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PREFACE

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EMERGING ISSUES IN DEMOGRAPHIC METHODOLOGY[1]

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During the 1980s, the most important innovations in demographic methodology took place in two fields: event history analysis, and non-linear models. Of these two, the latter has had (and will have) the greatest impact on demographic thinking, because it involves a shift in paradigm from analysing the predictable behaviour of linear models, to the investigation of the dynamics of non-linear models, some of which may show unpredictable equilibrium behaviour, even when they are completely deterministic. Event history analysis became widely accepted in demography in the last ten years, and the treatment of unobserved heterogeneity, and that of simultaneously interacting multiple states, may be considered important new contributions to demographic methodology. In addition to event history analysis and non-linear models, this brief review deals with a number of other developments that took place in demographic methodology during the previous decade: age-structured models, models of union formation and household dynamics, and projection methodology. We also discuss an important field which showed no progress: translation methods. The period which is covered in this paper is largely that of the 1980s, and the focus is on quantitative, purely demographic models and methods.

EVENT HISTORY ANALYSIS

Event history analysis became widely accepted in demography in the 1980s. Demographic literature on this topic was virtually non-existent ten years ago, although the technique (also known by such different labels as survival, renewal, hazard, time-to-failure, reliability, life-testing, and intensity regression analysis) had been applied in other fields by then for a number of years already. The article on proportional hazards written by Cox in 1972 is often taken as a starting point, and connections to counting processes were made by Aalen in 1978. In demography, pioneering articles were published by Menken et al. (1981) and Trussell and Hammerslough (1983). Many applications and methodological papers appeared since then, and the technique is now common practice among demographers. Recent overviews of issues of methodology and application that are relevant in demography have been compiled in book form not only in English (e.g. Trussell et al., forthcoming), but also in French (Courgeau and Lelièvre, 1989) and German (Blossfeld et al., 1986).

The key advantage of event history analysis is the possibility of classification by criteria which apply equally to the individual under investigation, and to a series of individual events, rather than focusing on one event, or on aggregated data. Another advantage is that appropriate techniques exist for the estimation of the parameters of the model (e.g. the effect parameters in a proportional hazards model) in the presence of right censored data. In recent years, two methodological innovations took place: the treatment of multistate models, and the analysis of unobserved heterogeneity. In the review below, we shall deal with continuous-time models only. For a review of issues of unobserved heterogeneity in the context of discrete-time models, see Wrigley (1990).

Multistate models

Many practical applications of event history analysis deal with one particular event: childbearing, death, migration, divorce, entrance into the labour market etc. Such models are known as single-spell models. But in demography, one often encounters situations in which one would wish to analyse two or more events simultaneously. Competing risks models (multiple decrements), for instance those describing cause-specific mortality can be mentioned here, but also more general multistate (or increment-decrement) models, such as those for interregional migration, in which an individual can jump ("migrate") from any of a set of n regions to any other of the (n-1) regions. Such multistate models include competing risks models and single-spell models as particular cases, and the results obtained for the multistate case apply equally well for the simpler models.

In the beginning of the 1980s, the demographic practice was such that multistate models were largely analysed by reducing them to single-spell models, using very restrictive assumptions (e.g. effects of covariates do not differ between events) or focusing on very special cases (semi-Markov models). But recently, more general multistate models have been investigated. Blossfeld et al. (1989, 60-79), Hamerle (1989), Petersen (1991), and Gill (forthcoming) review this issue. The focus is on Cox-type of models which trace the effect of an individual's characteristics on his or her hazards to experience the various events defined as jumps in the state space of the multistate model. For instance, we might wish to compare the hazards for moving between two pairs of states, after correcting for covariates; or compare the effects of the same covariate on different event hazards. The key problem is: how can we estimate a multistate model with a set of covariates specified for each of the events?

The results that have been obtained for multistate models can be summarized as follows. It can be shown that in the case of competing events, the partial likelihood to be evaluated is the product of the partial likelihoods for each of the events (and its accompanying effect vectors) separately. In the case of two events, say, for which the covariates are not linked in any way (i.e. state-specific), this means that just two separate partial likelihoods have to be evaluated; when they are connected, there is still one single joint partial likelihood. Another result is that in the parametric case (when the baseline hazard is modeled in some way), the partial likelihood can be derived, and that it is not much more complicated than in the case of one single event. Finally, expressions for the partial likelihood in the case of an increment-decrement model have been derived, again assuming that the covariates have no elements in common. Hamerle (1989) investigates these matters in the context of time-invariant covariates, and he contends that most of the results he obtains can be generalized to allow for time-dependent covariates as well.
Thus the usual Cox model and its partial likelihood analysis can be used in the more general multistate model rather straightforwardly, although there is an important assumption of independent censoring (Gill, forthcoming). It should be noted that as an alternative, one can maximize the full likelihood function, for a model much more general than the multistate model - for example one which accounts for unobserved heterogeneity (cf. below) and time-varying covariates as well - see Heckman and Singer (1984).

It will be clear, that with an increasing number of states, the number of model parameters grows exponentially, and this will decrease the precision of the estimates in a given data set. Therefore, in any multistate application the number of states and the number of covariates will be modest in general.

Unobserved heterogeneity

A second innovation that emerged in the 1980s is the treatment of unobserved heterogeneity. Unlike standard regression models, the usual Cox model has no error term. So when we omit important covariates, we can still have very small standard errors for the

effect estimators, and we get no warning of the misspecification. When no control is made for unobserved variables, the estimated hazard rate becomes biased towards negative duration dependence.
There are various ways to deal with this unpleasant property, see, for example, Gill (forthcoming), Trussell and Rodriguez (1990), and Yamaguchi (1986). The one most used by demographers is simply to add an "error term" ε to the (antilog of the) right-hand side of the standard Cox model. Next there are basically two strategies: a non-parametric approach, in which no assumptions on the distribution of ε are required; and one in which one assumes some particular distribution for ε. In the latter case, a gamma, a normal, and a lognormal distribution have been used. However, we often have not much a priori knowledge as to the specific form of the distribution for ε, and the results can be extremely sensitive to one particular choice (Heckman and Singer, 1982, 1984; Hobcraft and Murphy, 1986)[2]. Therefore, the non-parametric approach may be favoured.

Although the potential dangers of neglecting heterogeneity in event history models are well recognized, there is no general agreement as to which of the two approaches here, or which estimation technique should be preferred. For instance, it is always possible to find many models incorporating unobserved heterogeneity, and one model not incorporating unobserved heterogeneity, that fit equally well (as judged by the unconditional distributions of the endogenous variable), see Trussell and Rodriguez (1990, 118), and, for a proof, Hoem (1990c). External information is necessary to chose between different approaches. Indeed, Petersen (1991, 310) reports successful fitting of a parametric heterogeneity model on the basis of simulated data, for which he knew the correct distribution of ε, whereas Hoem (1990a) has less positive experiences (inclusion of unobserved heterogeneity didn't provide much additional insight, numerical problems with the CTM program especially written for the analysis of unobserved heterogeneity) with a non-parametric approach with relatively few observations and little a priori knowledge about his area of application, i.e. union dissolution of childless women in Sweden. The size of Hoem's data set contrasts with that of Aaberge et al. (1989), who investigate the divorce behaviour of 51,000 Norwegian women (at least ten times as many, approximately, as the number of observations in most other studies) - yet the latter authors share Hoem's conclusion that the covariate effect estimates are rather insensitive to the omission of unobservables[3]. As to the *distribution* of ε, there seems to be some agreement that as long as the baseline hazard is correctly specified, the choice for a particular distribution does not have much impact on the parameter estimates - thus very general baseline hazards may be preferred (Hoem, 1990, 138; Trussell and Rodriguez, 1990, 117).

Challenges

In spite of recent methodological progress in methods for event history analysis, a number of methodological problems have not been resolved yet in a satisfactory manner. Three issues will be discussed here: the quality of retrospective data, the problem of left censoring, and the treatment of interacting careers.

Most of the data that have been analysed with event history techniques were collected in a retrospective survey. Such an approach may have a profound impact on the *quality of the data*. In particular, it makes these data subject to errors of omission and misplacement - in particular for events which the respondent experienced as unpleasant (divorce, death of an infant), or for which the timing is not clear-cut (start of a consensual union, leaving the parental home), or for which the occurrence took place in the remote past. In addition, to collect reliable retrospective information on individual norms and values is next to impossible (except for those norms and values that are so fundamental to the individual - characterizing perhaps his or her personality - that they are constant or almost constant over time).
Poulain et al. (1991) tested the reliability of retrospectively collected data on the timing of marriage, birth of children, children's leaving home, and migration. Three strategies were used: (i) independent interviews of both spouses in a household; (ii) comparison of the two questionnaires for the two spouses in each household, and correction, by the spouses, of possible inconsistencies; and (iii) checks against population register data. Some 500 married persons aged 41-55 in 1986, who had always lived with their first marital partner, were interviewed. As could be expected, the timing of migration and of the dates at which children leave home was severely misreported (compared with the register), with a bias of more than one year (ante-dates and post-dates taken together) in 8 to 24 per cent of the cases. Marriage dates and birth dates were reported much more correctly. Yet Courgeau (1991) found little sensitivity for his model estimates of migration behaviour as a result of timing misspecification (see also Courgeau and Lelièvre, 1989, 19). Nevertheless, it may be recommended to take the dates at which events are reported in a retrospective survey not too literally, and to recognize that some of the timings may be very fuzzy. Several strategies to deal with such "fuzzy time" concept may be used. One is to introduce fixed (over individuals) time periods in which the event of interest took place, and to study the effects of varying the length of these periods (1, 3, 6, 12, ... months) - see, for example, Klijzing et al. (1988). A second strategy (less ad hoc and statistically more satisfactory than the former) is to write down the probability that an event takes place between two discrete time points t_1 and t_2, on the basis of the hazard rate model. As Petersen (1991, 312) argues, estimation of such a *model with grouped data* often involves maximization of the complete likelihood function, instead of separate factors. An obvious extension of the latter model would be to make the length of the time period t_2-t_1 depend on individual covariates, as recall lapse differs by sex (women report demographic events more accurately than men, see Poulain et al, 1991) and most probably also by age and length of

period until interview date.
If one would want to improve the quality of the data, one could rely on a prospective data collection strategy, such as a panel. This reduces memory effects clearly and it facilitates the collection of data on norms and values. Drawbacks, however, are that a panel is relatively costly, that panel drop-out may introduce a bias in the estimates, and that repeated interview may distort the answers, or even the actual behaviour of the respondents (for an overview of these and related issues, see the volume edited by Kasprzyk et al., 1989). It should be noted that the problem of selective panel drop-out may be handled with appropriate weighting procedures for each wave, provided that enough information at the population level is available. However, such *dynamic weights* greatly complicate the estimation of parameters of aggregate change and models of individual behaviour. It should be noted that in retrospectively collected data we have a similar problem of dynamic weights, because, even when the sample is representative for the population at the time the interview was taken, it is not necessarily representative for the population as of five years ago, or ten years ago. The reason is that selective mortality, migration, and other processes of entrance into and exit from the target population may have introduced a bias. Thus this calls for a modelling strategy in event history analysis which takes time varying weights into account.
Another possibility is to analyse population register data. There is no selectivity connected to this approach, but there are other problems. For instance, the *de jure* picture that the register reflects is sometimes only a crude approximation of the *de facto* situation that one is interested in. Furthermore, the number of variables is usually very limited - information on norms and values is clearly lacking altogether.

A second problem connected to event history analysis is that of *left censoring*. In many data collection strategies, we know the entire life history of the individuals. However, this is not always the case, for instance when measurement only applies to a fixed period in the past. In this situation, the state the individual is in at the first date to be recorded is known, but not the length of time that this person spent in that initial state (unless the latter duration was explicitly asked for). Usually it is not possible to calculate the effects of the unknown event history data upon future events, and therefore one often assumes that the previous history of state occupancies is irrelevant for the process to be analysed, or, equivalently, that the hazard function is time-invariant (e.g. Blossfeld et al., 1989, 29; Courgeau and Lelièvre, 1989, 52; Tuma and Hannan, 1984, 131). But frequently this assumption is not very satisfactory, for example when the sample is selected in terms of the endogenous variable - e.g. mortality analysis based on a sample of surviving persons only. Generally, left censoring is considered as a difficult problem.
A very useful iterative method handling left-censored data was recently suggested by Courgeau and Lelièvre (1989, 52-56). These authors estimated survival functions for two processes (birth of a first child, and a second migration) on the basis of an artificially left-censored sample, and compared their results with survival functions estimated for the whole sample (with complete fertility and migration histories). The idea, stemming from

Turnbull (1974) is to estimate an expected time of entry into the risk set for left-censored individuals, on the basis of the model which is initially estimated for event histories that are not censored to the left. Application of this technique to the artificially censored data set led Courgeau and Lelièvre to conclude that agreement with results obtained for the complete data set is very close, for various censoring times between 1943 and 1952 (the data were right-censored at 1965). In the case of birth of the first child, estimation on the basis of the sub-sample of complete life histories only, introduces a substantial bias.
The technique proposed by Courgeau and Lelièvre to handle left censored data is, in fact, a special case of the EM-algorithm for computation of maximum likelihood estimators with missing data, introduced by Dempster et al. (1977). The latter algorithm may be applied in parametric and semi-parametric models (Gill, forthcoming; Little, 1988). Among the advantages of the EM-algorithm are that it produces maximum likelihood estimates, that estimates of standard errors may be obtained in many practical cases, and that convergence properties are known (see the discussion following the article by Dempster et al.).

A final problem to be taken up in the future is that of *parallel life courses and interacting careers*. Individuals experience events in different arenas of life: the family (childbearing, union formation and dissolution), the labour market, the educational system, etc. The life course in one arena may influence that in another one, and vice versa. The best-known example is the reciprocal relationship between childbearing and a woman's labour market behaviour.
In traditional event history models, the focus is on a single event (or perhaps a sequence of similar events in a multistate set-up), and the impact of certain (possibly time-dependent) covariates on the occurrence of this event. The pattern of causation is from the covariates to the event hazard. However, to investigate parallel individual life courses would require a much more symmetric approach. It would imply treating events of type A as covariates for events of type B, and vice versa. Such models, for which a multistate perspective would be a natural way to proceed, are very rare in demography. The empirical studies in Courgeau and Lelièvre (1989, chapters V, VI, and IX) are a notable exception.
The non-parametric model proposed by Aalen et al. (1980) is a useful starting point. These authors investigated, for a medical application, the reciprocal influence between two events A and B by means of a four-state Markov process, where the four states are defined according to whether the two events have occurred or not (not A, not B; A, but not B; B, but not A; both A and B). This framework facilitates such statements as "occurrence of event A accelerates occurrence of event B, but not vice versa". Klijzing et al. (1988) applied this method to the interaction between female labour force participation and childbearing. A next step is to construct a (semi-)parametric model for two or more events. Klijzing (1990, 1991) formulated a semi-parametric hazard model for each of four processes: start of cohabitation, marriage, conjugal union dissolution, and childbearing. Dummy variables were included as covariates in each hazard to account for the possible

occurrence of the other three processes. In spite of these and other models which treat interactions, it should be noted that relationships between two or more events for an individual may be so complex that these cannot be reduced to simple interactions (Courgeau and Lelièvre, 1991).

The methods described above take account of the life courses of one and the same individual. However, two or more persons may also influence each other's life courses. For example, the labour market careers of two spouses may be interrelated; the housing and residential careers of a male and a female who intend to start a consensual union clearly depend on each other. Investigations of how the life course(s) of one person are related to that (those) of another person may have great potential for a better understanding of human behaviour (see also Courgeau and Lelièvre, 1991).

NON-LINEAR MODELS

Non-linear models are not a very recent phenomenon in demography. Lee (1974) introduced a formal model which is able to describe, among others, the feedback mechanisms specified by Easterlin. For some parameter values (i.e. when the feedback mechanisms are strong enough), the model produces sustained cycles in births. Contrary to linear stable population models, which produce cycles in births and age structures that vanish as the system reaches the stable situation, non-linear models such as the one proposed by Lee generate a persistent oscillating behaviour, even in the fertility rates. However, only recently the link was made between non-linear models and unpredictable behaviour of the endogenous variables.

The idea, which stems from weather forecasting, is that important parts of reality are inherently non-linear. Some non-linear systems behave erratically in certain critical areas of their parameter space. Such systems may display stable equilibrium behaviour, but once their parameters have surpassed so-called bifurcation points, the behaviour becomes chaotic, identification becomes impossible, and hence the models of such systems cannot be used for prediction purposes.

An illuminating example is provided by the simple logistic difference equation $x_{t+1}=\alpha \cdot x_t(1-x_t)$, for $0<x_t<1$, and $1<\alpha<4$. For $1<\alpha<3$, x_t tends to a single stable equilibrium $(\alpha-1)/\alpha$; for $\alpha>3$, x_t becomes periodic, first with two stable points, then with four stable points at $\alpha=1+\sqrt{6}=3.44949...$, and next with 8, 16, ... equilibrium values for higher α-values. Around $\alpha=3.57$, chaos ensues (except for some particular α-values generating more regular behaviour): we observe an unbounded number of stable points with different periodicities and an unbounded number of different periodic cycles.[4] In addition, there are an uncountable number of initial points x_0 whose trajectories are totally aperiodic.

It is not a possible random component in the logistic model, and other non-linear models, that causes chaotic behaviour, but it is the non-linear nature of these deterministic models. Thus, in spite of the fact that the model is fully specified, and that all the initial conditions are known, its behaviour cannot be predicted. This view involves a dramatic shift in theoretical thinking, perhaps even a change of paradigm. The current paradigm states that our understanding of many of the social phenomena we observe, including demographic behaviour, can be enhanced with more research. Our limited ability to predict demographic and other social behaviour is only temporary. The situation will improve when we put more effort into social science research. Until then, randomness accounts for the ignorance of true underlying mechanisms. This paradigm dates back to Laplace who stated already that for him probability arises from ignorance of true causes. The same attitude was implicit in other writings in the eighteenth century (Simpson, Lagrange, Gauss), and it persisted in the nineteenth century. Determinism, as opposed to probability, would be possible when all causes are known and their effects can be predicted. Indeed, the real test of determinism is predictability (Suppes, 1984, 31).
Contrary to this paradigm, the revolutionary new view connected to non-linear models is that very simple deterministic systems, for which we know the entire specification, can produce random behaviour, and thus cannot be predicted. The view that the limited predictability of certain processes is inherent to the phenomena involved, and not merely a consequence of our (hopefully temporary) ignorance, has slowly gained acceptance in such diverse fields as meteorology, population biology, economics, but also in specialized studies such as that into the dripping faucet, or the growth of a snowflake.

In demography, the study of non-linear models connected to chaotic behaviour is relatively new. Land (1986, 897) and Land and Schneider (1987, 17) discuss some of the aspects involved. Bonneuil (1990) constructs a non-linear model that replicates Coale's I_f index for the Pays de Caux during the years 1589-1700, and shows that mortality conditions exhibit a bifurcation point for the fertility index. The sustained cyclical behaviour, or "limit cycles" generated by the model of Lee (1974) and other fertility models (see, for instance, Frauenthal and Swick, 1983; Feichtinger and Sorger, 1990; Wachter and Lee, 1989) may be considered as a weaker form of chaos. Wachter (1991) proves that the existence of a pre-procreative age-span is a sufficient condition for bifurcation points in age-specific models of population renewal. Bonneuil (1989) investigates the shifts in fertility levels in nine European countries between the 1930s and the 1960s. Day et al. (1989) present an extensive non-linear model in which fertility and population size depend on such household factors as income, consumption, preference, and cost of childrearing, and they derive conditions under which sustained cycles and chaotic behaviour emerge.

Few non-linear demographic models displaying sustained cycles or chaotic behaviour are known outside the field of fertility. A notable exception is Courgeau (forthcoming a, forthcoming b), who studies an interregional migration model which is based upon the assumption that the migration between regions i and j in some time interval (t_0, t_1) can be described by means of a migration parameter defined as $M_{ij}/(P_i(t_0) \cdot P_j(t_1))$. Here M_{ij} is the number of migrants between regions i and j during the particular interval, $P_i(t_0)$ denotes the population in the region of origin at the beginning of the interval, and $P_j(t_1)$ represents the population in the region of destination at the end of the interval. Courgeau solves the resulting non-linear model for the P_i's, and he shows that in the case of three interacting regions, sustained cycles or even a chaotic behaviour may appear (depending on the values of the migration parameters) when the system is time-discrete. For the equivalent continuous-time system, the behaviour is much more regular.[5] An obvious extension of Courgeau's investigations would be to study to what extent the introduction of age increases or dampens this irregular behaviour.

Non-linear models may have great potential for the study of nuptiality (or, more generally, partnership formation), too. Instead of two or more interacting regions as in the previous example, we have here two interacting individuals. Two-dimensional marriage rates and probabilities, by ages of both spouses, involve the expression, in one denominator, of the time both spouses were exposed to the risk of marriage, or of the numbers of unmarried males and unmarried females. Traditional occurrence-exposure rates cannot be used. Proposals for "two-dimensional" rates and probabilities involve taking some average (arithmetic, geometric, harmonic mean) of the two exposure times or the two populations, or even more complicated functions, see, for example, Pollard (1977). In general, such two-dimensional measures can be written as $H_{ij}/f(M_x,F_y)$, where H_{ij} is the number of marriages between males aged i and females aged j, and f is some scalar function of the vectors defined by unmarried males and females of all ages x and y, respectively (including, at least M_i and F_j). Most of these two-dimensional measures would imply a non-linear model for males and females broken down by marital status (married vs. not married), and possibly by age. It will be clear that the whole range of two-dimensional marriage rate specifications, in particular the function $f(M_x,F_y)$, defines a new programme of research for nuptiality and partnership behaviour, based upon the theory of non-linear models. This issue is closely linked to the two-sex problem, cf. below.

In studying non-linear demographic models, we should be careful, however, to balance possible tendencies to discern chaos everywhere when explanation and understanding fails, as Bartlett stressed. First, chaos and randomness may be considered as complementary. The study of stochastic processes has revealed many useful results, when for instance part of an unknown process is modelled as noise. Second, it will be clear that the inclusion of relevant factors in any demographic system is a very legitimate goal to be pursued. It cannot be denied, of course, that studies which attempt to explain the demographic behaviour of individuals or groups are still most relevant. However, at the same time we should realise that demographic behaviour cannot be fully explained and understood, however great the effort in demographic research will be. One of the consequences is that uncertainty in demographic predictions should be accepted, and can be accepted, cf. below.

AGE-STRUCTURED MODELS

An important innovation in demographic methodology took place in 1982, when Preston and Coale published their new synthesis of population dynamics. The model they proposed is known as the "generalized stable population method", and the "variable r method". It is an extension to the age-structured modelling tradition starting with Lotka (with age as a continuous variable) and Leslie (with discrete age groups), which was further generalized by Rogers to include other dimensions than age, for instance place of residence, marital status, and labour market status. The models of Lotka, Leslie and Rogers deal with variation of demographic rates along the age dimension only. In the model proposed by Preston and Coale - which was first developed by McKendrick (1926) - *demographic rates are not only age-dependent, but also time-dependent.* It can be demonstrated that the instantaneous rate of change in the cohort direction is the sum of rates of change in the age direction and the time direction. The model has seen very useful applications in various fields, including the indirect estimation of parameters of demographic behaviour (Preston, 1987; Rundell, 1989). European demographers started to apply this approach only very recently, see the articles by Coale and Caselli (1990), Caselli and Vallin (1990) and Wunsch (1989). Willekens (1990) proposes to use the model for population forecasting, in particular to resolve controversies related to the choice between a period approach versus a cohort approach, or to isolate a true age pattern for graduation purposes.

A second innovation that took place in the construction of age-structured models was the inclusion of *immigration* from outside the population described by traditional models. Thanks to the work of Espenshade et al. (1982), Mitra (1983) and Cerone (1987), we now have a better understanding of the consequences of international migration for a country's population dynamics: fertility below replacement and a positive immigration surplus leads, when demographic parameters are kept constant, to a stationary population[6]; when fertility is at replacement, a constant flow of immigrants causes the population to grow linearly; and fertility above replacement combined with constant immigration results in a stable population with positive growth rate. Related to the studies mentioned here is the work of Willekens and Drewe (1984), Keilman (1985a), and Van Imhoff (1990), who developed expressions to include international migration in multidimensional demographic projection models of the cohort-component type, both for the linear and the exponential form that the population vector may take on during the unit projection interval.

MODELS OF UNION FORMATION AND HOUSEHOLD DYNAMICS

The two-sex problem

The two-sex problem has occupied many demographers for a long time already. The core of the problem is that when two persons interact (for example a female and a male, when nuptiality or fertility are investigated), the rate at which the event of interest (marriage, child birth) takes place should, somehow, take account of the exposure time of both individuals. The definition of a two-dimensional marriage rate (with one dimension for the age of each marriage partner), and, related the construction of a two-dimensional marriage model has made some progress during the 1980s. Most marriage models are non-linear, cf. above. Pollard (1977) and Wijewickrema (1980) review the conditions that any realistic marriage model should fulfil. The most problematic conditions are those of competition and substitution. Suppose that the number of marriages H_{ij} for males aged i and females aged j depend on the vector of eligible males of all ages, to be denoted by $\{M_x\}$ (with elements M_x), and the vector $\{F_y\}$ (with elements F_y) containing eligible females of all ages. The competition requirement states that H_{ij} must go down, when the supply of eligible males for ages other than i increases, and likewise for females. For instance, the marriage chances of a male aged 25 are decreased when more eligible males aged 28, or 35, become active in the marriage market. However, the extra supply of 28-year old males has a stronger impact on the marriage chances of the 25-year old male, than an additional number of males aged 35 has. This is the essence of the substitution (or relative competition) requirement: the negative effect on H_{ij} of an increase in M_x depends on how close x is to i, and similarly for the sexes interchanged.

A number of new marriage functions have been proposed in the last decade, for instance Schoen's harmonic mean model (Schoen, 1981), and the constant elasticity of substitution (CES) model of Pollak (1990). However, these two models fail to fulfil the competition axiom, let alone the substitution requirement. The marriage rule they model belongs to the class of zero spillover mating rules (Pollak, 1990, 326) - i.e. H_{ij} depends only on F_j and M_i, and not on the numbers of eligible persons in other age classes. The model which has been used by the Netherlands Central Bureau of Statistics (NCBS) since 1980 for projections of the population by age, sex and marital status does have appropriate competition effects (at least qualitatively). Moreover, it meets the substitution requirement for any triplet of ages at the right-hand side of the modal age at marriage, provided that the reference age is closest to the mode (Keilman, 1985b, 219).

Besides the construction of two-sex models, there has also been some progress in the link between such models and stable population theory. Wijewickrema (1980) proved that the existence of ergodicity is independent of the choice of a particular marriage function, and Pollak (1990) investigated the existence, uniqueness, and dynamic stability of a stable equilibrium situation.

Household models

Several operational household projection models were constructed during the 1980s. Reviews are given by Keilman (1988), De Vos and Palloni (1989), and Willekens (1990). Most models take the individual as the primary unit of analysis and modelling, but this creates problems in modelling interactions between household members. Interactions between individuals do not only arise in models of marriage and union formation. Household projection models which describe individual behaviour should also take account of the links between other members of one and the same household, for instance parents and children (see below). Recent developments in multidimensional household projection models have led to a very general treatment of a number of the aspects of the two-sex problem. Generally speaking, the endogenous variables of a household projection model should satisfy certain constraints. For instance, in any projection interval, the number of children born in or returning to a two-parent family should correspond to the numbers of female and male parents who experience an increase (by one) of the number of children present in a two-parent family. Van Imhoff (1992) constructed a very flexible algorithm to deal with these and similar consistency requirements in multidimensional models of individual household formation and dissolution behaviour. The algorithm is based upon a weighted linear least-squares optimization which minimizes the deviations between initial (inconsistent) and consistent numbers of events, given the consistency relations. The approach taken by the NCBS for their marital status projections is a special case of Van Imhoff's method.

Two main trends may currently be observed in the area of household modelling. First, although the static headship rate model and its extensions are still widely used (primarily because of its simplicity), dynamic models become more and more prominent. Second, the interest in household modelling has shifted from a description of numbers of households (of various types) to a description of (events occurring to) individuals (broken down by household position). At the same time, multidimensional demography provides an appropriate framework for the modelling of household dynamics at the individual level (Van Imhoff and Keilman, 1991).

In spite of the progress which was recently made, two major challenges exist. First, modern dynamic household models require much more detailed data than the traditional headship rate model. Often, such data are not available, and indirect estimation techniques have to be used to estimate the household parameters. The generalized stable population method and the EM-algorithm mentioned above hold a considerable promise for such indirect estimations. Second, household changes are closely linked to residential changes and migration, but very few models have been developed which take up this link. Thus, attempting to integrate household modelling and residential choice modelling would be a natural way to proceed.

FORECASTS AND PROJECTIONS

An important new trend in forecasting and projection making is the explicit recognition of forecast uncertainty. Since the early 1980s, systematic empirical assessments of the record of demographic forecasts have been carried out. Early contributions are those of Keyfitz (1981) and Stoto (1983), who analysed the ex-post observed error in the population's growth rate in a large number of countries. Ahlburg (1982) investigated the accuracy of birth forecasts produced by the United States Bureau of the Census (USBC). Also, the statistical agencies of several countries, including the US, the Netherlands, Sweden, Norway, and the FRG have taken up a strong interest in the ex-post accuracy of their population forecasts (for a review of forecast evaluations, see Keilman, 1990).
The largely *empirical* approach used in these investigations, focusing on ex-post errors, is in sharp contrast with the approaches pursued in the 1960s and the 1970s, when demographers attempted to *model* ex-ante errors in population forecasts using stochastic cohort-component models and ARIMA models. At the same time, it turned out that more sophisticated methods for projection models and assumption making (cohort analysis, parity-specific analysis, Delphi-approach, scenario writing) were unsuccessful in that they provided insufficient instruments to the forecasters in industrialized countries to foresee the dramatic fall in birth rates in the 1960s and 1970s. Therefore, the empirical analyses of ex-post observed errors, and the explicit recognition of forecast uncertainty by official forecasters can be considered a reaction to the poor records of sophisticated methodology. Indeed, one of the major findings of these empirical studies is that more advanced methods do not necessarily lead to more accurate forecasts (Ahlburg, 1982; Stoto, 1983; Keilman, 1990). The reason is that the cohort-component model, as well as its extensions proposed in the last few decades, is not a behavioural model. Therefore, its exogenous variables (fertility and mortality rates, migration parameters) determine to a large extent how close the model's forecasts will be to reality. But even a completely accurate specification of the future trajectories of the exogenous variables will be beyond the reach of demography, because a large part of individual human behaviour is essentially unpredictable (Boudon, 1986). Therefore, the proper attitude of demographers who want to forecast the future dynamics of a certain population is to indicate how large the *confidence intervals* around the projection results are - a mere calculation of more than one projection variant is insufficient, as long as no indication is given as to the expected level of error connected to the bounds defined by these variants.
Confidence intervals may be constructed in two different ways: following a model-based approach, or an empirical approach. The proper statistical point of view would be to construct a stochastic model for the population's dynamics, and to predict the variables of interest and their confidence intervals. The stochastic models developed in the 1960s and 1970s by demographers and statisticians such as Pollard, Sykes, Feichtinger, Schweder, and Le Bras produced confidence intervals that often were much narrower than what ex-post observed forecast errors indicated. The reason is many of these stochastic models focused on pure sample variance,, and not on the major source of errors, i.e. the basic

instability of average demographic parameters (age-specific rates for fertility and mortality). More recently, new models were developed by Stoto (1983), Alho and Spencer (1985), and Cohen (1986). For total population size, Stoto's method works well. In principle, that method states that future errors in a population's growth rate follow a normal distribution, and that historical errors can be used for an estimation of the mean and the variance of that distribution[7]. Application of Stoto's method leads to the conclusion that the high-low margins of official population forecasts correspond to approximate two-thirds confidence intervals for the US (up to 15 years, see Stoto, 1983, 18; USBC, 1989, 15) and 99 (!) per cent confidence intervals for the Netherlands (up to 40 years, Keilman, 1990, 168). Cohen's method assumes stochastic stationarity in observed matrices of transition probabilities. In both applications for total population size (Sweden for the period 1780-1980, and the Netherlands for the period 1830-1989), this assumption is violated: transition probabilities are not stationary, as the population growth rate declines more or less regularly (for Sweden since 1815, for the Netherlands since 1880), and relatively fast since the 1960s. Therefore, Cohen's method is inappropriate for most industrialized countries, as these went through similar demographic transitions as Sweden and the Netherlands (Keilman, 1990, 170).
Other methods than those of Stoto and Cohen have to be tested in practical situations. The focus should be on constructing confidence intervals around *age structures*, because most forecast users are interested in future numbers by age, and much less in total population counts. The approach of Williams and Goodman (1971) is an empirical method, which can be applied when a large number of forecasts are available. Alho and Spencer (1985) specify a model in which the vital rates evolve through time according to low-degree polynomials. Their method rests on a number of assumptions (e.g. about the autocorrelations and cross-correlations of age-specific vital processes), some of which are difficult to test. A detailed study is needed to assess the merits of this method.

TRANSLATION METHODS

In Sweden, the period Total Fertility Rate (TFR) has shown a continuous rise since the mid-1980s. Whereas many West-European countries display a similar pattern of fertility trends, the case of Sweden is remarkable: compared to these other countries, current TFR levels are relatively high (around 2.1 in 1990), and the increase is rather steep (the 1983-TFR was 1.6). As Hoem (1990b, 735) states "... (the) rise in period fertility reflects a change in the time pattern of cohort fertility. Ultimate cohort fertility may eventually also rise as a result of this change ...". He concludes that recent TFR trends in Sweden result, to a large extent, from initial postponement of childbearing, followed by a compensation for it, and he analyses in some detail birth rates of parities 1-3, controlling for age at previous births.

Dinkel (1985) observes a decline in the life expectancy of males in the Soviet Union between 1964 and 1980 by almost five years. In his discussion of the causes for this trend, he argues that two factors commonly mentioned (the Soviet health system, and alcohol abuse) can only be of minor importance. Instead he points out the weaknesses of a period life table, and he contends that selection effects due to World War II operating to successive birth cohorts offer a more reasonable explanation: less frail individuals were relatively often exposed to acts of war, and thus they were affected more severely than frail persons. In spite of a lack of proper cohort data, his hypothesis receives some support from the data.

What these two papers have in common is that they investigate *summary period and cohort indicators* for variables describing a *non-repeatable event*: births by parity, and death. Analytical treatments of the reciprocal impact of period and cohort indicators, known as *translation methods*, were provided by Ryder since 1964, and they have shed considerable light on the fine interplay between quantum and timing of both cohort and periods (Ryder, 1980). For example, these translation methods facilitate to isolate the impact of changes in cohort timing on period quantum, from the consequences exerted by changes in cohort quantum. One of Ryder's results (applied to fertility) is that when timing patterns in cohorts are constant, the TFR is (upon a linear approximation) a linear function of the completed cohort fertility.
Why can translation methods not be applied to such analyses of parity-specific fertility and mortality as referred to above? The answer is, that these techniques were developed for *repeatable events*, for instance age-specific fertility irrespective of parity. The salient feature of age-specific rates of repeatable events is that they simply add up (both for the cohort perspective and the period perspective) to quantum indicators (completed cohort fertility, and TFR, respectively). On the basis of this simple additive relationship, Ryder was able to derive his translation formulas. Age specific rates for non-repeatable events do not possess this simple additive relationships for the derivation of measures of quantum and tempo. For instance, when m(x) is a first-marriage rate for age x, and $n(x)=2\cdot m(x)/(2+m(x))$ is the corresponding probability, then the quantum of first marriage, e.g. the proportion ever-marrying (in the absence of mortality and emigration) is found as

$$1-\prod_{x}(1-n(x)).$$

This *multiplicative relationship* between age-specific measures and the quantum indicator is characteristic for non-repeatable events. Translation formulas for such non-repeatable events are not known of, but they would certainly enrich our insights in issues regarding period and cohorts as described by the two examples given above.[8] Translation methods for repeatable events have existed for more than 25 years now; extension of these techniques to non-repeatable events would fill an important gap in the collection of existing demographic methods.

DISCUSSION AND CONCLUSIONS

The list of demographic methods does not give a full picture of the relevant field, of course. One prominent topic which is missing is that of AIDS-models. Here only the first glimpses of a common structure in modelling are emerging. One may consult the proceedings of the recent Workshop on Modelling the Demographic Impact of the AIDS Epidemic (UN/WHO, 1991), or the special issue of *Mathematical Population Studies* (volume 3 number 3, 1992), devoted entirely to HIV- and AIDS-models. It seems that the most important methodological contribution by demographers to this new field is that of the modelling of pair formation and dissolution.
The focus in this review has been on methods, and that implies that very little attention was given to behavioural theories (although the distinction between method and theory is not always very sharp, see for instance, the model proposed by Day et al., 1989).
When thinking about "emerging methods", it is not always easy to distinguish between methods that are fashionable for some time, but which disappear from the literature, and new methods which acquire themselves a firm place in the practitioner's tool-kit. There are several criteria for choosing between the two groups. First, the extent to which a new method provides deeper or new insight into human behaviour is an important criterion. Second, one can look at the numbers of applications of various new methods. Any method which has seen wide-spread application in several fields will perhaps be used in the years to come as well. A third criterion is the possible emergence of extensions, generalizations, or sub-techniques. A method which has enough "critical mass" will be followed up this way, sooner or later. Fourth, the interplay between method and application is important when distinguishing between durable and passing new methods: new applications often pose new methodological challenges.

When judging the methods discussed in this paper by these four criteria, it will be clear that the introduction of techniques for *event history analysis* is by far the most important innovation that took place in demographic methodology in the 1980s. Event history techniques have been applied to such diverse fields as childbearing, union formation and dissolution, mortality, and migration. In the past decade generalizations (multivariate models) and extensions (unobserved heterogeneity) of the original technique became widely used. Moreover, various data situations (such as censored data, data aggregated over individuals, data grouped into observations for discrete time units) and various applications (e.g. parallel life courses) have posed new challenges to event history techniques. Finally, these techniques facilitate a much more profound investigation into the timing and the intensity of demographic processes than earlier methods could (e.g. classical analytical demographic measures, regression techniques, logit models). However, regarding the last point, it should be noted that modern models of event history analysis do not produce much better fits to data on individual human behaviour than traditional techniques do. Reported log-likelihood values, evaluated at the maximum of the likelihood function, lead to an average probability (per individual) that the model generates the data in the order of magnitude of 20-40 per cent, and this average probability may be interpreted as a goodness-of-fit measure[9]. The reason that event history models do not do better than traditional models when it comes to an explanation of individual demographic behaviour is that the former models, although the specifications may be different, include more or less the same covariates - a real improvement in our capability to understand demographic processes would require a satisfactory behavioural theory.

Generalized stable models and *multidimensional household models* have provided many new and useful insights, as these methods have been used for indirect estimation and for the description of household dynamics, respectively. The analysis of ex-post observed *forecast errors* has given us some quantified sense of the uncertainty connected to demographic forecasts. Much work remains to be done here, as was indicated above, but these three new demographic techniques have undoubtedly enriched the collection of demographic methods and approaches. Finally, the field of *non-linear modelling* in demography is largely unexplored, both in the methodological sense and with respect to applications. Therefore it is too early to state whether current interest in these models is merely a passing phenomenon, or, whether it will become a long-lasting tradition to use them.

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NOTES

1. Helpful comments from Øystein Kravdal, John Dagsvik, Lars Østby, Didier Blanchet and Jan Hoem are gratefully acknowledged. The usual disclaimer applies here, too.

2. Part of this sensitivity may be attributed to an incomplete specification of the model, see the remarks below concerning a correct specification of the baseline hazard.

3. Gill (forthcoming) notes that although adding heterogeneity does not much influence parameter estimates, it can dramatically increase their estimated standard errors, especially when the data is not well balanced. Thus, taking account of the fact that not all covariates are included shows that one has much less precise information about the effects of the covariates which have been included. Indeed, a comparison of estimated standard errors the models estimated by Aaberge et al. (1989) reveals that adding heterogeneity increases standard errors for nearly all parameter estimates. In this particular example however, the increase is not so dramatic as Gill suspects: roughly between 10 and 30 per cent.

4. This model can be analysed numerically with a programmable calculator, or in a spreadsheet. Chose an appropriate initial value for x_t, for example 0.25, and observe that when the parameter α is 2, say, the model will produce a stable equilibrium value of x=0.5. When α is increased, so does the equilibrium value: with α=2.5 we find x=0.6. But then suddenly, at α-values beyond 3, the system doesn't produce a stable equilibrium value any longer, but bounces indefinitely long between *two* values: 0.764567 and 0.538014 for α=3.1. The model produces a stable cycle between these two values. By turning up α even more, *four* equilibrium points appear at the next *bifurcation point* around α=3.45: 0.852443, 0.433954, 0.847451, and 0.446009. And next we observe 8 equilibrium values at α=3.54409. For higher α-values we will need a computer programme, because the bifurcation points come faster, and we need more iterations to obtain equilibrium. But we will see 16, 32, ... equilibrium values - and suddenly, beyond α=3.569946... the system breaks down and produces chaos. The difference between subsequent bifurcation points decreases in the limit with the same constant factor, namely 4.6692016. This constant, called Feigenbaum's number, is universal for all bifurcation processes. A very readable analytical account of the behaviour of this logistic model is given by Mickens (1990, 275-280).

5. Bartlett (1990, 324) notes that for continuous time systems, chaotic behaviour requires at least a three-dimensional phase space. Discrete-time systems can exhibit chaos in even one dimension, as is shown by the example of the logistic difference equation.

6. Coale (1972, 601) showed that a stationary population emerges when the net reproduction rates of native and immigrating women (R_n and R_i, respectively) are such that

$$R_n = 1 - a(I/B)R_i$$

in the case where a constant amount of I female immigrants enter the country each year; B is the annual number of births in the stationary population. The factor a is defined as the ratio in the stationary population between annual female births by immigrant women, and annual female immigrants. It summarizes the age pattern of immigrant women, and their fertility and mortality schedules. On the basis of Coale's formula one may easily determine the size of a stationary population in which immigrants adopt the fertility pattern of the population already present.
Now write the remaining life expectancy after immigration as e_i, and the life expectancy at birth as e_0. Then it follows for the stationary population P that

$$e_0B + e_iI = P, \quad I > 0$$

Elimination of B from these two expressions results in the following formula for the stationary population size

$$P = I\{(1-R_n)e_i + a.R_i.e_0\}/\{1-R_n\}$$

In case immigrants adopt the fertility level of the population already present, we set R_i equal to R_n, and find (approximating R_n by TFR/2.1)

$$P = I\{(2.1\text{-TFR})e_i + a.\text{TFR}.e_0\}/\{2.1\text{-TFR}\}$$

For his illustrative calculations, Coale used a value of a equal to 0.521 (correcting at the same time for the fact that the analysis is made for both sexes). Hence a population with a constant TFR of 1.5, an annual immigration surplus of 100,000, and a mortality pattern such that $e_0 = 75$ and $e_i = 50$ would ultimately reach a stationary size of 14.8 million persons.

7. Stoto assumes serial independence of forecast errors. De Beer (1991) argues that for many countries this is an unrealistic assumption, because growth rates decrease or increase systematically. In that case Stoto's method would underestimate uncertainty. De Beer illustrates this underestimation convincingly in a comparison between Stoto's method and one assuming a first-order autocorrelation structure, both fitted to data from the Netherlands for the period 1950-1990.

8. Ryder (1980, 47-52) derived expressions for the impact that births of various orders may have on the quantum and the tempo of cohort fertility. However, the age- and parity-specific birth rates he uses do not have person years of mothers *by parity* in their denominators - only age is considered. Hence they are not proper occurrence-exposure rates, and to use such indicators for translation purposes may introduce a certain amount of bias in the results, very much the same as the distortions caused by the Period Total First Marriage Rate when interpreted as an indicator of period quantum for first marriage (Wunsch and Termote, 1978, 55).

9. When L is the log-likelihood value evaluated at its maximum, and n is the number of observed individuals, then the geometric average of individual contributions to the overall likelihood is exp(L/n). This measure may be interpreted as an average (over all individuals) probability that the model produces the data. It will be clear that this is a very approximate measure: for a precise comparison between various models one ought to take the number of covariates into account. Nonetheless the heuristic measure described here may be considered to give an appropriate indication of the order of magnitude of the fit.