon (1938). By the Simon definition, income equals potential consumption, or consumption plus saving. Hence, positive consumption implies positive income[4], although not necessarily in cash. A transfer in kind is tantamount to an income equalling the cash value of the transfer, with perhaps a discount to allow for lack of free choice.

I was looking for a concept that could distinguish between income as economic autonomy and the money-value of passive consumption, when I decided to look up Simon's original definition, as distinct from various paraphrases.

I found the following:

> Personal income connotes, broadly, the exercise of control over the use of society's scarce resources. It has to do not with sensations, services, or goods, but rather with rights which command prices (or to which prices may be imputed). Its calculation implies estimate (a) of the amount by which the value of a person's store of property rights would have increased, as between the beginning and end of the period, if he had consumed (destroyed) nothing, or (b) of the value of the rights which he might have exercised in consumption without altering the value of his store of rights. In other words, it implies estimate of consumption and accumulation.... [5]

It turns out that power (exercise of control, rights) is crucial to Simon's original definition of income. Judging by the last clause, Simon himself did not see the same implications of this as I do. It seems obvious to me that

[4]or, of course, negative saving

[5]Simon (1938), page 49

consumption does not necessarily entail the 'exercise of control over the use of resources'. Income, then, should be regarded as conceptually distinct from consumption.

Therefore, analysis of the distribution of individual income is not a poor approximation to the analysis of household income, as some authors imply. Household income is an indicator of the (potential) standard of living, consumption or material well-being of each individual household member: man, woman or child. But it does not measure their economic autonomy, which is the concern of this paper.

4 Inequality Measure

The inequality measure used belongs to the class of generalised entropy measures. Let the income of unit j be Y_j, and let m denote mean income. The generalised entropy class is generated by:

$$I(\alpha) = \frac{1}{\alpha(1-\alpha)}\left[\frac{1}{n}\sum_j \left(\frac{Y_j}{m}\right)^{\alpha} - 1\right] \tag{1}$$

The parameter α may be any real number.[6] For $\alpha = 2$, $I(\alpha)$ is a simple transformation of the familiar coefficient of variation, v, thus:

$$I(2) = 0.5v^2$$

The lower bound of $I(\alpha)$ is 0 (all incomes are equal); its upper bound depends on α, but in all cases exceeds 1.

I have chosen $\alpha = 1/2$. The deliberate inclusion of zero incomes restricts α to strictly positive values. The greater α, the greater weight is given to large incomes. The coefficient of variation, corresponding to $\alpha = 2$, is too heavily influenced by the upper tail of the income distribution according to several authors. The value 1/2 lies close to the lower limit of possible α's, and tends to give about the same kind of orderings as the Gini coefficient.

The generalised entropy measures have the attractive property of being additively decomposable by group.

[6]For proof of this and further discussion, see e.g. Shorrocks (1980).

Let the population consist of g groups and let p_g and m_g be group g's proportion of the population and its mean income respectively. Furthermore, let $I_g(\alpha)$ be the inequality within group g.

The between-group inequality, I_B is defined as the inequality that would have obtained if the inequality within each group were zero:

$$(2) \qquad I_B(\alpha) = \frac{1}{\alpha(1-\alpha)} \left[\sum_g p_g \left(\frac{m_g}{m} \right)^\alpha - 1 \right]$$

Additive decomposability means that total inequality in the population can be expressed in terms of the within-group inequalities and the between-group inequality as follows:

$$(3) \qquad I(\alpha) = \sum_g v_g I_g(\alpha) + I_B(\alpha)$$

Here, $v_g = p_g(m_g/m)^\alpha$.

The decomposition formula 3 enables us to interpret changes in total inequality as due either to changes in the composition of the population (the p_g's) or in the relative incomes of the various groups ($I_B(\alpha)$) or in the internal inequality of one or more groups ($I_g(\alpha)$). As will be seen in section 8, the internal inequality of women is greater than that of men even though women show smaller inequality than men within each of several categories of occupational status. The explanation is that more women are found in the occupational categories with high internal inequalities.

Formula 3 implies that a decrease in the internal inequality of one group always leads to a reduction in total inequality (all other group inequalities and all group means and proportions being constant). Inequality measures outside the class of the generalised entropy measure do not possess this property.[7] That is why I have chosen to measure inequality by $I(\alpha)$ instead of the more familiar Gini.

5 Some definitions

The measure of income used (gross taxable income) is very broad, comprising i.a. profits, earnings and pensions from the National Insurance.[8]

[7] Shown by Shorrocks (1980)

[8] For further information, see section 9, page 14

Non income earners (Non I E) received no income of any kind in the relevant year. Persons receiving an income of 1 krone or more, but less than the minimum old age pension from National Insurance, are classified as 'Other income earners'. Persons in this group have part time occupations of various kinds. The majority are married women, but many students of either sex are also found in this category. Note that many persons working part time may be classified as 'Employes' or 'Self employed'.

Pensioners receive their living from the National Insurance and other private or governmental pension schemes. The group includes old age pensioners, handicapped persons and some single mothers.

Non-pensioners with incomes above the minimum old age pension are deemed by the income surveys to be economically active. The income surveys have no independent information of e.g. working hours, hence this less than happy definition of economic activeness. All persons in full time paid work belong to the categories 'Self-employed' or 'Employees'. But many persons in these categories are part-time workers, particularly among women employees, whose working hours are very varied.

The classification according to size of income works well, however, as a criterion of economic autonomy. In Norway, the minimum old age benefit is an unofficial poverty line, and many Norwegians actually live on it. But it is not possible to be economically self-supporting on an income below this level.

The population studied consists of all Norwegian adults, that is persons 17 years old or more. Data for non income earners were not available previous to 1982. I have been able to estimate the number of women non- income earners for these years, but not the number of men. Computations concerning men in 1982 have been made twice, on the basis of the old population (adult I E) and the new (all adults).

6 Structural changes

Tables 1 and 2 show how changes in labour force participation and demographic developments are reflected in the income surveys.

The proportion of adult women who are non income earners, has fallen from an estimated 45 % in 1970 to around 5 % at the end of the eighties. The proportion seems to have become stabilised in the latter half of the

Table 1: Adult women by occupational status. Per cent

	Self Empl	Em–ployees	Pensi–oners	Other I E	All I E	All Women
1970	1	25	17	12	55	100
1973	1	27	20	14	62	100
1976	2	32	27	20	81	100
1979	2	36	27	20	76	100
1982	2	41	24	23	90	100
1984	2	39	29	20	91	100
1986	3	45	27	19	94	100
1987	3	46	27	19	95	100
1988	3	47	27	17	94	100
1989	3	44	30	17	95	100

I E = Income Earner

Table 2: Adult men by occupational status. Per cent

	Self Empl	Em–ployees	Pensi–oners	Other I E	All I E	All men
1970	14	64	15	6	100	::
1973	13	62	18	6	100	::
1976	13	62	18	7	100	::
1979	13	63	18	6	100	::
1982	12	64	17	9	100	::
1982	10	63	17	9	99	100
1984	10	60	20	9	99	100
1986	10	62	18	9	99	100
1987	10	63	18	9	99	100
1988	10	62	18	8	98	100
1989	9	60	19	10	98	100

I E = Income Earner

:: not available

eighties. The corresponding increase is approximately equal for the three main categories of income earners.

For men, there is the faintest trend in the opposite direction: a decrease in paid economic activity, mainly due to an increasing number of pensioners and 'Other income earners'. The increasing number of male and female pensioners is mainly demographic: they are mainly old age pensioners, a majority of whom are women because women live longer than men.

Table 3: Women's mean income in percent of men's

	Self Empl	Em- ployees	Pensi- oners	Other I E	All I E	All Adults
1970	69	59	69	108	49	27
1973	57	63	70	178	53	33
1976	56	64	70	82	48	39
1979	52	63	71	75	50	42
1982	54	64	71	98	50	45
1982						46
1984	47	65	67	125	51	47
1986	57	62	65	108	52	49
1987	61	65	70	106	54	52
1988	58	63	67	102	54	51
1989	61	64	67	110	55	53

Gross Income.

Table 3 shows women's relative mean income. There is no trend either way within each separate occupational group. There is a slight increase in the relative mean income of all income earners. Tables 1 and 2 suggest the increase stems from an increase in the proportion of pensioners as compared with 'Other income earners' on the part of women, while men have shifted in the opposite direction.

The average wage rate of Norwegian women is about 80% of men's average wage rate.[9] Female employees have an *income* of only about 65% of male employees because women's working hours are shorter. Women work more part-time and less overtime.

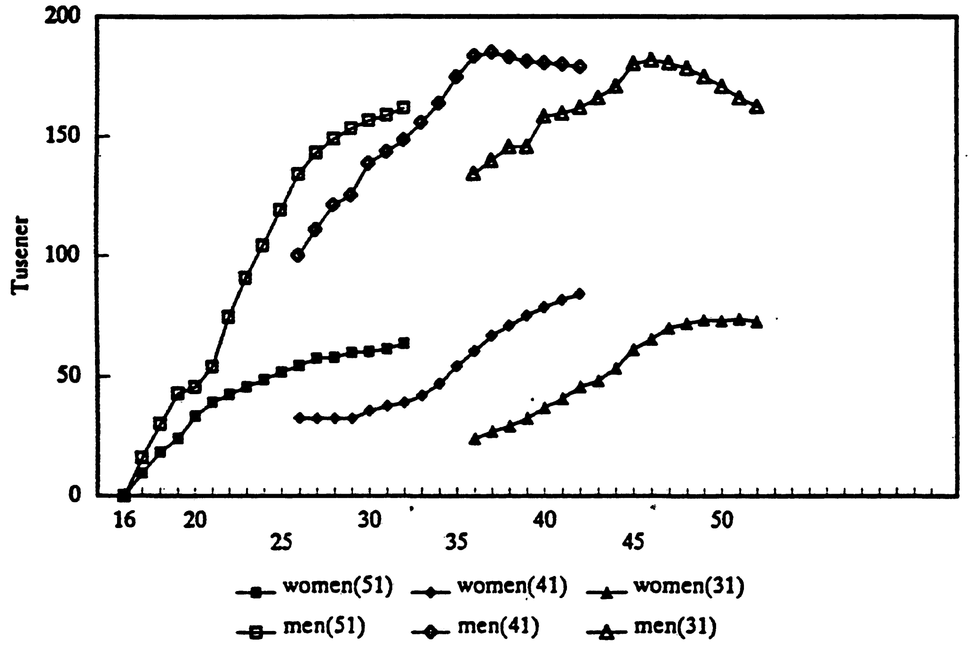
[9]According to Barth (1992) page 19.

The relative mean income of all adult women has nearly doubled, showing an increase from 27% to 53%.

7 Income by age

As shown in Bojer (1988), the variation of income with age for women is very different from the usual inverted U of men. It is well known that in cross-section data, variations with age will be distorted by cohort differences. Partial age-income curves for three cohorts of men and women are shown in figure 1. The data cover 16 years, from 1968 to 1984.[10]

Figure 1: Real income by age. Men and women; 3 cohorts



The down-turn of the curves for two male cohorts probably reflects decreasing real wages in the beginning of the eighties rather than an age-effect.

[10]For details, see section 9

For women, the curves reflect the strong influx of every cohort into the labour market in the period covered. We may perhaps guess at a future inverted U for women, but with a dip in the late twenties, the typical child-bearing age of the two younger cohorts shown.

8 Inequality

Table 4: Inequality of income earners.

	1970	1973	1976	1979	1982	1984	1986	1987	1988	1989
Men	0.24	0.30	0.26	0.22	0.23	0.24	0.21	0.21	0.22	0.24
Women	0.25	0.26	0.31	0.25	0.28	0.26	0.24	0.25	0.23	0.24
All I E	0.29	0.33	0.33	0.28	0.30	0.30	0.27	0.27	0.27	0.28

Table 4 shows inequality by gender for adult income earners. I find no discernible trend in this table.[11] Note that inequality among women is greater than among men; total inequality is greater than either.

Tables 5 and 6 show inequality for all adults, and by occupational category. The strongest result is the remarkable decline in women's internal inequality; the decline is entirely due to the increasing number of women income earners, as we see by comparing with table 4. Women's internal inequality is approaching that of men, but is still greater.

I have not computed total inequality; it follows from the decomposability of $I(\alpha)$ that total individual inequality has decreased in the period since the inequality of women has decreased, men's being more or less constant.

It is sometimes taken for granted that income dispersion must be smaller among women than among men, since women fill a narrower range of occupations, and since there are very few women high earners. I have previously shown (Bojer (1988)) that men's and women's Lorenz curves in 1982 intersect above the 95th centile. The ordering of inequality by gender therefore depends on the measure chosen; an inequality measure strongly influenced

[11] The formulas for computing standard errors are cumbersome, and I have so far computed standard errors for 1982 only (see Bojer (1988)). The standard errors were then of magnitude < 0.01; we can take differences of 0.02 or more to be significant.

by incomes in the top range may show greater inequality for men. As indeed did the coefficient of variation in 1982.

Other inequality measures, such as the Gini-coefficient and the present generalised entropy measure, show women to have the greater inequality overall. But within each occupational category, women tend to have smaller inequality than men do.

The decomposition formula 3 shows total inequality to be a weighted sum of the within-group inequalities plus the between-group inequality. Women's overall inequality is greater than men's because more women than men belong to the strongly dispersed group of 'Other employees', while fewer women than men are 'Employees', where internal inequality is small.[12]

Tables 7 and 8 show inequality for employees and pensioners decomposed into within-group and between-group inequalities. These are the most stable and homogeneous groups, and they now account for about 3/4 of the adult population.

The difference in internal inequality between men and women employees is too small to be significant for any single year. Given the extremely segregated Norwegian labour market, we might expect women employees to have smaller internal inequality men employees. But earnings are determined by hours worked as well as the wage rate. About half of economically active women work part-time. As previously explained (see section 5), the income surveys are not able to distinguish between full timers and part timers. The short part-timers are grouped as 'Other income earners', but many women employees also work part time. Part time working is still quite rare among men taken as a whole, though common for male students. Income inequality for men employees are caused by wage differentials. Internal inequality for women employees are caused by dispersion in working hours as well.

Women's smaller internal inequality is more marked for pensioners. All Norwegians become entitled to the minimum old age pension at 67 years of age. Whether we receive more than the minimum, depend upon dues paid out of income during active working life. Most men, but very few women, receive more than the minimum, which explains why there is more dispersion among male pensioners.

[12]In Bojer (1988), I have also shown that women's between-group inequality is larger than men's in a decomposition by occupational status, but smaller in a decomposition by age.

Table 5: Women's internal inequality

	Self Empl	Emp-loyees	Pens ioners	Other I E	All I E	All women
1970	0.21	0.09	0.13	0.35	0.25	1.21
1973	0.19	0.10	0.12	0.35	0.26	1.12
1976	0.22	0.09	0.15	0.36	0.31	0.69
1979	0.14	0.07	0.11	0.30	0.25	0.52
1982	0.19	0.08	0.10	0.34	0.28	0.46
1984	0.15	0.09	0.09	0.40	0.26	0.44
1986	0.19	0.09	0.10	0.31	0.24	0.36
1987	0.15	0.10	0.12	0.29	0.25	0.35
1988	0.16	0.08	0.11	0.30	0.23	0.34
1989	0.14	0.07	0.10	0.45	0.24	0.34

Gross Income

Table 6: Men's internal inequality

	Self Empl	Emp-loyees	Pens ioners	Other I E	All I E	All men
1970	0.18	0.09	0.18	0.50	0.24	::
1973	0.26	0.09	0.15	::	0.30	::
1976	0.24	0.08	0.18	0.57	0.26	::
1979	0.14	0.08	0.15	0.47	0.22	::
1982	0.28	0.08	0.14	0.33	0.23	0.25
1984	0.25	0.09	0.14	0.29	0.24	0.26
1986	0.17	0.09	0.12	0.30	0.21	0.23
1987	0.16	0.10	0.12	0.29	0.21	0.23
1988	0.17	0.11	0.11	0.39	0.22	0.25
1989	0.21	0.11	0.11	0.43	0.24	0.27

Gross Income

:: not available

Table 7: Inequality of employees

	1970	1973	1976	1979	1982	1984	1986	1987	1988	1989
All employees	0.12	0.12	0.10	0.10	0.10	0.10	0.12	0.12	0.12	0.12
Men	0.09	0.09	0.08	0.08	0.08	0.09	0.09	0.10	0.11	0.11
Women	0.09	0.10	0.09	0.07	0.08	0.09	0.09	0.10	0.08	0.07
Inequality between	0.03	0.02	0.02	0.02	0.02	0.02	0.03	0.02	0.02	0.02

Gross Income. Persons over 16.

Table 8: Inequality of pensioners

	1970	1973	1976	1979	1982	1984	1986	1987	1988	1989
All pensioners	0.17	0.15	0.18	0.14	0.13	0.13	0.13	0.13	0.13	0.12
Men	0.18	0.15	0.18	0.15	0.14	0.14	0.12	0.12	0.11	0.11
Women	0.12	0.12	0.15	0.11	0.10	0.09	0.10	0.12	0.11	0.10
Inequality between	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02

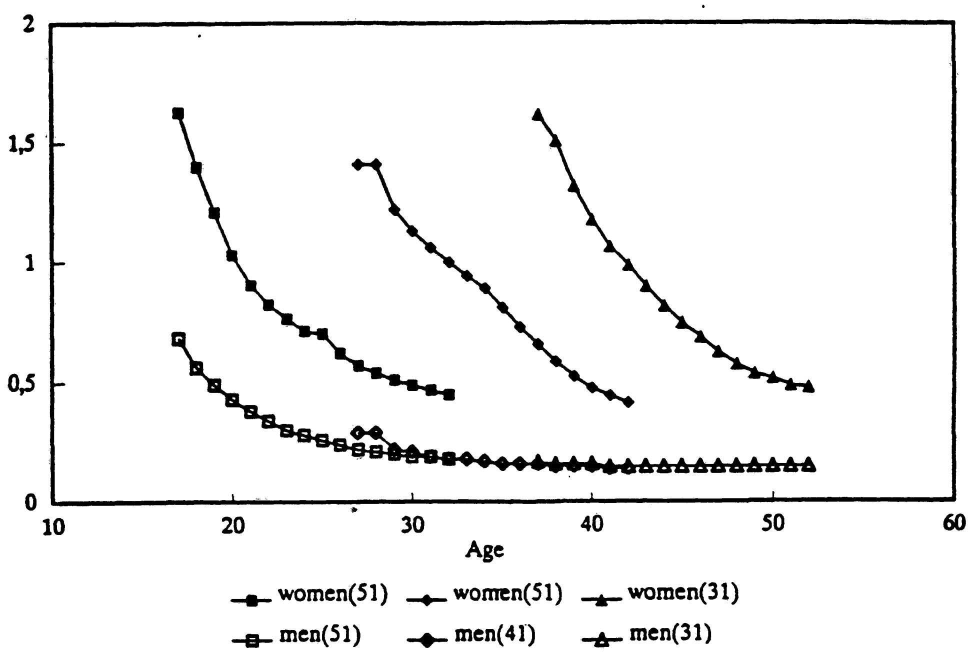
Gross Income. Persons over 16

Lastly, figure 2 shows the effect of the accounting period on the internal inequality of men and women. For each cohort, the first point on the curve shows inequality in the incomes of the first year. The second point shows inequality in incomes of the first plus second year, and so on; the last point shows the inequalities when incomes have been added over all the 16 years observed.

Income inequality is usually assumed to decline with the number of years included, and so it does here; markedly so for women and for young men. Note, however, that for men, inequality does not seem to be affected by accounting period from about 30 years of age to the late forties. The sharp decline from the teens to the mid-twenties must surely be due to steadily greater labour force participation. For women, of course, the influx into paid work during the period explains most of the change.

The rate of change in inequality with the length of the accounting period is sometimes taken as a measure of mobility in the population. In this case, mobility is best understood as transition in and out of paid work. This mobility seems small for men between 30 and 50, but is otherwise fairly

Figure 2: Inequality by accounting period and age. Men and women; 3 cohorts



9 Definitions. Data

Gross income is gross taxable income as declared on income tax returns. Gross taxable income includes net entrepreneurial income, profits, earnings, pensions and benefits from the National Insurance. The concept of taxable income has broadened somewhat in the period studied. Child benefit, scholarships to students and certain other social transfers are not included.

Income earner is a person with at least NKr 1 in taxable income during the year. A non income earner has received no taxable income of any kind.

Occupational status The classification is made according to size and composition of gross income. A person is economically active if the sum of entrepreneurial income and earnings exceeds the minimum old age pension to which every Nowegian is entitled. The economically active are subdivided into

Self Employed and

Employees according to whether entrepreneurial income or earnings is the greater component of gross income.

Pensioners are persons to whom more than 50 % of taxable income accrues from private insurance or social security.

Other income earners receive taxable income smaller than the minimum old age pension, from whatever source. They are students, young men in military service and short part-time workers, the last group consisting mainly of married women.

I have redefined all categories wherever necessary to obtain comparability throughout the period studied. The definitions may not therefor conform to the the corresponding characteristic in the printed publications from the Central Bureau of Statistics.

The data are taken from the Central Bureau of Statistics, Surveys of Income and Property. The surveys were carried out every third year in the period 1970–1982, and every year from 1984 on. From 1982 on, the surveys are random sample surveys, drawn according to the standard sampling procedure of the CBS, and constituting a random sample of the whole non-institutionalised population.

The surveys from the years 1970 to 1979 comprised income earners only, and the sampling procedure was not strictly random. For these years, I have estimated the number of female non income earners by subtracting the estimated number of female income earners aged over 16 years from the total number of females over 16 years. This method did not give reasonable results when applied to males; therefore no data for male non income earners in the seventies are given.

The income and property surveys base their data for taxable income on official income tax returns. There is therefore a built in bias downwards, although errors in the data for earnings and pensions should not be exaggerated. On the other hand, non-response is nil.

Income data for self-employed are strongly influenced by changing tax laws, particularly rules on depreciation allowances. This group is also very heterogeneous, and is included for the sake of completeness only.

Further information on the data and sampling procedure may be found in the publications from CBS listed in the references.

The cohort data are random samples taken from the registers of the Norwegian National Insurance. 4 cohorts were studied: individuals born in 1951, 1941, 1931 and 1921. The income concept used is so called *pensionable income*; i. e. gross income from wages and from entrepeneurial activities. Pure capital income and income from transfers (e.g. pensions) are not included. For details, see Zimmer (1989). The sample was taken, and the data prepared, by Kari Skrede of the Norwegian Institute of Applied Social research (INAS).

10 References

Barth, E. (1992) *Lønnsforskjeller mellom kvinner og menn i Norge*, Rapport 92:7, Institute for Social Research, Oslo. (Wage differentials between women and men in Norway)

Bojer, H. (1987) 'Personlig inntektsfordeling i Norge 1970–1984', *Tidsskrift for Samfunnsforskning*, vol. 28, pp 247–258. (Personal distribution of income in Norway 1970—1984)

Bojer, H. (1988) *Occupational Status, Sex and Income Inequality in Norway 1982*, Memorandum No 2 1988, Department of Economics, University of Oslo.

Brown, J.A.C. (1976) 'The Mathematical and Statistical Theory of Income Distribution' in Atkinson, A. B. (Ed): *The Personal Distribution of Income*, Allen and Unwin, London.

CBS 1973 *Inntektsstatistikk 1970*, NOS A 543, Central Bureau of Statistics, Oslo.

CBS 1977 *Inntektsstatistikk 1973*, NOS A 892, Central Bureau of Statistics, Oslo.

CBS 1979 *Inntektsstatistikk 1976*, NOS B 94, Central Bureau of Statistics, Oslo.

CBS 1982 *Inntektsstatistikk 1979*, NOS B 267, Central Bureau of Statistics, Oslo–Kongsvinger.

CBS 1985 *Inntektsstatistikk 1982*, NOS B 569, Central Bureau of Statistics, Oslo–Kongsvinger.

CBS 1987 *Inntektsstatistikk 1984*, NOS B 678, Central Bureau of Statistics, Oslo–Kongsvinger.

CBS 1988 *Inntektsstatistikk 1985*, NOS B 784, Central Bureau of Statistics, Oslo–Kongsvinger.

CBS 1990 *Inntekts- og formuesstatistikk 1986*, NOS B 867, Central Bureau of Statistics, Oslo–Kongsvinger.

CBS 1990 *Inntekts- og formuestatistikk 1987*, NOS B 885, Central Bureau of Statistics, Oslo–Kongsvinger

CBS 1990 *Inntekts- og formuesstatistikk 1988*, NOS B 948, Central Bureau of Statistics, Oslo–Kongsvinger.

Shorrocks, A. F. (1980) 'The Class of Additively Decomposable Inequality Measures', *Econometrica,* vol 48, pp. 613–625.

Simons, H. (1938) *Personal Income Taxation,* University of Chicago Press, Chicago.

Zimmer, G. (1990) *Ulikhet og beregningsperiode* (Inequality and accounting period) Unpublished Masters Thesis, University of Oslo.

STATISTICAL INFERENCE AND THE MEASUREMENT OF POVERTY

Markus Jäntti
Åbo Akademi University
Department of Statistics
SF - 20500 Turku
358-21-654 161 (tel)
358-21-654 497 (fax)
`mjantti@aton.abo.fi`

Leif Nordberg
Åbo Akademi University
Department of Statistics
SF - 20500 Turku
358-21-654 163 (tel)
358-21-654 497 (fax)
`lnordberg@linus.abo.fi`

Preliminary – not for citation

Abstract

This paper studies the measurement of poverty in finite populations based on complex sample surveys and related statistical inference. The literature on poverty indices has hitherto been little concerned with estimation and statistical properties, although when more than a head-count of the poor is performed, these can be quite complicated.

From a statistical point of view, most poverty indices can be interpreted as means of some suitable variables, or as functions of generalized means of the kind used in the theory of U-statistics. For measures of the first type we can use standard sampling theory for the estimators of means. For measures of the second type the situation is more complicated as we have to rely on finite population U-statistics theory.

We review the definitions and properties of some the more well-known poverty measures. Then we discuss the estimation of those measures that can be treated as "traditional" means of functions of variables or functions of means. We present the basics of finite population U-statistics theory and illustrate with an application to the Sen measure. We also discuss some of the complications that arise when the poverty line is a function of the sample median.

Poverty measurement consists of identifying the poor and aggregating the relevant information into an index. The literature on poverty indices has hitherto been little concerned with estimation and statistical properties, although when more than a head-count of the poor is performed, these can be quite complicated. This paper addresses the problem of estimating and studying the sampling properties of two large classes of poverty indices.

There exists an extensive literature on suitable functional forms for poverty measures. However, from a statistical point of view, most can be interpreted as means or functions of means of some suitable variables, or as functions of generalized means of the kind used in the theory of U-statistics. For measures of the first type we can use standard sampling theory for the estimators of means. For measures of the second type the situation is more complicated as we have to rely on finite population U-statistics theory.

The structure of the paper is the following. We begin by reviewing the definitions and properties of some the more well-known poverty measures. Then we discuss the estimation of those measures that can be treated as "traditional" means of functions of variables or functions of means. We then present the basics of finite population U-statistics theory and illustrate with an application to the Sen measure. Finally, we discuss some of the complications that arise when the poverty line is a function of the sample median.

1 Measures of poverty

In the assessment of overall poverty, two distinct (although not unrelated) phases can be pointed out. In the first phase, the poor are identified (*identification*). This can be done in a number of ways with differences emanating from varying concepts of poverty, uses of different indicators of poverty (incomes, consumption bundles etc) and so on. Once the identification phase is complete (in the sense that the criteria for identification have been set) there is the phase of ***aggregation***. This enables us to construct a measure that is sensitive to the extent of poverty among the poor through the "distance" of the poor incomes from a pre-defined threshold level.[1]

In the literature that concerns the aggregation of information about poverty, much stress has (naturally) been laid on different ways of operationalizing the poverty concept into an aggregate measure. A great number of different measures of poverty have been suggested by various authors from which one can choose. An aspect of poverty measurement that has received far less attention is, however, the estimation of poverty measures from sample survey data, especially when it comes to assessing the degree of uncertainty (ie. error) in the estimated measures.

A number of different measures of poverty have been suggested. A commonly used measure is the "head count"-ratio which is simply the proportion of the poor of the total population. Another simple measure is the income-gap-ratio, which measures the average income short-fall from a specified poverty line among the poor. Though especially the head count- ratio is probably the most commonly used measure of poverty it as the income-gap-ratio – fails to satisfy some quite reasonable, yet elementary requirements on a poverty measure as formulated by Sen (1976). These are that a measure of poverty should depend on (1) poor incomes alone, on (2) how poor the poor are and also on (3) differences in poverty among the poor (ie. income inequality among the poor). Condition (3) was formulated as a transfer axiom by Sen (1976) meaning that a measure of poverty should register an increase in poverty if a regressive transfer of income occured among the poor. This last requirement turns out to be important *vis-á-vis* the derivation of a measure.

If the level of living is adequately expressed by the income of households y and there exists some income level z called the poverty line, a household is said to be poor if $y < z$. A population consists of N income receiving units, and $\mathbf{Y} = \{y_1, \ldots, y_N\}$ is the ordered vector of incomes . We assume that $\mathbf{Y} \in \mathcal{R}^+$, i.e., that all incomes are non-negative, and that $y_i \leq y_j$ for all $i \leq j$, i.e., the population has been ordered in ascending order (this involves no loss in generality).

[1]See Sen (1982), especially ch. 3.

A poverty index is a function $P : \{\mathcal{R}^+, z\} \rightarrow \mathrm{R}^+$ such that for $\mathbf{Y}, \mathbf{X} \in \mathcal{R}^+$, if $\mathbf{X}$ has (in some sense) more poverty than $\mathbf{Y}$, then $P(\mathbf{X}, z) > P(\mathbf{Y}, z)$. Let Q be the number of poor in the population, $Q = \sum_{i=1}^{N} 1[y_i < z]$, where $1[\cdot]$ takes the value of one when the condition in the brackets is fulfilled and zero otherwise (the indicator function). The the proportion of poor in the population (the head-count-ratio) is

$$H = Q/N. \tag{1}$$

The income gap – ratio is defined by

$$I = Q^{-1} \sum_{i=1}^{Q} \frac{z - y_i}{z} = 1 - \frac{\mu_p}{z} \tag{2}$$

where $\mu_p = Q^{-1} \sum_{i=1}^{Q} y_i$ is the mean of poor incomes. Neither of these measures satisfy the all of the above requirements, but some suitable combination of H and I (for instance their product, HI) would lead to a measure that satisfied requirements (1) and (2). It is thus condition (3) that leads us to consider other measures.

The measure suggested by Sen is the following:

$$S = \frac{2}{(Q+1)Nz} \sum_{i=1}^{Q} (z - y_i)[Q + 1 - i], \tag{3}$$

which can also be expressed as

$$H[I + (1 - I)G_p], \tag{4}$$

where G_p is the Gini-coefficient of the poor incomes. The first form can be interpreted as the normalized weighted expectation of the individual poverty gaps, with the weights given by an equidistance weighting function. Sen argues that the relative deprivation felt by a poor person is dependent on the position of that person among the poor. Hence an equidistance weighting of the poverty-gaps. To arrive at the second form some arithmetic is involved. The second form demonstrates a nice interpretation of the Sen measure: it is a function of the income-gap- and the head-count - ratios "corrected" for the income inequality.

A number of measures have been suggested that modify the approach taken by Sen. We shall not review these measures at length in this paper.[2] We shall rather concentrate on the properties of some measures which have some attractive attributes. A property that is satisfied by a class of the measures is additive decomposability (AD). Thus, following Kakwani (1989), we can write a general form of poverty measures as

$$P = N^{-1} \sum_{i=1}^{Q} \theta(z, y_i), \tag{5}$$

where $\theta(z, y_i)$ is a function of the poverty line and income. Consider a population consisting of i sub-populations. We can then express the measured poverty in the total population as a weighted sum of the sub-group poverty or

$$P = \sum_{j=1}^{m} \lambda_j P_j \tag{6}$$

where λ_j and P_j are the sub-population weight and poverty index. By specifying appropriate forms for the function $\theta(z, y_i)$ we can obtain different decomposable measures of poverty. By substituting $\theta(z, y_i) = 1$ in (5) we obtain the head-count-ratio.

A parametric class of measures that satisfy the decomposability condition proposed by Foster et al. (1984) is obtained when $\theta(z, y_i)$ is defined as $[(z - y_i)/z]^\alpha$, where $\alpha \geq 0$. This yields a class of poverty measures

$$F_\alpha = N^{-1} \sum_{i=1}^{Q} \left[\frac{z - y_i}{z} \right]^\alpha \tag{7}$$

F_0 is the head-count-ratio and F_1 is equal to HI. The third requirement is satisfied for $\alpha > 1$.

Thus, the literature on suitable functional forms for P is extensive. However, from a statistical point of view, most can be expressed in either of two forms:

$$P_I[\mathbf{Y}, z] = h[H, \bar{\theta}_I(\mathbf{Y}, z)] \tag{8}$$

$$P_{II}[\mathbf{Y}, z] = h[H, \bar{\theta}_{II}(\mathbf{Y}, z)] \tag{9}$$

[2]See Foster (1984) and Seidl (1989) for reviews.

where $\bar{\theta}_I(\mathbf{Y}, z) = N^{-1} \sum_{i=1}^{N} 1[y_i < z]\theta_I(y_i, z)$ and $\bar{\theta}_{II}(\mathbf{Y}, z) = [N(N-1)]^{-1} \sum_{1 \leq i \neq j \leq N} 1[y_i \& y_j < z]\theta_{II}(y_i, y_j, z)$, and h, θ_I and θ_{II} are known functions. Most of the measures belong to the first class while the measure suggested by Sen (1976) is a member of the second class.

2 Estimating poverty measures

When measuring and comparing poverty in real populations we clearly have to rely on sample estimates of the measures above. This raises two technical points. First, we have to construct an estimator of the measure at hand which applies to discrete populations. Secondly, we must ask ourselves if the ordering we obtain by the estimated values of a measure of poverty is "real", ie. consistent with the ordering of populations from which our samples emanate. In other words, we are concerned with if the difference in poverty as measured by a specific poverty measure is real (as opposed to the possibility that an observed difference is due to sampling error). At this instance a few words of caution are at place. It should be remembered that at least the Sen measure is an *ordinal* measure, ie. that we obtain an ordering of populations according to the extent of poverty in each, but we cannot in fact tell *how much* more poverty there is in a one population compared with another. The interpretation of a "significant" difference must therefore be made with caution.

Let $y_1, y_2, \ldots, y_n$ denote n observations drawn by simple random sampling without replacement (SRS) from $\mathbf{Y}$. Then the proportion of poor units in the sample $q = \sum_{i=1}^{n} 1[y_i < z]/n$ is a consistent estimate of H, while $\bar{\theta}_I$ and $\bar{\theta}_{II}$ can be consistently estimated by

$$\widehat{\theta_I} = \sum_{i=1}^{n} \frac{\theta_I(y_i, z)1[y_i < z]}{n}$$

$$\widehat{\theta_{II}} = \sum_{1 \leq i \neq j \leq n} \frac{\theta_{II}(y_i, y_j, z)1[y_i \& y_j < z]}{n(n-1)}.$$

respectively. Substituting these estimates in P_I, P_{II} will in general yield consistent estimates of the specific poverty indices. We will first discuss the sampling properties of measures belonging to P_I.

3 Sampling properties of AD measures

Simple random samples

In the case of simple random sampling (SRS) the γ:th moment of a variable x is usually estimated by the corresponding sample moment, $m(\gamma) = n^{-1}\sum_{i=1}^{n} x_i^{\gamma}$. Further, as $\sigma^2(x^{\gamma}) = \mu(2\gamma) - \mu(\gamma)^2$, the variance of $m(\gamma)$ is usually estimated by

$$\widehat{\sigma}^2(\gamma) = \frac{m(2\gamma) - m(\gamma)^2}{n}. \tag{10}$$

Here we have assumed that the ratio n/N is small so that the finite population correction is negligible. These results can be applied to the poverty measures by assuming that $x = \theta(y, z)$. If we define the function

$$\theta(y_i, z) = 1 \tag{11}$$

the sample estimator of the head count - ratio can be written as

$$\widehat{H} = \frac{1}{n}\sum_{i=1}^{n} \theta_i 1[y_i < z]. \tag{12}$$

This is simply

$$\widehat{H} = \frac{q}{n} \tag{13}$$

where q is the number of the poor in the sample (that is, individuals with incomes less or equal to the poverty line) and n is the sample size.

The natural estimator of the above defined additively decomposable poverty measures is obviously

$$\widehat{P} = \frac{1}{n}\sum_{1\leq i\leq q} \theta(y_i, z). \tag{14}$$

Using the indicator function we can write (15) as

$$\widehat{P} = \frac{1}{n}\sum_{i=1}^{n} \theta(y_i, z). \tag{15}$$

This yields the following estimators for the Foster measure:

$$\widehat{F}_{\alpha} = \frac{1}{n}\sum_{i=1}^{n} 1[y_i < z]\left[\frac{z - y_i}{z}\right]^{\alpha} \tag{16}$$

Estimating the variance of the above estimator is straightforward. By applying (10) we get

$$Var(\widehat{P}) = \frac{1}{n}\left(n^{-1}\sum_{i=1}^{n} 1[y_i < n]O(y_i;z)^2 - \left[n^{-1}\sum_{i=1}^{n} 1[y_i < n]O(y_i;z)\right]^2\right). \tag{17}$$

This yields the following expression for the variance estimator of the Foster measure:

$$Var(\widehat{F}_\alpha) = \frac{1}{n}\left(n^{-1}\sum_{i=1}^{n} 1[y_i < z]\left[\frac{z-y_i}{z}\right]^{2\alpha} - \left[n^{-1}\sum_{i=1}^{n} 1[y_i < z]\left[\frac{z-y_i}{z}\right]^{\alpha}\right]^2\right). \tag{18}$$

More complex sampling designs

Linear estimation

It is very common to face a situation where the sample has not been drawn by SRS. In fact, very few large samples are drawn by this method. A number of methods have been developed in order to estimate statistics with high precision in more complex designs. We shall apply some of these findings to the data set we are working with (ELINOLO-1986).[3]

If we estimate the statistics as if they were obtained by simple random samples, we might obtain grossly biased estimates. Thus to get unbiased estimates we must take into account true sample design. Neglecting the fact that the number of households is unknown, the class of measures under consideration presently have the convenient property that we may obtain fairly simple expressions of their variances, even in complex settings. For nonlinear estimators the analytical expressions for the sample variances are difficult to derive and hence approximative methods have to be employed.

A complication arises from the fact that the frame lists individuals, whereas we are mainly interested in the study of households. This slightly alters the standard procedures for estimation from stratified samples. The probability that a household will be included in the sample is not constant (as it would if we were to study individuals) but varies with the household size. The following notation will be used:

[3] Tilastokeskus, 1988.

m_{hi} number of people in household i in stratum h (in the sample)
m_h sample size of stratum h
M_h total size of stratum h
M size of total population.

Assuming that sampling is with replacement but that the probability for unit to be drawn more than once is negligible, the probability for household i in stratum h to be selected is approximately

$$\pi_{hi} = \frac{m_h m_{hi}}{M_h}, \tag{19}$$

where notation is as above. Thus the probability of a household to be drawn is a function of its size. An important aspect of some sampling designs is that the number of households is unknown (we only know the total number of people, and observe m_{hi} from the sample) and must be estimated. We shall return to this complication later. We begin by abstracting from the fact that the number of households can be unknown and has to be estimated simultaneously. Generally, to estimate the mean of a variable θ for all households in a stratified sample we calculate

$$\hat{\bar{\theta}} = M^{-1} \sum_{h=1}^{H} \sum_{i=1}^{m_h} \frac{\theta_{hi}}{\pi_{hi}} \tag{20}$$

or

$$\hat{\bar{\theta}} = M^{-1} \sum_{h=1}^{H} \frac{M_h}{m_h} \sum_{i=1}^{m_h} \frac{\theta_{hi}}{m_{hi}}. \tag{21}$$

The estimated variance of this estimator is

$$\widehat{Var}(\hat{\bar{\theta}}) = \frac{1}{M^2} \sum_{h=1}^{H} \sum_{i=1}^{m_h} \left(\frac{\theta_{hi}}{\pi_{hi}} - \frac{1}{m_h} \sum_{h=1}^{H} \sum_{i=1}^{m_h} \frac{\theta_{hi}}{\pi_{hi}} \right)^2 \tag{22}$$

Applying this to the formula for the e.g. Foster measure we obtain the following expression:

$$\widehat{Var}(\hat{F}_\alpha) = \frac{1}{M^2} \sum_{h=1}^{H} \sum_{i=1}^{m_h} \left(\frac{1[y_i < z][(z - y_{hi})/z]^\alpha}{\pi_{hi}} - \frac{1}{m_h} \sum_{h=1}^{H} \sum_{i=1}^{m_h} \frac{1[y_i < z][(z - y_{hi})/z]^\alpha}{\pi_{hi}} \right)^2 \tag{23}$$

If we now take account of the fact that we are also estimating the number of households, we shall have a nonlinear estimation problem.

Nonlinear estimation

We now face the problem of estimating a ratio in a sample drawn from a stratified population. There are two courses of action to estimate the variance of the estimator, namely (a) to find an approximative expression for the variance estimator by some linearization of the ratio estimator or (b) to use resampling methods in the estimation of the ratio and its variance. We shall deal with the use of a specific resampling method, the bootstrap.[4]

The ratio to be estimated is

$$P(y,z) = \frac{\widehat{\Theta}}{\widehat{N}} \tag{24}$$

where $\Theta = \sum_h \sum_i \frac{\theta_{hi} 1[y_i < z]}{\pi_{hi}}$ is the sum total of $\theta_i 1[y_i < z]$ and $N = \sum_h \sum_i \frac{1}{\pi_{hi}}$ is the estimated number of households. The variance of this estimator can, using a Taylor expansion, be approximated by the linearization

$$\widehat{Var}(\widehat{P}) = \left(\frac{\widehat{\Theta}}{\widehat{N}}\right)\left(\frac{\widehat{Var}(\widehat{\Theta})}{\widehat{\Theta}^2} + \frac{\widehat{Var}(\widehat{N})}{\widehat{N}^2} - 2\frac{\widehat{Cov}(\widehat{\Theta},\widehat{N})}{\widehat{\Theta}\widehat{N}}\right). \tag{25}$$

where

$$\widehat{Cov}(\widehat{\Theta},\widehat{N}) = \sum_{h=1}^{H}\sum_{i=1}^{m_h}\left(\frac{\theta_i 1[y_i < z]}{\pi_{hi}} - \frac{\widehat{\Theta}}{m_h}\right)\left(\frac{1}{\pi_{hi}} - \frac{\widehat{N}}{m_h}\right). \tag{26}$$

We shall simulate a sample distribution by using the bootstrap method, the basic idea being that we can "estimate" the sampling distribution of a statistic by drawing K random samples (with replacement) from the given master sample. The resample size is the same as the master sample size. The statistic (in this case the poverty measure) will be calculated for each sample yielding $\widehat{P}^1, \widehat{P}^2, \ldots, \widehat{P}^K$ estimates. The mean of these, $\bar{P} = K^{-1}\sum_{i=1}^{K} \widehat{P}^i$ is a consistent estimate of the statistic P. The variance of the estimates is a consistent estimator of the variance of $\widehat{P}$, ie.

$$\widehat{\sigma^2(\widehat{P}^i)} = \frac{1}{K-1}\sum_{i=1}^{K}[\bar{\widehat{P}} - \widehat{P}^i]^2 \tag{27}$$

gives a consistent estimate of the sampling variance of $\widehat{P}$.

[4] Pahkinen and Lehtonen (1989), Efron (1982).

We estimate the poverty measure from each of the K bootstrap samples, obtaining $\widehat{P}_1, \ldots, \widehat{P}_K$ statistics, and apply the above formula for estimating the variance, thus yielding an approximative variance estimate. The estimated measures for standardized and unstandardized household incomes using both the non-linear and bootstrap estimators for standard errors are reported in Table 1. The measures are estimated for the poverty line $z = 22285$, which is the lower bound of the administrative poverty lines in Finland (see above).

The motivation for using the bootstrap method is that if the distribution of the variable in the population is unknown then the best we can do is to use the information we have about the distribution in the sample by trying to 'reconstruct' the population. In the present application, where we are dealing with a stratified sample, we have in each stratum a sample of the size m_h and the probability that an observation i is drawn is π_{hi}. Thus the total number of units in the population is $M = \sum_{h=1}^{H} \sum_{i=1}^{m_h} \pi_{hi}^{-1}$. To emulate the population we can let the sample represent the population. Then each observation in the sample represents M_h/m_h units of the total M_h in the stratum. By drawing a bootstrap-sample, ie. a random sample of the size m_h, with replacement, from every stratum of the 'reconstructed' population and repeating this a number of times, we start to obtain some variation in the parameters that are of interest. If we assume that the sample distribution is a good approximation to the population distribution, and repeat the procedure a large number of times, then the distribution of the estimated parameter will converge towards the 'true' distribution as $K \rightarrow \infty$. The number of bootstrap-samples drawn in the present study, $K = 1000$ is not very high in terms of the number of observations in the samples, but should be adequate for the purpose at hand.

A number of remarks concerning the present study should be noted. The disaggregated populations (sub-samples) are treated as independent samples. The original sampling probabilities have been recalculated, using the information about each sub-population in the original sample to obtain those probabilities that apply to the 'new' population. This means that we have calculated the probability of a unit to be drawn on the basis of the size of each sub-population (in stratum h) and its size in the sample. Thus the estimated poverty measures in each of the disaggregated samples are unbiased. Another item is that the bootstrap-samples are not drawn at random *from the whole* material, but the drawing in ***each stratum*** is random. This is how the 'master-sample' has been drawn. Thus the bootstrap-sample resembles more closely the original than if we were merely to draw at random from the entire sub-sample.

Sampling distribution

In this section we shall demonstrate the use of the bootstrap method in deriving the sampling distribution of the poverty measures suggested above. The "master-samples" we have chosen for this demonstration are two subsamples from the ELINOLO data, namely single adults (HHT1) and two non-aged adults with no children (HHT4).

The estimated values of the Foster measures for $\alpha = 0, 1, 2$ for the master samples are given in Table 1. The purpose of this simulation exercise is mainly to obtain some sort of picture of the sampling distributions of the poverty measures. The results should, of course, be regarded with some reservation as one cannot make very far going conclusions on the basis of only one population. However, we believe that the results do give some qualitative indications about the sampling properties of the relevant measures. The method employed is, however, applicable even in more general circumstances.

To give an idea of the sample distribution we have plotted the empirical frequency distributions of the measures in Figures 1-6. To make the issue clearer the scale of the horizontal axis is set to the estimated value in a bootstrap sample divided by the bootstrap-estimate of the measure. Beside the figures we have written the minimum, maximum, and standard deviation of the measure and their estimated value (the mean). The procedure was done for $K = 1000$.

Visual inspection of the sampling distributions of the poverty measures in the two subsets give rise to some quite general reflections.

Table 1: Estimated poverty measures and standard errors.

Measure	Single adult HHT1	Two adults HHT4
F_0	0.1240	0.0355
STD_{NL}	0.0053	0.0042
STD_{BS}	0.0032	0.0032
F_1	0.0483	0.0110
STD_{NL}	0.0034	0.0018
STD_{BS}	0.0018	0.0015
F_2	0.0567	0.0117
STD_{NL}	0.0060	0.0041
STD_{BS}	0.0027	0.0014

(1) The relative precision of the measures clearly depends on the proportion of the poor, in the sense that the more probable it is that a randomly selected person (household) will be poor the greater the precision of the estimate. This is intuitively appealing: consider an urn with one red ball and a hundred white ones. If the information we are seeking lies with the red ball, the probability that we obtain this information by drawing, say one hundred balls, at random is quite small. This, of course, is analogous with the fact that in a population where the poor are scarce, we face a greater risk of not drawing them in our sample. Therefore the relative precision of the estimates is greater in populations where poverty is abundant. Thus, if poverty is widespread, the variance of the estimators is small and our estimates are more reliable.

(2) The more complicated measure, F_2, (ie. that is distribution sensitive) is far less precise than the simpler ones (eg. the head count - ratio). Even this result is quite trivial, as we face several sources of variation in the more complicated measures compared with those that only take account of one or two population parameters. The more information we wish to obtain, say not only the relative number of the poor, but how poor, on average, they are and the inequality among them, the less precise will our estimates be.

(3) The precision of the measures, or rather their imprecision is of such a magnitude that

fairly large samples seem motivated to obtain good estimates. This, again, lies in the nature of poverty measurement; we shall (hopefully!) often have quite a small number of poor in a sample from a population. Therefore quite large samples are required for us to be confident of our results.

A general conclusion of the results obtained in this section is that as conceptual detail in a poverty measure increases so does the sampling error. The more complex measures seem to be better suited to situations where we may use fairly large samples and where we might expect to find wide spread poverty. In small samples from populations with very few poor households we can not expect to obtain very precise results. The problem of sampling error might also be far larger than is normally thought. One might feel confident, having a large sample (eg. the ELINOLO-sample, $m = 12057$) that sampling error is no problem, even if the sample is disaggregated by some criteria. But if the proportion of poor people in the population is only 0.02, and the information we wish to retain is about poverty, the reliability of our measurements can be seriously questioned. The considerations of statistical inference might be very important.

4 Sampling properties of Sens measure

U-statistics

In a famous paper from 1948 W. Hoeffding introduced the concept of so called U-statistics. Let us assume that we have n independent observations $x_1, x_2, ..., x_n$ on a stochastic variable x with the distribution F, and that we want to estimate a parameter $\phi(F)$ that can be expressed as

$$\phi(F) = \int\int ... \int g(x_1, x_2, ..., x_m)dF(x_1)dF(x_2)...dF(x_m) = E_F[g(x_1, x_2, ..., x_m)]. \qquad (28)$$

where g is a symmetric Borel measurable function called a kernel. Assuming that $n \geq m$, the corresponding U-statistic is defined as

$$U_n = \sum_{1 \leq i_1 \neq \ldots \neq i_m \leq n} g(x_{i_1}, x_{i_2}, \ldots . x_{i_m})/C(n, m). \tag{29}$$

where $C(a, b) = a(a-1)\ldots(a-b+1)$ for all positive integers a and b such that $a > b$.

Obviously U_n is an unbiased estimator of ϕ. To derive the large sample properties of U_n Hoeffding used the projection method. If we assume that

$$q(x_i) = E[U_n|x_i], \quad i = 1, 2, \ldots, n, \tag{30}$$

and that

$$\hat{U}_n = \sum_{1 \leq i \leq n} q(x_i)/n, \tag{31}$$

then it can, under certain regularity conditions, be established that

$$U_n - \phi = m(\hat{U}_n - \phi) + R_n \tag{32}$$

where $E(R_n^2) = 0(n^{-2})$. Using this result and the fact that the summands in $\hat{U}_n$ are independent random variables, it can be shown that under appropriate moment conditions on $q(x_i)$, asymptotically

$$\sqrt{n}(\hat{U}_n - \phi) \sim N(0, m^2\zeta_1). \tag{33}$$

where

$$\zeta_1 = Var[q(x_i)]. \tag{34}$$

Corresponding results can be derived for sets of U-statistics calculated from the same sample.

In the classical U-statistics theory it is assumed that we have samples from a probability distribution. However, as shown by e.g. Sen (1972) and Sen (1988) the theory can also be extended to cover the case of probability sampling without replacement from finite populations. Let us assume that our populations consists of N units with values $\mathbf{Y} = [y_1, y_2, \ldots, y_N]$ and that we are interested in p parameters $\phi_1, \phi_2, \ldots, \phi_p$, which can be expressed as

$$\phi_j(\mathbf{Y}) = \sum_{1 \leq i_1 \neq i_2 \neq \ldots \neq i_{m_j} \leq N} g_j(y_{i_1}, \ldots, y_{i_{m_j}})/C(N, m_j). \tag{35}$$

where $g_1, g_2, ..., g_p$ denote the corresponding symmetric kernel functions.

For a SRS of size n ($\max(m_1, ..., m_p) \leq n \leq N$) and consisting of the observations $y_1, y_2, ..., y_n$ the optimal, unbiased estimators for the parameters are the corresponding U-statistics given by (Sen, 1988)

$$\widehat{\phi}_j = \sum_{i \leq i_1 \neq ... \neq i_{m_j} \leq n} g_j(y_{i_1}, ..., y_{i_{m_j}})/C(n, m_j), j = 1, 2, ..., p. \tag{36}$$

The expressions for the exact moments of the $\widehat{\phi}_j$:s are obviously quite complicated. However assuming that the population and sample sizes are fairly large we can use the asymptotic results derived by Nandi and Sen (1963), Sen (1972) and others. The main idea is again to express the U-statistics in terms of their projections.

Let us assume a sequence $\{\mathbf{Y}_N\}$ of populations and a sequence $\{s_n\}$ of samples such that as N increases then $n/N \rightarrow \alpha$. Then if $\widehat{\phi}_1$ and $\widehat{\phi}_2$ denote a pair of U-statistics calculated from the same sample it can be shown that asymptotically

$$\sqrt{n}\left[\begin{array}{c} \widehat{\phi}_1 \\ \widehat{\phi}_2 \end{array} - \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right] \sim N\Big(0, \left[\begin{array}{cc} m_1^2 Var(q_1(y_{i_1})) & m_1 m_2 Cov(q_1(y_{i_1}), q_2(y_{i_1})) \\ m_2 m_1 Cov(q_2(y_{i_1}), q_2(y_{i_1})) & m_2^2 Var(q_2(y_{i_1})) \end{array} \right]\Big)(1-\alpha), \tag{37}$$

where

$$q_j(y_{i_1}) = \Sigma^*_{i_2, ..., i_{m_j}} g_j(y_{i_1}, y_{i_2}, ..., y_{i_{m_j}})/C(N-1, m_j - 1). \quad j = 1, 2. \tag{38}$$

and the summation Σ^* extends over all distinct combinations $i_2, ..., i_{m_j}$ over the set $\{1, 2, ..., N\} \backslash i_1$. Thus

$$Var(\widehat{\phi}_j) = (1-\alpha) m_j^2 (\sum_{1 \leq i \leq N} q_j(y_i)^2/N - \phi_j^2), j = 1, 2 \tag{39}$$

and

$$Cov(\widehat{\phi}_1, \widehat{\phi}_2) = (1-\alpha) m_1 m_2 (\sum_{1 \leq i \leq N} q_1(y_i) q_2(y_i)/N - \phi_1 \phi_2). \tag{40}$$

By substituting the sample variances and covariances for the corresponding population parameters we can estimate the asymptotic variance-covariance matrix for the U-statistics.

For probability sampling with unequal probabilities the situation is more complicated especially with respect to the asymptotic distributions of the estimates. Our approach in this paper is mainly heuristic, as we assume that the basic results concerning large sample behaviour of U-statistics in the case of SRS-sampling can under mild conditions be generalized to apply also for the case of probability sampling. Folsom (1984) and Williams (1988) have studied the use of U-statistics in this situation more thoroughly. Williams showed e.g. that central limit theorems for linear statistics from unequal probability samples can be extended to U-statistics.

Let as assume that we have drawn the sample by some type of fixed size, varying probability sampling without replacement. As usually we define the first order inclusion probability of unit i as the probability π_i of obtaining a sampling that includes the i:th unit and the second order inclusion probability of a pair of units k and l as the probability π_{kl} of obtaining a sample that includes both units k and l. Higher order inclusion probabilities are defined in the same way. We assume that all inclusion probabilities are positive for all the combinations of units in the population under study.

Using the traditional Horvitz-Thompson approach it is easily seen that

$$\widehat{\phi}_j = \sum_{1 \le i_1 \ne \ldots \ne i_{m_j} \le n} [g_j(y_{i_1}, \ldots y_{i_{m_j}})/\pi_{i_1 i_2 \ldots i_{m_j}}]/C(N, m_j), \tag{41}$$

where the summation is again over all distinct ordered sets of m_j element in the sample and $\{\pi_{i_1 i_2 \ldots i_{m_j}}\}$ denote the m_j-order inclusion probabilities, is an unbiased estimator for ϕ_j. In this case the projection is

$$\begin{aligned} q_{\pi_j}(y_{i_1}) &= E[g_j(y_{i_1}, \ldots y_{i_{m_j}})|y_{i_1}] & (42) \\ &= \frac{\sum^*_{i_2, \ldots, i_{m_j}} \pi_{i_2 \ldots i_{m_j}|i_1} g_j(y_{i_1}, \ldots y_{i_{m_j}})}{\sum^*_{i_2, \ldots, i_{m_j}} \pi_{i_2 \ldots i_{m_j}|i_1}} & (43) \end{aligned}$$

where $\pi_{i_2 \ldots i_{m_j}|i_1}$ denote the joint probability that also units $i_2, \ldots i_{m_j}$ are included in the sample given that unit i_1 is included, and where the summation $\sum^*$ is over all distinct $i_2, \ldots i_{m_j}$ over the set $\{1, 2, \ldots, N\} \backslash i_1$.

Thus assuming that $Var(\widehat{\phi}_j) \simeq Var(m_j \sum_{1 \le i \le n} (1/N) q_{\pi_j}(y_i)/\pi_i)$ we get, by standard paper is mainly heuristic, as we assume that the basic results concerning large sample behaviour of U-statistics in the case of SRS-sampling can under mild conditions be generalized to apply also for the case of probability sampling. Folsom (1984) and Williams (1988) have studied the use of U-statistics in this situation more thoroughly. Williams showed e.g. that central limit theorems for linear statistics from unequal probability samples can be extended to U-statistics.

Let as assume that we have drawn the sample by some type of fixed size, varying probability sampling without replacement. As usually we define the first order inclusion probability of unit i as the probability π_i of obtaining a sampling that includes the i:th unit and the second order inclusion probability of a pair of units k and l as the probability π_{kl} of obtaining a sample that includes both units k and l. Higher order inclusion probabilities are defined in the same way. We assume that all inclusion probabilities are positive for all the combinations of units in the population under study.

Using the traditional Horvitz-Thompson approach it is easily seen that

$$\hat{\phi}_j = \sum_{1 \leq i_1 \neq \ldots \neq i_{m_j} \leq n} [g_j(y_{i_1}, \ldots y_{i_{m_j}})/\pi_{i_1 i_2 \ldots i_{m_j}}]/C(N, m_j). \tag{41}$$

where the summation is again over all distinct ordered sets of m_j element in the sample and $\{\pi_{i_1 i_2 \ldots i_{m_j}}\}$ denote the m_j-order inclusion probabilities, is an unbiased estimator for ϕ_j. In this case the projection is

$$q_{\pi_j}(y_{i_1}) = E[g_j(y_{i_1}, \ldots y_{i_{m_j}})|y_{i_1}] \tag{42}$$

$$= \frac{\sum^{*}_{i_2,\ldots,i_{m_j}} \pi_{i_2 \ldots i_{m_j}|i_1} g_j(y_{i_1}, \ldots y_{i_{m_j}})}{\sum^{*}_{i_2,\ldots,i_{m_j}} \pi_{i_2 \ldots i_{m_j}|i_1}} \tag{43}$$

where $\pi_{i_2 \ldots i_{m_j}|i_1}$ denote the joint probability that also units $i_2, \ldots i_{m_j}$ are included in the sample given that unit i_1 is included, and where the summation $\sum^{*}$ is over all distinct $i_2, \ldots i_{m_j}$ over the set $\{1, 2, \ldots, N\} \backslash i_1$.

Thus assuming that $Var(\hat{\phi}_j) \simeq Var(m_j \sum_{1 \leq i \leq n} (1/N) q_{\pi_j}(y_i)/\pi_i)$ we get, by standard

Horvitz-Thompson theory, that

$$Var(\hat{\phi}_j) = Var(m_j(1/N) \sum_{1 \le k \le n} q_{\pi_j}(y_k)/\pi_k) = (m_j^2/N^2) \sum_{1 \le k,l \le N} (\pi_{kl} - \pi_k\pi_l) q_{\pi_j}(y_k) q_{\pi_j}(y_l)/\pi_k\pi_l \tag{44}$$

for which an estimator is given by

$$\widehat{Var(\hat{\phi}_j)} = (m_j^2/N^2) \sum_{1 \le k,l \le n} (\pi_{kl} - \pi_k\pi_l) \hat{q}_{\pi_j}(y_k) \hat{q}_{\pi_j}(y_l)/\pi_k\pi_l\pi_{kl}, \tag{45}$$

where the summation is now over all pairs of units in the sample. For the covariance between the estimates $\hat{\phi}_i$ and $\hat{\phi}_j$ we get the expression

$$\begin{aligned} Cov(\hat{\phi}_i, \hat{\phi}_j) &= Cov[m_i(1/N) \sum_{1 \le k \le n} q_{\pi_i}(y_k)/\pi_k, m_j(1/N) \sum_{1 \le k \le n} q_{\pi_j}(y_k)/\pi_k] \\ &= (m_i m_j/N^2) \sum_{1 \le k,l \le N} (\pi_{kl} - \pi_k\pi_l) q_{\pi_i}(y_k) q_{\pi_j}(y_l)/\pi_k\pi_l \end{aligned} \tag{46}$$

with the estimator

$$\widehat{Cov(\hat{\phi}_i, \hat{\phi}_j)} = (m_i m_j/N^2) \sum_{1 \le k,l \le n} (\pi_{kl} - \pi_k\pi_l) \hat{q}_{\pi_i}(y_k) \hat{q}_{\pi_j}(y_l)/\pi_k\pi_l\pi_{kl}. \tag{47}$$

Application to Sens measure

It is easily seen that many of the poverty indices can be written in the "kernel form" as

$$P = h(\phi_1, \phi_2, ..., \phi_p), p > 1, \tag{48}$$

where h is a known function and $\phi_j, j = 1, 2, ...p$, denote the population means for some symmetric kernels $g_j(y_{i1}, ..., y_{im_j}), m_j > 1$. For instance, if $p = 1$ and g_1 is poverty intensity function $\theta(y_i, z)$ and h is the identity relation, we get the class of additively decomposable poverty measures. Perhaps more interestingly, also the Sen measure (and measures of the Sen-type) can be expressed in kernel form.

As shown in e.g. Jäntti (1991) the Sen measure can be expressed as

$$P_S = H[1 - (\mu_p/z)(1 - G_p)], \tag{49}$$

where H is the proportion of poor units μ_p is the mean income among the poor and G_p is the Gini coefficient for the income distribution among the poor. Further, as

$$G_p = (NH-1)/NH - \sum_{1\leq i\neq j\leq N} 1[y_i\&y_j < z]\min(y_i, y_j)/(N^2H^2\mu_p) \tag{50}$$

and thus, if the number of poor units is not very small,

$$G_p = 1 - \sum_{1\leq i\neq j\leq N} 1[y_i\&y_j < z]\min(y_i, y_j)/(N^2H^2\mu_p) \tag{51}$$

we see that

$$P_S = (1/H) \sum_{1\leq i\neq j\leq N} 1[y_i\&y_j < z](1-\min(y_i, y_j)/z)/(N(N-1)) \tag{52}$$

if also $(N-1)/N \simeq 1$. Thus P_S may be expressed as

$$P_S = \phi_2/\phi_1, \tag{53}$$

where the corresponding kernel functions are

$$\begin{aligned} g_1(y_i) &= 1 \quad \text{if} \quad y_i < z && (54)\\ &= 0 \quad \text{in other cases}, && (55)\\ g_2(y_i, y_j) &= 1-\min(y_i, y_j)/z, \quad \text{if} \quad y_i\&y_j < z && (56)\\ &= 0 \quad \text{in other cases}. && (57) \end{aligned}$$

Assuming that we have obtained the sample $y_1, y_2, ..., y_n$ by a sampling design characterized by the first and second order inclusion probabilities $\{\pi_1, ..., \pi_N, \pi_{12}, ..., \pi_{N-1,N}\}$, P_S can be consistently estimated by

$$\widehat{P}_S = \widehat{\phi}_2/\widehat{\phi}_1, \tag{58}$$

where

$$\begin{aligned} \widehat{\phi}_1 &= (1/N) \sum_{1\leq k\leq n} 1[y_k < z]/\pi_k = \widehat{H} && (59)\\ \widehat{\phi}_2 &= 1/(N(N-1)) \sum_{1\leq k\neq l\leq n} 1[y_k\&y_l < z](1-\min(y_k, y_l)/z)/\pi_{kl}. && (60) \end{aligned}$$

Further

$$Var(\widehat{\phi}_1) = \frac{1}{N^2} \sum_{1 \le k,l \le N} (\pi_{kl} - \pi_k \pi_l) \frac{q_{\pi_1}(y_k)}{\pi_k} \frac{q_{\pi_1}(y_l)}{\pi_l}, \tag{61}$$

where

$$q_{\pi_1}(y_r) = 1[y_r < z], \quad r = 1, \dots, N \tag{62}$$

and

$$Var(\widehat{\phi}_2) = \frac{4}{N^2} \sum_{1 \le k,l \le N} (\pi_{kl} - \pi_k \pi_l) \frac{q_{\pi_1}(y_k)}{\pi_k} \frac{q_{\pi_1}(y_l)}{\pi_l}. \tag{63}$$

where

$$q_{\pi_2}(y_r) = \frac{\sum_{1 \le r' \ne r \le N} \pi_{r'|r} 1[y_r \& y_{r'} < z](1 - \min(y_r, y_{r'})/z)}{\sum_{1 \le r' \ne r \le N} \pi_{r'|r}}. \tag{64}$$

Finally,

$$Cov(\widehat{\phi}_1, \widehat{\phi}_2) = \frac{2}{N^2} \sum_{1 \le k,l \le N} (\pi_{kl} - \pi_k \pi_l) \frac{q_{\pi_1}(y_k)}{\pi_k} \frac{q_{\pi_2}(y_l)}{\pi_l}. \tag{65}$$

By substituting for the "population sums" the "sample sums" with the summands expanded by the corresponding inclusion probabilities we get estimators for the variances and the covariance. Then, using the traditional "linearization approach" $Var(\widehat{P}_S)$ may be estimated by

$$\widehat{Var}(\widehat{P}_S) = (1/\widehat{\phi}_1)^2 \widehat{Var}(\widehat{\phi}_2) + (\widehat{\phi}_2/\widehat{\phi}_1^2)^2 \widehat{Var}(\widehat{\phi}_1) - 2\widehat{\phi}_2/\widehat{\phi}_1^3 \widehat{Cov}(\widehat{\phi}_1, \widehat{\phi}_2). \tag{66}$$

The results above are obviously applicable as such if we want to estimate e.g. the Thon-measure, but can also be used for more complicated measures.

To further illustrate the results we have undertaken two small simulation studies using a real population. In the first simulation, we have used a SRS sampling design. In the second, we divided the population in two strata with different sampling fractions. In both cases the sample size is 100. The results are reported in Appendix 1. The estimated variance calculated by the above formulas seems in general to give a fairly good estimate of the variance of the sampling distribution.

5 The poverty line as a parameter

One of the problematic aspects of using poverty measures is the choice of a poverty line. In the analysis above we have assumed the poverty line to be a predetermined constant. However, in practice it is not unusual that the poverty line is defined as some ratio of the estimated median (or mean) income. This means that in a strict sense, the poverty line should also be treated as a parameter that is estimated from the sample. However, as is well-known, already the estimation of the median and its sampling variance is quite complicated for even moderately complex sampling designs (see e.g. Sedransk and Smith, 1988).

One possible approach is the following. Consider (P, z), a two-dimensional parameter which is estimated by $(\widehat{P}, \widehat{z})$. Then

$$Var(\widehat{P}) = E[Var(\widehat{P}|\widehat{z})] + Var[E(\widehat{P}|\widehat{z})] \tag{67}$$

Thus, if we use the sample to estimate the sampling distribution of $\widehat{z}$ on one hand, and the sampling distribution of $\widehat{P}$ for a set of probable values of z on the other, it is possible to use (65) in order to estimate $Var(\widehat{P})$. Another, perhaps more practical method would be to use bootstrap- or jackknife-techniques.

Appendix

To study the properties of the variance estimator for the Sen measure we conducted two small simulation studies. In both cases the population consisted of a sample of single person households from the 1985 census with data on taxable income drawn from tax records. The size of the population was $N = 4272$ households. Using a poverty vline of FIM= 30000 (approximately 50% of the median), the proportion of poor units in the population was 27%, and the value of the Sen measure was $P_S = .17$.

In the first study we draw by simple random sampling without replacement 200 samples of size $n = 100$. For each sample we estimate P_S by (58) and $\widehat{Var}(\hat{P}_S)$ by (66). In the first row of Table A1 we show the means and the variances of the estimates.

In the second simulation the population was split into two strata with $N_1 = 1000$ and $N_2 = 3272$. The proportions of poor units in the two strata were 40% and 23%. Again, we draw 200 samples using simple random sampling without replacement within strata and sample sizes $n_1 = n_2 = 50$. The means and the variances of the estimates of P_S and $\widehat{Var}(\hat{P}_S)$ are given in the second row of Table A1. In both cases the variance estimator seems to provide a fairly accurate approximation of the sampling variance, despite the small sample sizes.

Table A1: Means and variances of $\hat{P}_S$ and $\widehat{Var}(\hat{P}_S)$

Simulation	$\hat{P}_S$		$\widehat{Var}(\hat{P}_S)$	
	Mean	Variance	Mean	Variance
I	.17	.0011	.0010	4.84×10^{-8}
II	.17	.0012	.0012	1.23×10^{-7}

References

Efron, B. (1982): *The jackknife, the bootstrap and other Resampling Plans*, Society for Industrial and Applied Mathematics 38, Philadelphia, Pennsylvania.

Fishburn, P. (1980): Stochastic dominance and moments of distributions, *Mathematics of Operations Research*, Vol. 5, No. 1, pp. 94-100.

Folsom, R.E.(1984): Probability sample U-statistics: Theory and applications for complex sample designs. Ph.D. dissertation. University of North Carolina, Chapel Hill, USA.

Foster, J. (1984): On economic poverty, *Advances in Econometrics*, vol. 3. pp. 215-251. JAI press.

Foster, J. , Greer, J. & Thorbecke, E. (1984): A class of decomposable poverty measures, *Econometrica*, Vol. 52 no. 3, pp. 761-766.

Hoeffding, W.(1948): A class of statistics with asymptotically normal distributions. Annals of Mathematical Statistics. Vol. 19. p. 293-325.

Jäntti, M. (1991): On the measurement of poverty: Conceptual issues and estimation problems. *Meddelanden från Ekonomisk-Statsvetenskapliga Fakulteten vid Åbo Akademi*. A:328, Turku, Finland.

Kakwani, N. (1989): Testing for Significance of Poverty Differences: With Applications to Côte d'Ivoire, *LSMS Working Paper* no. 62, Washington, World Bank.

Malmberg, J.-O.(1988): A.K. Senin köyhyysmitan estimointi. Depatment of Statistics, University of Helsinki. (mimeographed, in finnish)

Pahkinen & Lehtonen (1989): *Otanta-asetelmat ja tilastollinen analyysi*, Helsinki, Gaudeamus.

Sedransk, J. and Smith, P.J. (1988): Inference for finite population quantiles. In Krishnaiah, P.R. and Rao, C.R. (eds.): *Handbook of Statistics*, Vol. 6, Amsterdam: North-Holland.

Sen, A.K. (1976): Poverty: An ordinal approach to measurement, *Econometrica*, Vol. 44, pp. 219-231.

Sen, A.K. (1979): Issues in the measurement of poverty, *Scandinavian Journal of Economics*, 81, pp. 285-307.

Sen, A.K. (1982), *Choice, Welfare and Measurement*, Oxford, Basil Blackwell.

Sen P.K.(1972): Finite population sampling and weak convergence to a Brownian bridge. Sankhya. Vol. A 34, p. 85-90.

Sen P.K.(1988): Asymptotics in finite population sampling. In P.R. Krishnaiah and C.R. Rao (eds.): Handbook of Statistics. Vol. 6. Amsterdam: Elsevier Science Publishers. p. 291-333.

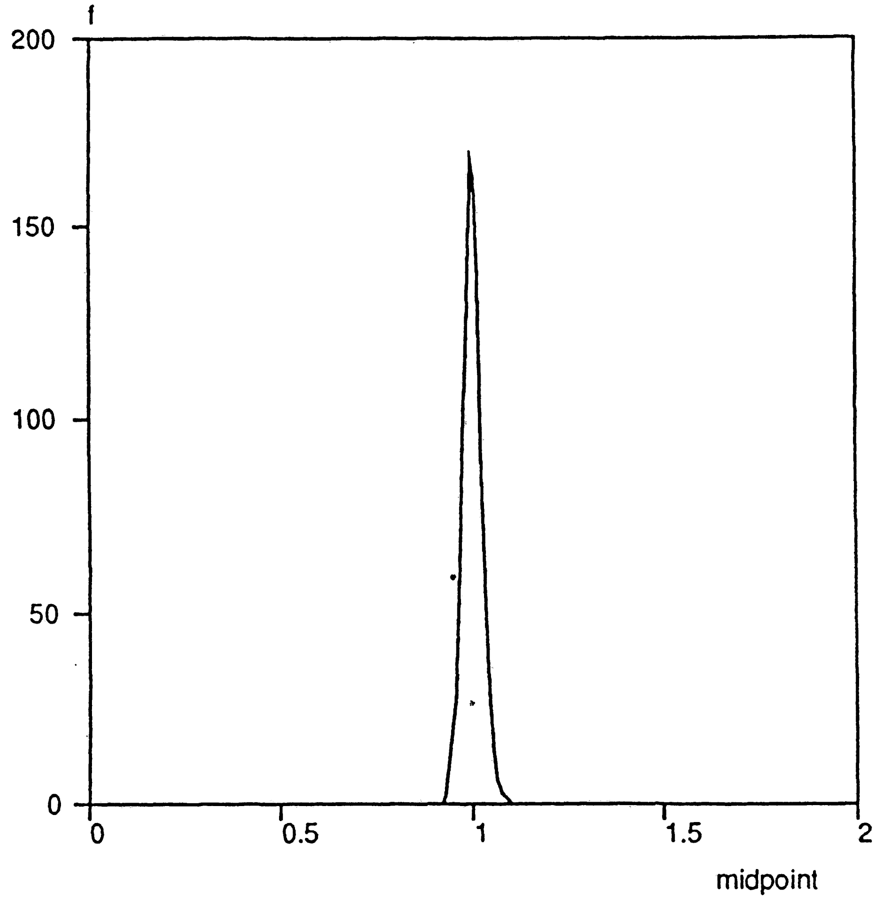
Seidl, C. (1988): Poverty measurement: A survey. In Bös, D., Rose, M. and Seidl, C. (eds.): *Welfare and efficiency in public economics.*, Heidelberg, Springer, p. 71-147.

Thon, D. (1979): On measuring poverty, *Review on income and wealth*, vol. 25, pp. 429-439.

Tilastokeskus (1988): *Elinolotutkimus 1986*, Aineiston laatu; tietoa tyoprosessista laadun kannalta, Raportti 1988:12, Helsinki.

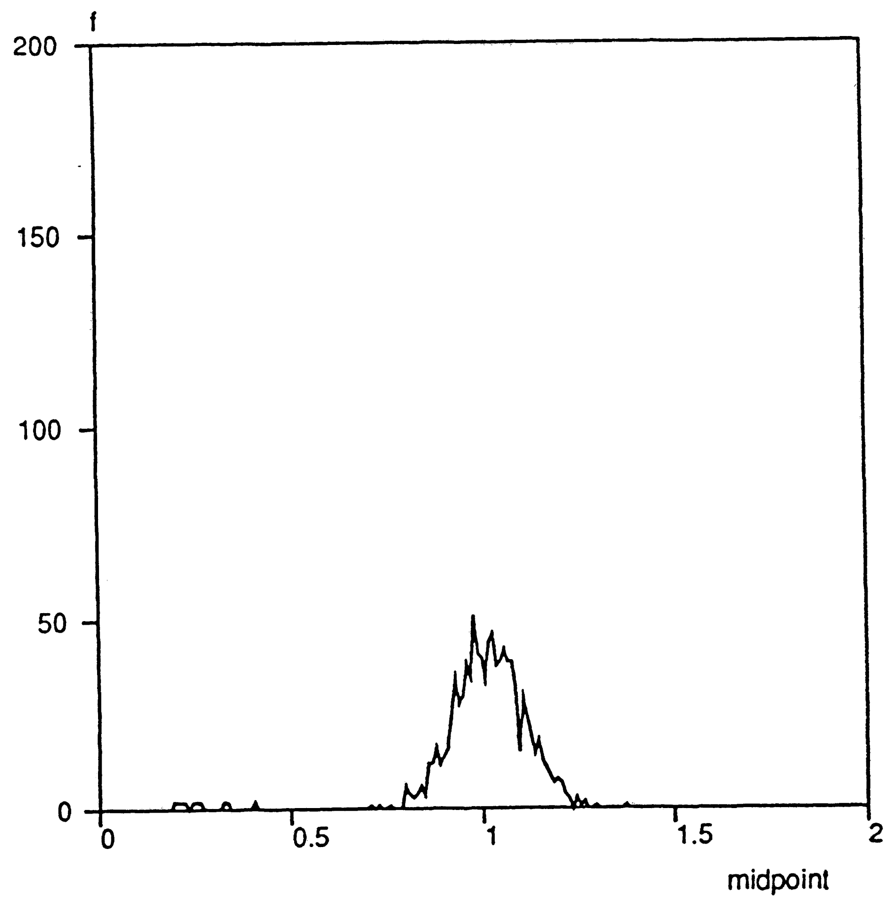
Williams R.L.(1988): Large sample theory for U-statistics in unequal probability samples. Ph.D. dissertation. University of North Carolina, Chapel Hill, USA.

Figure .1. Sampling distribution of poverty measure F_0, Household type 1



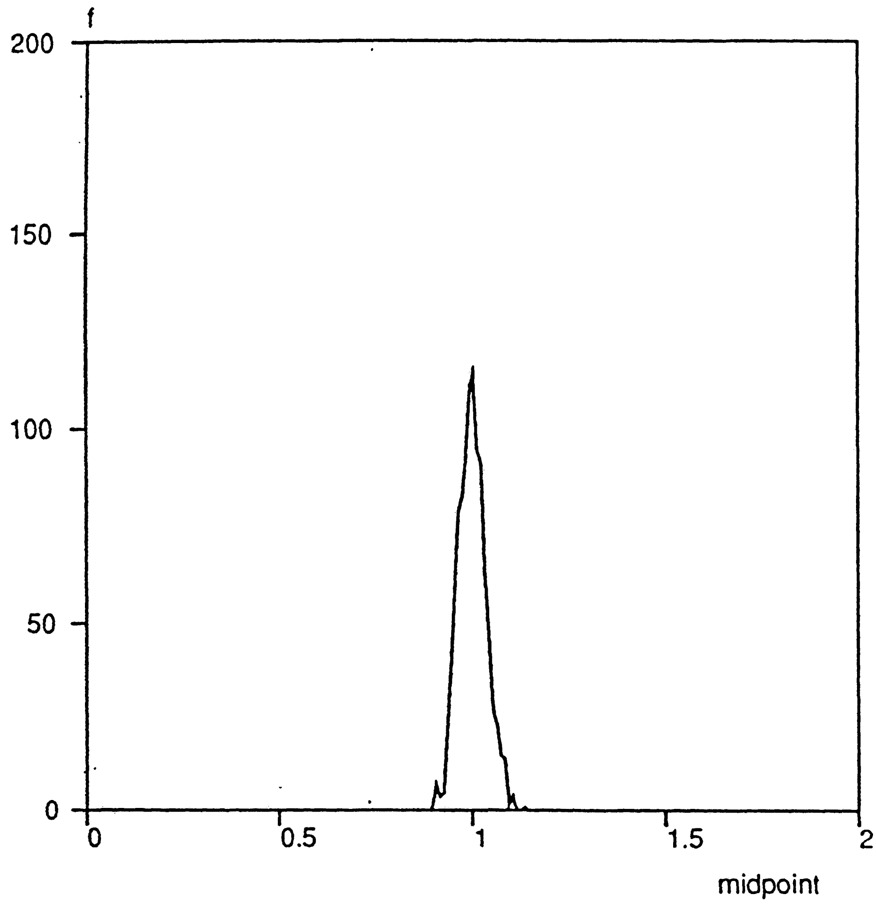
mean	=	0.1240
std	=	0.0031
min	=	0.1150
max	=	0.1354

Figure .2. Sampling distribution of poverty measure F_0, Household type 4



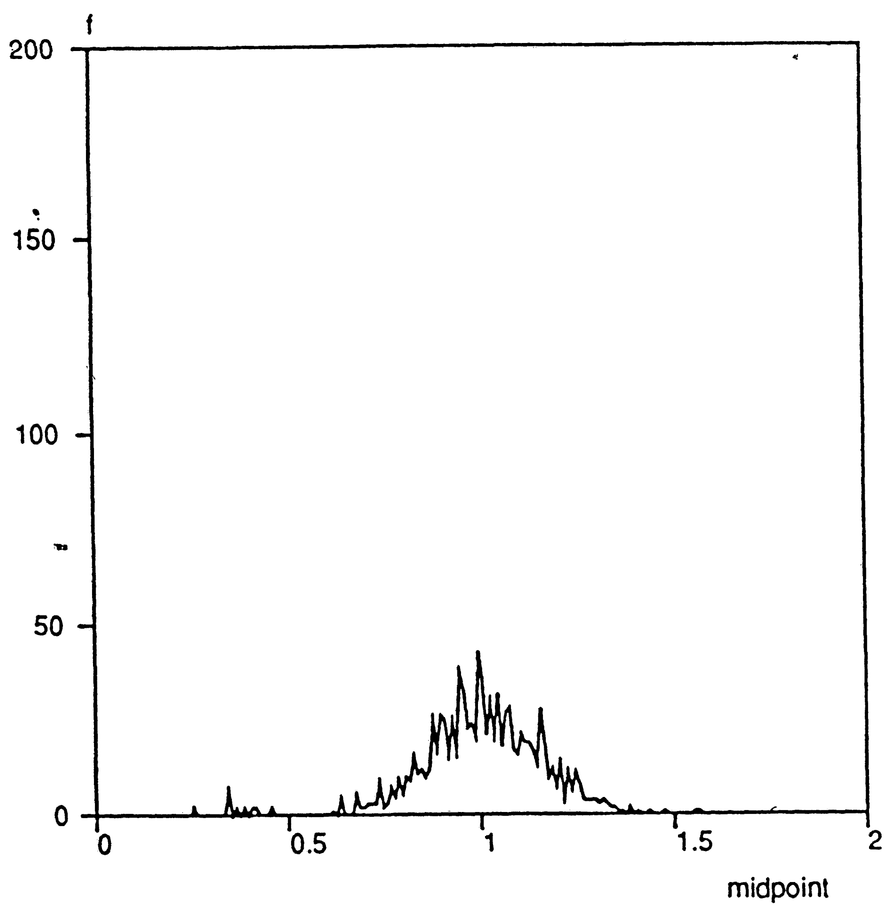
mean	=	0.0349
std	=	0.0048
min	=	0.0096
max	=	0.0481

Figure .3. Sampling distribution of poverty measure F_1, Household type 1



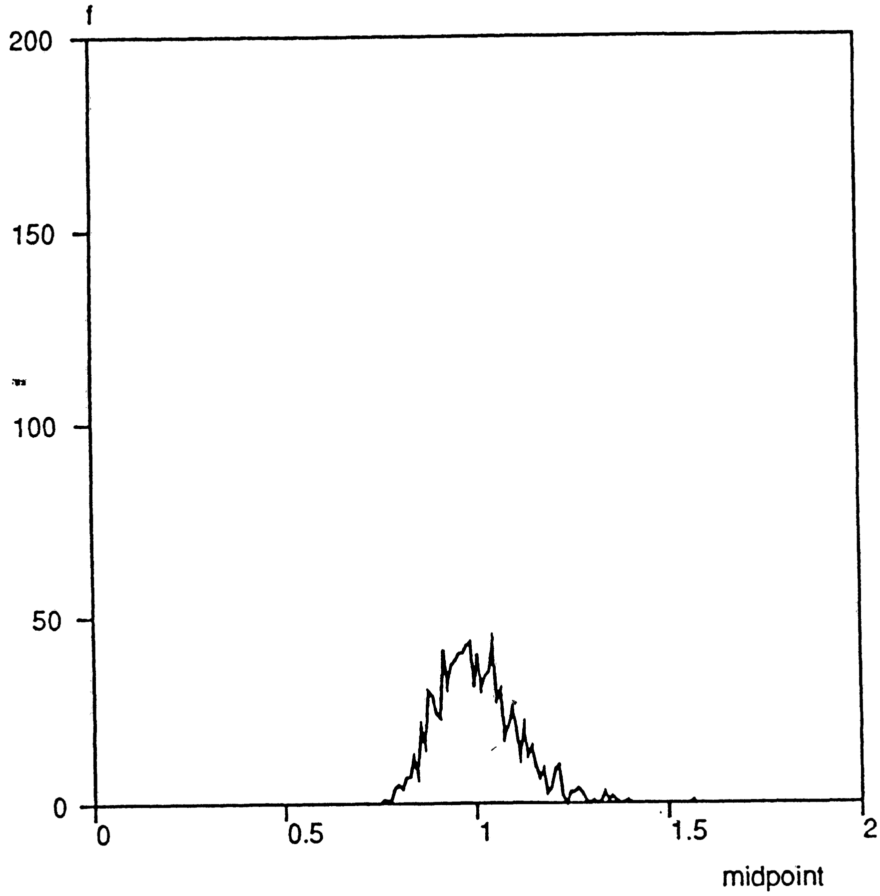
mean = 0.0484
std = 0.0018
min = 0.0433
max = 0.0552

Figure .4. Sampling distribution of poverty measure F_1, Household type 4



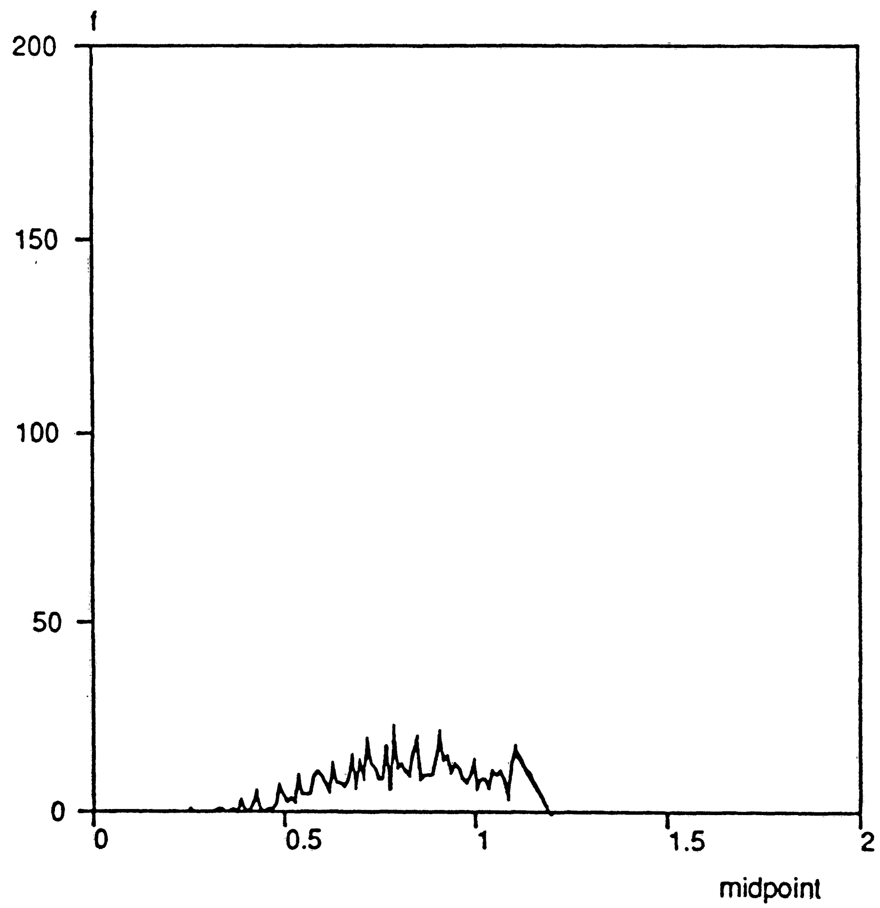
mean = 0.0109
std = 0.0018
min = 0.0027
max = 0.0171

Figure 5. Sampling distribution of poverty measure F_2, Household type 1



mean = 0.0567
std = 0.0060
min = 0.0426
max = 0.0885

Figure 6. Sampling distribution of poverty measure F_2, Household type 4



mean = 0.0117
std = 0.0041
min = 0.0030
max = 0.0337

THE LEYDEN POVERTY LINE WHEN PRICES ARE INCOME-DEPENDENT

Jolanda M. van Leeuwen,
Tinbergen Institute, Erasmus University Rotterdam,
P.O.Box 1738, 3000 DR Rotterdam, The Netherlands

Abstract

In modern welfare states, introducing income-dependent prices is a popular government strategy to support people with low incomes. Throughout this paper income-dependent prices are non-decreasing functions of income. One can think of rent subsidy on housing, charges for legal assistance, tuition fees, charges for health care and charges for child care. In the Netherlands the most important income-dependent price is rent for housing; about 12% of Dutch households receive an income-dependent rent subsidy which can be close to 50% of (market-value) rent.

B.M.S. Van Praag and M.R. Baye incorporated income-dependent prices in objective poverty lines in their theoretical paper "The poverty concept when prices are income-dependent" published in the first 1990 issue of the Journal of Econometrics. In addition to that research, this paper will deal with the effects of income-dependent prices on subjective poverty lines, more precise the Leyden poverty line, and presents some empirical results.

A major finding is that people do not bear in mind income-dependent prices when they answer the income-evaluation question, i.e. they do not mention extra low incomes evaluated with "very bad" nor do they mention extra high incomes evaluated with "very good", because of the higher, respectively lower, subsidies in those income situations. Therefore, we modify the incomes mentioned before they are used to calculate poverty lines.

Van Praag and Baye concluded that when income-pricing is ignored one overstates the "true" number of households living in poverty. To find out how many households are falsely considered as being poor, we will compare the outcomes of three methods: the first takes into account income-dependent prices as they really are; the second considers income-dependent subsidies as extra income; and the last one ignores the existence of income-dependent prices.

November, 1992

A MICROSIMULATION MODEL OF CONSUMER BEHAVIOR FOR TAX ANALYSIS

by

Jørgen Aasness and Jing Li

ABSTRACT

A microsimulation model of consumer behavior, named KONSUM, is presented and applied to analyze distributional effects of changes in direct and indirect taxes. The starting point of KONSUM is a household's utility function, with parameters depending of the number of children and adults in the household, and corresponding demand functions for 13 commodity groups. Corner solutions are taken into account. Utility based price indexes are defined for each household. Alternative measures of equivalence scales, inequality, poverty, and welfare are defined. KONSUM is connected to LOTTE which provides a minipopulation of Norwegian households with detailed income accounts and a complete set of updated rules of direct taxation. The effects of indirect taxation on consumer prices are modelled using a version of the price block of the macroeconometric model MODAG, taking into account detailed institutional rules and the input-output structure of the Norwegian economy. Results from a specific application: Doubling the rates of child allowances reduces the number of poor children in Norway from 101 000 to 42 000. Reduced VAT of food contributes to further reduction of the number of poor children.

Central Bureau of Statistics, Research Department, P.B. 8131 Dep., N-0033 Oslo, Norway.
tel: 47-2-864823, fax: 47-2-111238, E-mail: J2A@SSB.NO.

Abstract of paper presented at the conference on Poverty and Distribution, Oslo, November 16-17, 1992. A handout was distributed at the seminar. A final version of the paper will later be sent to those interested.

AN APPLICATION OF LATENT MARKOV MODELS TO ESTIMATE RESPONSE ERRORS FROM REPEATED SURVEYS

By Ib Thomsen and Dinh Quang Pham

1. INTRODUCTION

In this paper we shall use a latent Markov model to estimate response errors from panel data. Early applications of this approach is given in Lazarsfeld and Henry (1968) and Wiggin (1973). However, it seems as if these results have been given little attention by researchers concerned with estimation of response errors in sample surveys. This we feel is a petty, especially because new estimation methods have been developed recently. In Bye and Schechter (1968) maximum likelihood estimates are calculated by means of the Newton-Raphson algorithm, and in Langeheine and Van de Pol (1990) a very general program for estimation by means of the E-M algorithm in latent Markov models, is presented. At the Central Bureau of Statistics of Norway we have used the E-M algorithm to estimate response errors in connection with several panel surveys. The purpose of this note is to present some results and experiences from this work.

2. THE MODEL AND ESTIMATION PROCEDURES

We shall use a standard response error model for categorical variables. We shall limit our attention to cases where the categorical variable takes two values, but the extension to other variables is straight forward.

At any point in time the population is divided into two latent classes, for instance in the labour force and out of the labour force. Let $l_i(t)$ denote the latent probability of belonging to class i at time t. Assume that individuals move from one latent class to the other, following a simple Markov Process. Let m_{ij} denote the probability of being in latent class j at time (t+1) given latent class i at time t. Due to response error membership in latent class i is not directly observable. Let q_{ij} denote the probability of responding class j given that the true class is i.

After having observed the same individuals at two points in time, the observations can be presented in a table, P_{ij} (1,2), where P_{ij} (1,2) denotes the proportions of individuals observed as members of class i at time 1 and class at time 2.

Assuming that the response probabilities, q_{ij}, are constant over time, and that movements between the classes over time follows a simple Markov model, we have that

$$E\,(P_{ij}(1,2)) = \sum_{a=1}^{2}\sum_{b=1}^{2} l_a(1) q_{ai} m_{ab} q_{bj} \quad (1)$$

After having inserted the observed values of $P_{ij}(1,2)$ on the left side in equations (1) we have three equations with five parameters. In order to make the parameters identifiable, further assumptions have to be made. Such an approach is used in Wiggin (1973) and Chua and Fuller (1987). See also Poterba and Summers (1986) for a discussion of these assumptions.

If data are available for more than two points in time, and the response and transition probabilities remain constant over time, it is possible to estimate the parameters. Let $P_{ijk}(1,2,3)$ denote the proportion of individuals belonging to class i,j and k at times 1,2 and 3. Then it follows that

$$E\,(P_{ijk}(1,2,3)) = \sum_{a=1}^{2}\sum_{b=1}^{2}\sum_{c=1}^{2} l_a(1) q_{ai} m_{ab} q_{bj} m_{bc} q_{ck}$$

$$(i=1,2; j=1,2; k=1,2) \quad (2)$$

Again we insert the observed values of $P_{ijk}(1,2,3)$ on the left side in equations (2), and obtain seven equations with five parameters. However, the equation are not linear, and therefore difficult to solve. In Lazarsfeld and Henry (1968) is shown how to solve them, but the solutions are hard to generalize. Furthermore, the solutions sometimes give negative estimates. As far as we know no soft-ware has been developed. In Bye and Schechter (1986) maximum likelihood estimates are found by means of the Newton-Raphson algorithm.

We shall use the E-M algorithm as described in Langeheine and Van de Pol (1990).

3. ESTIMATION OF RESPONSE ERRORS IN THE NORWEGIAN LABOUR FORCE SURVEYS

As in many other countries the Norwegian labour force surveys use rotated samples. A person is in the sample for two consecutive surveys, and returns into the sample for two consecutive surveys one year later. In the table below data from only three points in time is used. Furthermore, it is not realistic to assume that the transition probabilities are constant, and we therefore allow them to be different. From equations (2) is seen that we now have 7 equations and 7 parameters. Applying a simple version of the E-M algorithm we have estimated the 7 parameters. Till now the experiences with the E-M algorithm have been very positive. We have had no problems with convergence. However, the number of iterations is often very high.

In table 1 response probabilities, latent and observed proportions in the labour force are presented for five age and sex groups. It is seen that the bias introduced by response errors among young individuals is substantial, while it is small for individuals aged 25 years and above.

Latent transitions probabilities are also calculated and compared to the observed ones. As expected the model results show much less real change than indicated from inspection of the observed turnover tables.

Table 1. Observed and latent proportions in the labour force, and response probabilities. 5 age and sex groups.

Age and sex group	Observed proportions in th labour force	Latent proportions in the labour force	Response probabilities q_{11}	q_{22}
16-19 years	49,50	47,0	92,5	85,6
20-24 years	81,3	77,6	97,1	73,6
Men 25-54 years	94,5	93,9	99,9	89,6
Women 25-54 years	79,8	80,1	98,0	93,8
55-74 years	44,7	44,9	97,7	98,6

References.

Bye, B.V. and Schechter, E.S: A latent Markov Model Approach to the estimation of response error in multivariate panel data. J. of Amer. Stat. Assn.81. pp. 375-380.

Chua, T.C. and Fuller, W.A. (1987): A Model for Multinomial Response Error Applied to Labour Flows. ASA, Vol 872, No. 397, pp. 46-51.

Haberman, S.J. (1979): Analysis of Quantitative Data, Vol.2, New Developments. New York. Academic Press.

Langeheine, Rolf and Pol Frank van de (1990): A Unifying Framework for Markov Modelling in Discrete Space and Discrete Time. Soc. Methods and Research. 18 pp. 416-441.

Lazarsfeld, P.F. and Henry N.W. (1968): Latent Structure Analysis. Boston: Houghton-Miffin.

Poterba, J.M. and Summers, L.H. (1986): Reporting Errors and Labour Market Dynamics. Econometrica, vol. 54 No. 6.

Wiggins, L.M. (1973): Panel Analysis. Amsterdam: Elsevier.