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Choice Probabilities and Equilibrium Conditions in a Matching Market with Flexible Contracts

by

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Abstract

The purpose of the paper is to develop discrete and continuous probabilistic choice models for a matching market of heterogeneous suppliers and demanders.

The point of departure is similar to that of Tinbergen (1956) which considers equilibrium conditions in a matching market with a particular continuous distribution of preferences and attributes of the agents. The present paper extends Tinbergen's analysis to allow for rather general specifications of the preferences and the distribution of agent-specific attributes.

Keywords: two-sided matching models, discrete choice, market equilibrium, marriage models, the Golden Section.

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1. Introduction

Many important areas of individual behavior involves the search for a partner in a matching market. Typical examples of matching phenomena are the process of marriage formation, the admission process of students into colleges, and the matching of employees and workers in the labor market. A particular important challenge is to obtain a tractable econometric framework for analyzing matching behavior in a population of heterogeneous agents.

A game-theoretic analysis of the matching problem relevant for matching markets started with Gale and Shapley (1962) and Shapley and Shubik (1972). See Roth and Sotomayor (1990) for an overview of the literature as well as a theoretical analysis of marriage markets under particular assumptions about the rules of the game.

Becker (1981) applies a matching model to study marriage and household economics. His concern is to analyze which men are married to which women under the assumption that the couple derives utility from attributes of the man and the woman.

None of the authors mentioned above consider the problem of developing a framework that yields a mathematically tractable expression for the probability distribution of the number pf realized matches as a function of the agent's preferences and the size of the relevant population groups. In the demographic literature, however, several authors have formulated more or less ad hoc models for the number of marriages formed as a function of the number of unmarried males and females in each age group (cf. Hoem, 1969, McFarland, 1972, Pollard, 1977, and Schoen, 1977). The only contributions we know of that have attempted to derive a structural matching model for the distribution of the number of realized matches, is Tinbergen (1956) and Hartog.

The goal of the present paper is the same as in Tinbergen and Hartog, op cit. We consider an economy with a large number of suppliers and demanders. Each agent wishes to form a match with a potential partner which includes specific terms of a contract (such as price, for example). The agents are heterogeneous with respect to characteristics (attributes). An agent's characteristics affect his own preferences and enter as attributes in the utility functions of other agents. However, only some of the attributes are observed by the analyst. Each agent has preferences over all potential partners and over different contracts. The utility functions depend on observed as well as unobserved attributes (relative to the analyst). There are no search costs. The distribution of the preferences has a particular form which yields convenient formulae for aggregate supply, demand and realized matches. In particular, the structure of the probability density of realizing a particular match is consistent with the functional form obtained by Dagsvik and Strøm (1992) and Ben-Akiva et al. (1985). However, while these authors do not explicitly discuss the relationship between supply and demand, the present paper discusses how the probability density of realizing a match is determined in equilibrium.

Similarly to Tinbergen op cit., the framework developed here assumes rather stylized behavior on the part of the agents. We refer to Dagsvik (1993b) for a micro-theoretic formulation that allows agents to be uncertain about their opportunities in the matching market and face search costs.

The organization of the paper is as follows: In Section 2 and 3 the matching model with a finite number of agents is analyzed. In Section 4 and 5 we extend the model to allow for flexible contracts and a finite number of observable (to the econometrician) categories of suppliers and demanders. In Section 6 we consider the special case of a market for (indivisible) products characterized by price and other attributes. Here we allow for constraints in the sense that not every possible attribute combination is feasible in the market and we also assume that the agents have preferences over latent attributes of their potential trading partners. In Section 7 we consider the case with infinitely many agents with continuous attributes. In the final section we consider a market with differentiated products with continuous attributes and where the agents only have preferences over product attributes.

2. Demand, supply and realized matches: description of the game and derivation of choice probabilities

We consider a market with suppliers and demanders (agents) that wish to form a match with a partner to exchange services. Typical examples are the flow of services that follows from a job-match, a marriage, and the like. The agents are heterogeneous with respect to unobserved characteristics called attributes and they have preferences over attributes of their potential trading partners.

Let N be the number of suppliers and M the number of demanders. In the following we shall employ small superscripts s and d as indices for a particular supplier, s, and demander, d, and sometimes capital superscripts, S and D, to indicate supply and demand. We will adopt the convention that a person who does not engage in a match is self-matched. Let U^{sd} be the utility of supplier s of a match with demander d. Let U_0^s be the utility of supplier s of being self-matched. Similarly let V^{ds} be the utility of demander d of a match with supplier s and let V_0^d be the utility of demander d of being self-matched. We assume that

$$U^{sd} = \varepsilon^{sd}, \tag{2.1}$$

$$U_0^s = \alpha \varepsilon_0^s, \tag{2.2}$$

$$V^{ds} = \eta^{ds} \tag{2.3}$$

and

$$V_0^d = \beta \eta_0^d \tag{2.4}$$

where α and β are systematic terms that are common to all the suppliers and demanders, respectively, while ε^{sd}, ε_0^s, η^{ds} and η_0^d are i.i.d. random tasteshifters (random to the observer). Moreover, we assume that

$$P\left(\varepsilon^{sd}\leq y\right) = P\left(\eta^{ds}\leq y\right) = P\left(\varepsilon_0^{s}\leq y\right) = P\left(\eta_0^{d}\leq y\right) = \exp\left(-\frac{1}{y}\right). \tag{2.5}$$

The assumption (2.5) is consistent with the "Independence from Irrelevant Alternatives" (IIA) axiom, cf. Ben-Akiva and Lerman (1985). If we had chosen an additive formulation in (2.1) to (2.4) then the c.d.f. in (2.5) would have had to be replaced by $\exp(-e^{-y})$ to be consistent with IIA. This is easily realized by taking the logarithm of (2.1) to (2.4). The choice between the multiplicative and the additive formulation is only a matter of taste since they are, from a theoretical point of view, completely equivalent.

We are now ready to consider the agents behavior in the matching market. We shall consider a particular market adjustment process towards equilibrium that is perceived as taking place in several stages indexed by τ. The rule of the game assumed here differs from the "deferred acceptance" procedure considered in Roth and Sotomayor (1990). To describe the agents behavior it is useful to introduce some additional notation. Let $\{B_\tau^s\}$ and $\{A_\tau^d\}$ be families of sets defined sequentially by

$$A_{\tau+2}^{d} = \left\{s : U_{\tau+1}^{sd} = \max\left(\max_{r\in B_{\tau+1}^{s}} U_{\tau+1}^{sr}, U_{0\tau+1}^{s}\right)\right\} \tag{2.6}$$

and

$$B_{\tau+1}^{s} = \left\{d : V_{\tau}^{ds} = \max\left(\max_{r\in A_{\tau}^{d}} V_{\tau}^{dr}, V_{0\tau}^{d}\right)\right\}. \tag{2.7}$$

for τ=1,2,..., where A_1^d consists of all the demanders. We shall call A_τ^d and B_τ^s , τ=1,2,..., conditional choice sets for demander d and supplier s, respectively. Specifically, $A_{\tau+2}^d$ is the set of all the suppliers that rank demander d on top in stage τ+1, given that supplier s has choice set $B_{\tau+1}^s$, s=1,2,...,N. Similarly, $B_{\tau+1}^s$ is the set of all the demanders that rank supplier s on top in stage τ, given that demander d has choice set A_τ^d, d=1,2,...,M. The random tasteshifters are assumed to be i.i.d. across stages while the structural terms α and β are independent of τ. In stage one none of the agents have information about their opportunities in the market. The demanders start by choosing the best alternative from A_1^d, d=1,2,...,M. This generates the sets $\{B_2^s\}$ which consist of all the suppliers that are ranked on top by the demanders in stage one. In stage two the suppliers take the conditional choice sets, $\{B_2^s\}$, as given and they make new rankings (offers) by maximizing utility given these conditional choice sets. This produces new conditional choice sets, $\{A_3^d\}$. This process continuous until convergence is attained. Here, convergence is understood as convergence of the corresponding probability distributions. Below we shall demonstrate that the process described above will converge. Let A^d and B^s denote the corresponding conditional equilibrium choice sets. A match between supplier s and demander d occurs if

$$U^{sd} = \max\left(\max_{r \in B^s} U^{sr}, U_0^s\right) \tag{2.8a}$$

and

$$V^{ds} = \max\left(\max_{r \in A^d} V^{dr}, V_0^d\right), \tag{2.8b}$$

The index τ is absent in (2.8) since (2.8) presupposes that τ is so large that equilibrium has been attained. Note that in the adjustment process described above the agents in stage τ only need to have information about the potential partners in the respective conditional choice sets as of stage τ. It is important to note that the conditional equilibrium choice sets $\{A^d\}$ and $\{B^s\}$ may differ from the corresponding sets of agents that realize a match. This is so because several agents may rank the same potential partner on top.

Let us now introduce the notion of conditional supply and demand probabilities. Specifically, define

$$g_\tau^S = P\left(U_\tau^{sd} = \max_{r \in B_\tau^s} U_\tau^{sr} \mid d \in B_\tau^s\right), \tag{2.9}$$

$$g_\tau^D = P\left(V_\tau^{ds} = \max_{r \in A_\tau^d} V_\tau^{dr} \mid s \in A_\tau^d\right), \tag{2.10}$$

and

$$g^S = \lim_{\tau \to \infty} g_\tau^S \quad \text{and} \quad g^D = \lim_{\tau \to \infty} g_\tau^D.$$

Since the distribution of the utilities does not depend on characteristics of the individual agents, (2.9) and (2.10) therefore are independent of s and d. From assumptions (2.1) to (2.5) it follows that

$$P\left(U_{\tau+1}^{sd} = \max_{r \in B_{\tau+1}^{s}} U_{\tau+1}^{sr} \middle| d \in B_{\tau+1}^{s}, \tilde{m}_{\tau+1}^{sd}\right) = \frac{1}{1+\alpha+\tilde{m}_{\tau+1}^{sd}} \tag{2.11}$$

where $\tilde{m}_{\tau+1}^{sd}$ is the number of agents in $B_{\tau+1}^{s}$-{d}. From (2.7) we realize that the variable $\tilde{m}_{\tau+1}^{sd}$ is binomially distributed with probability g_{τ}^{D} and the largest number $\tilde{m}_{\tau+1}^{sd}$ can attain is M-1. It thus follows that

$$P\left(\tilde{m}_{\tau+1}^{sd} = k\right) = \binom{M-1}{k}\left(g_{\tau}^{D}\right)^{k}\left(1-g_{\tau}^{D}\right)^{M-1-k} \tag{2.12}$$

for k=0,1,..., M-1. From (2.9) and (2.11) we get

$$g_{\tau+1}^{S} = E\left(\frac{1}{1+\alpha+\tilde{m}_{\tau+1}^{sd}}\right). \tag{2.13}$$

Similarly, we get from (2.10) that

$$g_{\tau+1}^{D} = E\left(\frac{1}{1+\beta+\tilde{n}_{\tau+1}^{ds}}\right) \tag{2.14}$$

where $\tilde{n}_{\tau+1}^{ds}$ is the number of agents in $A_{\tau+1}^{d}$-{s}, and has the c.d.f.

$$P\left(\tilde{n}_{\tau+1}^{ds} = k\right) = \binom{N-1}{k}\left(g_{\tau}^{S}\right)^{k}\left(1-g_{\tau}^{S}\right)^{N-1-k}. \tag{2.15}$$

An alternative expression for (2.13) and (2.14) is obtained as follows: If X is a non-negative discrete random variable it is immediately verified that

$$E\left(\frac{1}{a+X}\right) = \int_0^1 z^{a-1} E\left(z^X\right) dz \tag{2.16}$$

for any positive constant a. Since

$$E\left(z^{\tilde{m}^{sd}_{\tau+1}}\right) = \left(1 - g_\tau^D + z g_\tau^D\right)^{M-1} \tag{2.17}$$

and

$$E\left(z^{\tilde{n}^{ds}_{\tau+1}}\right) = \left(1 - g_\tau^S + z g_\tau^S\right)^{N-1}, \tag{2.18}$$

(2.17), (2.18) and (2.16) imply that (2.13) and (2.14) can be expressed as

$$g^S_{\tau+1} = \int_0^1 z^\alpha \left(1 - g_\tau^D + z g_\tau^D\right)^{M-1} dz \tag{2.19}$$

and

$$g^D_{\tau+1} = \int_0^1 z^\beta \left(1 - g_\tau^S + z g_\tau^S\right)^{N-1} dz. \tag{2.20}$$

<u>Lemma 1</u>

Eq. (2.19) and (2.20) imply that

$$\frac{1}{1+\alpha+(M-1)g_\tau^D} \leq g^S_{\tau+1} \leq \frac{1}{\alpha+\max\left(1,(M-1)g_\tau^D\right)} \tag{2.21}$$

and

$$\frac{1}{1+\beta+(N-1)g_\tau^S} \leq g_{\tau+1}^D \leq \frac{1}{\beta+max\left(1,(N-1)g_\tau^S\right)}. \tag{2.22}$$

The proof of Lemma 1 is given in Appendix 1.

Theorem 1

The system of equations (2.19) and (2.20) converges towards a unique solution, $g^S, g^D \in (0,1)$.

Proof:

From (2.19) and (2.20) it is easily veryfied that the sequences $\{g_\tau^S\}$ and $\{g_\tau^D\}$ are increasing. By assumption, $\alpha>0$ and $\beta>0$ and (2.21) and (2.22) therefore imply that $\{g_\tau^S\}$ and $\{g_\tau^D\}$ are bounded and accordingly they must be convergent.

Q.E.D.

We shall now study the equilibrium solution when N and M are large. Specifically, we shall allow N and M to increase such that N/M tends towards a constant.

From (2.21) and (2.22) the next result is immediate.

Corollary 1

Suppose that N and M increase such that M/N tends towards a constant. Moreover, assume that α and β depend on N(M) such that $\bar{\alpha} = lim_{N\to\infty}\alpha(N)/\sqrt{N}$ and $\bar{\beta} = lim_{N\to\infty}\beta(N)/\sqrt{M}$ exist. Then

$$v \equiv \lim_{N\to\infty} g^S\sqrt{N} \quad and \quad u \equiv \lim_{M\to\infty} g^D\sqrt{M}$$

are determined by the system of equations

$$v = \frac{\sqrt{N/M}}{\bar{\alpha} + u} \tag{2.23}$$

and

$$u = \frac{\sqrt{M/N}}{\bar{\beta} + v}. \tag{2.24}$$

We conclude this section by deriving the asymptotic expression for the probabilities of realizing a match. Let q^S be the probability that a supplier shall realize a match with any demander and let q^D be the probability that a demander shall realize a match with any supplier. From (2.8) we realize that the probability of realizing a match with a particular demander equals $g^S \cdot g^D$. Since there are M demanders we must have that

$$q^S = g^S \cdot g^D M. \tag{2.25}$$

Similarly, we have

$$q^D = g^S \cdot g^D N. \tag{2.26}$$

Note that our equilibrium concept that follows from (2.19), (2.20), (2.25) and (2.26) is a probabilistic concept. This means that unless M and N are large the corresponding fractions of realized matches will not necessarily be equal to their respective mean values (2.25) and (2.26).

By taking (2.23) and (2.24) into account we obtain the next result.

Corollary 2

When N and M are large then the assumptions above imply that q^S and q^M are (asymptotically) equal to

$$q^S = \left(\bar{\alpha}\bar{\beta} + \psi + 1/\psi - \sqrt{(\bar{\alpha}\bar{\beta} + \psi + 1/\psi)^2 - 4}\right)/2\psi \tag{2.27}$$

and

$$q^D = \left(\bar{\alpha}\bar{\beta} + \psi + 1/\psi - \sqrt{(\bar{\alpha}\bar{\beta} + \psi + 1/\psi)^2 - 4}\right)\psi/2 \tag{2.28}$$

where $\psi = \sqrt{N/M}$.

From (2.27) and (2.28) we realize that when N=M and $\bar{\alpha}=\bar{\beta}=0$ then q^S and q^D are close to one when N is large. At first glanse this may seem surprising, since the population of suppliers and demanders have the same size. The explanation is that since the utility functions have i.i.d. random tasteshifters then for sufficiently large N the probability that a supplier will find a very attractive demander which ranks the supplier on top (among all suppliers) will be close to one.

Above we have not given any rationale for why α and β should increase when the population of suppliers and demanders increases. Let us therefore give one possible story as an example. An equivalent representation of the preferences (2.1) and (2.2) is $U^{sd} = a\varepsilon^{sd}$ and $U_0^s = \varepsilon_0^s$, where $a=1/\alpha$. Suppose now that the agents are uncertain about the duration of a match in the long run. Let a be interpreted as the mean of the suppliers expected utility where the expectation is taken with respect to the duration of the match. When the population increases one may argue that the uncertainty increases since the information the suppliers have about each demander may decrease. Consequently, the (mean) expected value, a, will decrease with the population size, and thus $\alpha=1/a$ will increase. The same argument applies for β.

3. A special case: The Golden Section

It has long been realized that certain shapes of rectangle seem to the human eye to be aesthetically more satisfactory than others. Indeed, given a large range of rectangular shapes to choose from, most people, it is said, will tend to choose as most satisfactory one which length bears to its width the same ratio as the sum of the length and the width bear to the length alone. The resulting ratio is called the Golden Section (φ) and it is determined by the equation

$$\varphi^2 = \varphi + 1 \tag{3.1}$$

i.e., $\varphi = (1+\sqrt{5})/2$.

The Golden Section is exhibited in the Athenian Parthenon and a number of other buildings of classical antiquity and it is also found in the Egyptian Great Pyramid as the ratio between the slope-height and the half-base (within .001 of the Golden Section). Moreover, it is found at the entrance of the tomb of Ramses IX and on the walls of the colonnade of Amon in the Temple of Luxor (cf. Schwaller de Lubicz, 1985 and Lemesurier, 1977).

The Golden Section is also linked to the so-called Fibonacci Series where each number equals the sum of its two predecessors. It is found with surprising frequency in nature, for example in pattern of plant growth, in flower-petal arrangements, in the laws of Mendelian heredity and in the ratios between planetary orbits.

It is intriguing that the (inverse) Golden Section also emerges as a solution of (2.27) and (2.28) in the following special case with M=N and $\bar{\alpha}=\bar{\beta}=1$, which means that $\alpha=\beta=\sqrt{N}$. When N=1 the last assumption, $\bar{\alpha}=\bar{\beta}=1$, means that the probability of preferring a match over being self-matched is equal to 1/2. Thus, when only one potential partner is present the agents are, on average, indifferent between the two alternatives "being matched" and "self-matched". In addition the last assumption ensures that the choice probabilities q^S and q^D remain invariant with respect to the population size, N, when N is large. Under these assumptions it follows that $g^S=g^D$ and $q^S=q^D$. From (2.27) and (2.28) we obtain that for large N

$$q^S = q^D = 1/\varphi^2. \tag{3.2}$$

The probability of being self-matched equals

$$1 - q^{S} = 1 - 1/\varphi^{2} = 1/\varphi. \tag{3.3}$$

The last equality in (3.3) follows from (3.1).

4. Introducing flexible contracts

In this section we shall modify the description on page 5 so as to allow for flexible contracts. Relevant examples are tuition fees and grades in the market for educaton, and wages and non-pecuniary conditions in the labor market. Let t=1,2,...,C, index the contracts, and assume for simplicity that the set of possible contracts is finite. Let $U_{\tau}^{sd}(t)$ denote the utility function of supplier s of a match with demander d under contract t. Similarly, let $V_{\tau}^{ds}(t)$ denote the corresponding utility of demander d. Similarly to (2.6) and (2.7) define

$$A_{\tau+2}^{d}(t) = \left\{s : U_{\tau+1}^{sd}(t) = \max\left(\max_{k}\left(\max_{r \in B_{\tau+1}^{s}(k)} U_{\tau+1}^{sr}(k)\right), U_{0\tau+1}^{s}\right)\right\} \tag{4.1}$$

and

$$B_{\tau+1}^{s}(t) = \left\{d : V_{\tau}^{ds}(t) = \max\left(\max_{k}\left(\max_{r \in A_{\tau}^{d}(k)} V_{\tau}^{dr}(k)\right), V_{0\tau}^{d}\right)\right\} \tag{4.2}$$

for τ=1,2,..., where $A_{1}^{d}(t)$ consists of all the demanders that rank contract t on top. Thus the interpretation of $B_{\tau}^{s}(t)$ is as the set of demanders that rank a match with supplier s under contract t on top in stage τ. Similarly, $A_{\tau}^{d}(t)$ is the set of suppliers that rank a match with demander d under contract t on top in stage τ. The agents update their conditional choice sets according to (4.1) and (4.2) similarly to the case discussed in Section 2. Let $A^{d}(t)$ and $B^{s}(t)$ be the corresponding conditional equilibrium choice sets. A match with contract t between supplier s and demander d will occur if

$$U^{sd}(t) = \max\left(\max_{k}\left(\max_{r \in A^{d}(k)} U^{sr}(k)\right), U_{0}^{s}\right) \tag{4.3}$$

and

$$V^{ds}(t) = \max\left(\max_k\left(\max_{r\in A^d(k)} V^{dr}(k)\right), V_0^d\right). \tag{4.4}$$

In the next section we shall investigate whether the process described above converges to equilibrium under assumptions that are similar to those introduced in Section 2.

5. The general case with flexible contracts and several observable categories of suppliers and demanders

In the present section we assume that a subset of the attributes are observable to the econometrician. We also assume that the number of observable categories of suppliers and demanders is finite. Specifically, let M_j be the number of demanders of type j, j=1,2,...,Δ, and N_i the number of suppliers of type i, i=1,2,...,Z. The total number of possible contracts is also finite and equal to C, the total number of suppliers and demanders are N and M. In Section 7 we shall consider the case with continuous attributes.

Let $U_{ij}^{sd}(t)$ be the utility of supplier s of type i of a match with demander d of type j with contract t. Let U_{i0}^{s} be the utility of supplier s of type i of being self-matched. We assume that

$$U_{ij}^{sd}(t) = a_{ij}(t)\varepsilon_{ij}^{sd}(t) \tag{5.1}$$

and

$$U_{i0}^{s} = a_{i0}\varepsilon_{i0}^{s}, \tag{5.2}$$

where $a_{ij}(t)$ and a_{i0} are systematic terms and $\varepsilon_{ij}^{sd}(t)$ as well as ε_{i0}^{s} are i.i.d. random tasteshifters

On the demand side the description is completely analogous. Thus

$$V_{ji}^{ds}(t) = b_{ji}(t)\eta_{ji}^{ds}(t) \tag{5.3}$$

and

$$V_{j0}^{d} = b_{j0}\eta_{j0}^{d} \tag{5.4}$$

is the utility function of demander d that corresponds to (5.1) and (5.2).

Let $B_{ij}^{s}(t)$ be the set of demanders (in equilibrium) of type j that wish to form a match with supplier s under contract t. Similarly, let $A_{ji}^{d}(t)$ be the set of suppliers (in equilibrium) of type i that wish to form a match with demander d of type j under contract t. When a demander d of type j decides which of the sets $B_{ij}^{s}(t)$, $s=1,2,\ldots,N_i$, $i=1,2,\ldots,Z$, $t=1,2,\ldots,C$, he belongs to, he takes the sets $A_{ji}^{d}(t)$, $i=1,2,\ldots,Z$, $t=1,2,\ldots,C$, as given and ranks a match with supplier s of type i under contract t on top if this maximizes his utility given that $s \in A_{ji}^{d}(t)$. As in Section 2 and 4 we shall call $\{B_{ij}^{s}(t)\}$ and $\{A_{ji}^{d}(t)\}$ conditional choice sets. Let $m_{ij}^{s}(t)$ and $n_{ji}^{d}(t)$ be the number of demanders in $B_{ij}^{s}(t)$ and suppliers in $A_{ji}^{d}(t)$. We shall call $m_{ij}^{s}(t)$ and $n_{ji}^{d}(t)$ the conditional demand and supply, relative to supplier s and demander d. We shall investigate below the conditions under which market equilibrium exists.

Similarly to (2.5) we assume that $\varepsilon_{ij}^{sd}(t)$, $\varepsilon_{ji}^{ds}(t)$, ε_{i0}^{s}, ε_{j0}^{d}, $s=1,2,\ldots,N_i$, $i=1,2,\ldots,Z$, $d=1,2,\ldots,M_j$, $j=1,2,\ldots,\Delta$, $t=1,2,\ldots,C$, are i.i.d. with

$$P\left(\varepsilon_{ij}^{sd}(t)\le y\right) = P\left(\eta_{ji}^{ds}(t)\le y\right) = P\left(\varepsilon_{i0}^{s}\le y\right) = P\left(\eta_{j0}^{d}\le y\right) = \exp\left(-\frac{1}{y}\right). \tag{5.5}$$

Consider the behavior of supplier s conditional on the choice set $B_{ij}^{s}(t)$. Let $g_{ij}^{S}(t)$ be the probability that supplier s of type i will prefer demander d of type j under contract t. We shall call $\{g_{ij}^{S}(t)\}$ the conditional supply probabilities. Specifically, for $d \in B_{ij}^{s}(t)$

$$g_{ij}^{S}(t) \equiv P\left(U_{ij}^{sd}(t) = \max_{k,r}\left(\max_{q\in B_{ik}^{s}(r)} U_{ik}^{sq}(r)\right) \middle| d\in B_{ij}^{s}(t)\right)$$

$$= E\left(\frac{a_{ij}(t)}{a_{i0} + a_{ij}(t) + \left(m_{ij}^{s}(t) - 1\right)a_{ij}(t) + \sum_{(k,r)\neq(j,t)} m_{ik}^{s}(r)\,a_{ik}(r)} \middle| d\in B_{ij}^{s}(t)\right) \tag{5.6}$$

$$= E\left(\frac{a_{ij}(t)}{\sum_{k}\sum_{r>0} a_{ik}(r)\,m_{ik}^{s}(r) + a_{i0}} \middle| d\in B_{ij}^{s}(t)\right).$$

The derivation of (5.6) is completely analogous to the derivation of the choice probabilities of the extreme value random utility model, see Ben-Akiva and Lerman op cit. Similarly, the conditional demand probabilities are given by

$$g_{ji}^{D}(t) = E\left(\frac{b_{ji}(t)}{b_{j0} + \sum_{k}\sum_{r>0} n_{jk}^{d}(r)\,b_{jk}(r)} \middle| s\in A_{ji}^{d}(t)\right). \tag{5.7}$$

Obviously, we have

$$E\left(m_{ij}^{s}(t)\right) = M_j\, g_{ji}^{D}(t) \tag{5.8}$$

and

$$E\left(n_{ji}^{d}(t)\right) = N_i\, g_{ij}^{S}(t). \tag{5.9}$$

In order to simplify the analysis we shall assume that N_i and M_j are large for all i=1,2,...,Z, and j=1,2,...,Δ. Specifically, we shall assume that N_i and M_j increase such that N_i/N, M_j/M and N/M tend towards positive constants. Let

$$u_{ij}(t) \equiv \lim_{M\to\infty} g_{ji}^{D}(t)\,M_j/\sqrt{M} \tag{5.10}$$

and

$$v_{ji}(t) \equiv \lim_{N\to\infty} g_{ij}^{S}(t) N_i/\sqrt{N} \,. \tag{5.11}$$

As in Section 2 we assume that (asymptotically) a_0 and b_0 are proportional to $\sqrt{M}$ and $\sqrt{N}$, respectively. Let

$$\bar{a}_{i0} = \lim_{M\to\infty}\left(a_{i0}(M)\sqrt{M}\right) \tag{5.12}$$

and

$$\bar{b}_{j0} = \lim_{N\to\infty}\left(b_{j0}(N)\sqrt{N}\right. \,. \tag{5.13}$$

When N and M are large we obtain from (5.6), (5.7), (5.8) and (5.9), similarly to (2.23) and (2.24) that

$$v_{ji}(t) = \frac{a_{ij}(t) N_i/\sqrt{MN}}{\bar{a}_{i0} + \sum_k \sum_{r>0} a_{ik}(r) u_{ik}(r)} \tag{5.14}$$

and

$$u_{ij}(t) = \frac{b_{ji}(t) M_j/\sqrt{MN}}{\bar{b}_{j0} + \sum_k \sum_{r>0} b_{jk}(r) v_{jk}(r)} \,. \tag{5.15}$$

The system of equations (5.14) and (5.15) for i=1,2,...,Z, j=1,2,...,Δ, and t=1,2,...,C, represents the equilibrium conditions for the asymptotic expressions (5.10) and (5.11) of the conditional supply and demand probabilities for all i, j and t. Recall that conditional supply and demand are different from the conventional definitions of supply and demand. In particular, conditional supply will not coincide with conditional demand in equilibrium. For later reference it will be convenient to rewrite the equilibrium conditions as follows. Let

$$A_i \equiv \bar{a}_{i0} + \sum_k \sum_{r>0} a_{ik}(r) u_{ik}(r) \tag{5.16}$$

and

$$B_j \equiv \bar{b}_{j0} + \sum_k \sum_{r>0} b_{jk}(r)\, v_{jk}(r). \tag{5.17}$$

Then by (5.14) and (5.15)

$$v_{ji}(t) = \frac{N_i a_{ij}(t)}{A_i\sqrt{MN}} \tag{5.18}$$

and

$$u_{ij}(t) = \frac{M_j b_{ji}(t)}{B_j\sqrt{MN}}. \tag{5.19}$$

When (5.18) and (5.19) are inserted in (5.16) and (5.17) we get

$$A_i = \bar{a}_{i0} + \sum_k \frac{M_k c_{ik}}{B_k\sqrt{MN}}, \tag{5.20}$$

and

$$B_j = \bar{b}_{j0} + \sum_k \frac{N_k c_{kj}}{A_k\sqrt{MN}} \tag{5.21}$$

where

$$c_{ij} \equiv \sum_{r>0} a_{ij}(r)\, b_{ji}(r). \tag{5.22}$$

In appendix 1 we prove that the system of equations (5.20) and (5.21) for i=1,2,...,Z, and j=1,2,...,Δ, has a unique solution for A_i and B_j, for all i and j, provided $\bar{a}_{i0}>0$ and $\bar{b}_{j0}>0$ for all i and j. We summarize the results obtained above in the following theorem.

Theorem 2

Under the assumptions stated above the asymptotic conditional supply, $\{v_{ji}(t)\}$, and demand, $\{u_{ij}(t)\}$, satisfy (5.14) and (5.15).

If $\bar{a}_{i0}>0$, $\bar{b}_{j0}>0$ for all i and j then the system of equations (5.14) and (5.15) has a unique solution.

Let us now consider the probability of realizing a match with a particular contract. The probability that a specific supplier of type i shall realize a match with a specific demander of type j with contract t is obviously given by

$$g_{ij}^{S}(t) \cdot g_{ji}^{D}(t).$$

Thus, the probability, $q_{ij}^{S}(t)$, that a specific supplier of type i shall form a match with any demander of type j with contract t is equal to

$$q_{ij}^{S}(t) = g_{ij}^{S}(t) \cdot g_{ji}^{D}(t) M_{j}. \tag{5.23}$$

Similarly, the probability, $q_{ji}^{D}(t)$, that a demander of type j shall realize a match with any supplier of type i with contract t, equals

$$q_{ji}^{D}(t) = g_{ji}^{D}(t) \cdot g_{ij}^{S}(t) N_{i}. \tag{5.24}$$

From (5.8), (5.9), (5.14), (5.15), (5.23) and (5.24) we obtain the following result.

Corollary 3

Assume that N and M are large. Then the (asymptotic) probability that a given supplier of type i shall obtain a match with any demander of type j with contract t is given by

$$q_{ij}^{S}(t) = \frac{a_{ij}(t)u_{ij}(t)}{\bar{a}_{i0} + \sum_{k}\sum_{r>0} a_{ik}(r)u_{ik}(r)} = \frac{a_{ij}(t)b_{ji}(t)M_j}{A_i B_j \sqrt{MN}}. \tag{5.25}$$

The (asymptotic) probability that a given demander of type j shall obtain a match with any supplier of type i with contract t, equals

$$q_{ji}^{D}(t) = \frac{b_{ji}(t)v_{ji}(t)}{\bar{b}_{j0} + \sum_{k}\sum_{r>0} b_{jk}(r)v_{jk}(r)} = \frac{a_{ij}(t)b_{ji}(t)N_i}{A_i B_j \sqrt{MN}}. \tag{5.26}$$

The respective (asymptotic) probabilities of being self-matched equal

$$q_{i0}^{S} = \frac{\bar{a}_{i0}}{A_i} \tag{5.27}$$

and

$$q_{j0}^{D} = \frac{\bar{b}_{j0}}{B_j}. \tag{5.28}$$

Note that in this economy there is no excess supply nor excess demand.

From (5.25) we observe that with $f_{ij}(t) \equiv \log a_{ij}(t) + \log u_{ij}(t)$ and $f_{i0} \equiv \log a_{i0}$ we obtain that $q_{ij}^{S}(t)$ has the structure of the familiar extreme value random utility model where $f_{ij}(t)$ is the systematic component of the utility function. However, in contrast to the standard formulation $f_{ij}(t)$ no longer depends solely on the agent's (of type i) preferences for a demander of type j under contract t, due to the fact that agents operate in a two-sided matching market. Fortunately, the complex interdependencies in the market can be conveniently accounted for by modifying the preference term $\log a_{ij}(t)$ through the addition of $\log u_{ij}(t)$. By symmetry, the same argument applies for $q_{ji}^{D}(t)$.

From the viewpoint of empirical applications it is interesting to note that the probability of realizing a particular match, given that some match is realized within a specific category, has a particularly simple form. Let $w^S_{ij}(t)$ and $w^D_{ji}(t)$ be the respective conditional probabilities of realizing a particular match given that some match of type (i,j) is realized. From (5.25) and (5.26) we immediately get that

$$w_{ij}^S(t) \equiv \frac{q_{ij}^S(t)}{\sum_{r>0} q_{ij}^S(r)} = w_{ji}^D(t) \equiv \frac{q_{ji}^D(t)}{\sum_{r>0} q_{ji}^D(r)} = \frac{a_{ij}(t)\, b_{ji}(t)}{\sum_{r>0} a_{ij}(r)\, b_{ji}(r)}. \tag{5.29}$$

Moreover, note also that by (5.27) and (5.28) we can express (5.25) and (5.26) as

$$q_{ij}^S(t) = \frac{a_{ij}(t)\, b_{ji}(t)\, M_j\, q_{i0}^S\, q_{j0}^D}{\bar{a}_{i0}\, \bar{a}_{j0} \sqrt{MN}} \tag{5.30}$$

and

$$q_{ji}^D(t) = \frac{a_{ij}(t)\, b_{ji}(t)\, N_i\, q_{i0}^S\, q_{j0}^D}{\bar{a}_{i0}\, \bar{a}_{j0} \sqrt{MN}}. \tag{5.31}$$

These expressions imply that when N_i and M_j are observed for all i and j, $a_{ij}(t)b_{ji}(t)/\bar{a}_{i0}\bar{b}_{j0}$ can be estimated directly from (5.30) or (5.31) by applying estimates of the equilibrium choice probabilities.

Let us finally consider the elasticities of the choice probabilities with respect to N_i and M_j for all i and j.

Let Q^S and Q^D be matrices with elements

$$Q_{ij}^S = \sum_{r>0} q_{ij}^S(r) \quad \text{and} \quad Q_{ij}^D = \sum_{r>0} q_{ij}^D(r).$$

Furthermore, let $\partial_M Q_0^S$, $\partial_N Q_0^S$, $\partial_M Q_0^D$ and $\partial_N Q_0^D$ denote matrices with elements

$$\partial_M Q^S_{0ij} = \frac{\partial \log q^S_{i0}}{\partial \log M_j}, \quad \partial_M Q^D_{0ij} = \frac{\partial \log q^D_{i0}}{\partial \log M_j},$$

$$\partial_N Q^S_{0ji} = \frac{\partial \log q^S_{i0}}{\partial \log N_j} \quad \text{and} \quad \partial_N Q^D_{0ij} = \frac{\partial \log q^D_{i0}}{\partial \log N_j}.$$

From (5.27), (5.28), (5.20) and (5.21) the next result follows by implicit differentiation.

Corollary 4

The equilibrium elasticities of the probabilities of being self-matched, with respect to the number of suppliers and demanders of each type, are given by

$$\partial_M Q_0^S = -(I - Q^S Q^D)^{-1} Q^S, \tag{5.32}$$

$$\partial_N Q_0^S = (I - Q^S Q^D)^{-1} Q^S Q^D, \tag{5.33}$$

$$\partial_M Q_0^D = -(I - Q^D Q^S)^{-1} Q^D \tag{5.34}$$

and

$$\partial_N Q_0^D = (I - Q^D Q^S)^{-1} Q^D Q^S. \tag{5.35}$$

The elasticities of $q^S_{ij}(t)$ follow immediately from the formulas above, since by (5.30)

$$\frac{\partial \log q^S_{ij}(t)}{\partial \log M_r} = \frac{\partial \log q^S_{i0}}{\partial \log M_r} + \frac{\partial \log q^D_{j0}}{\partial \log M_r}, \tag{5.36}$$

for any r. Thus, to obtain estimates of the elasticities above we only need estimates of the equilibrium probabilities of realizing a match.

6. Markets for products or services with latent exogenous constraints on contract and where suppliers (demanders) have preferences over latent demander (supplier) attributes

In this section we consider the particular case where the structural parts of the utility functions of the suppliers (demanders) do not depend on the attributes of the demanders (suppliers) and where each supplier only produces one unit of a good or a service. This case is relevant in situations where the agents wish to exchange goods and services. Moreover, we shall assume that, due to exogenous regulations, not all types of contracts can be realized. Note that we still allow the tasteshifters in the utility functions of the suppliers (demanders) to depend on the demanders (suppliers). The motivation for this is that a supplier (demander) may find it more convenient to trade with some demanders (suppliers) than others, due to location, etc. Also the preferences of the agents may depend on the attributes of their potential trading partners because some agents may advertise their supply and demand more efficiently than others. See Anderson and Palma (1988) where they discuss a model where firms preferences depend on consumer location.

In Section 8 we demonstrate that under analogous assumptions to those of the present section, the structure of the choice probabilities are independent of whether or not the agents preferences depend on latent attributes of the potential trading partners.

Eriksen (1986) also discusses equilibrium conditions in the context of discrete choice modeling with exogenous restrictions. However, in contrast to the present paper, he only considers equilibrium when aggregate supply is exogenously given.

Notice first that when the agents utility functions only depend on the contract attributes, (5.18) and (5.19) reduce to

$$v_{ji}(t) = \frac{N_i a_i(t)}{A_i\sqrt{MN}} \tag{6.1}$$

and

$$u_{ij}(t) = \frac{M_j b_j(t)}{B_j\sqrt{MN}}, \tag{6.2}$$

where A_i and B_j, $i=1,2,...,Z$, and $j=1,2,...,\Delta$, are determined by (5.20) and (5.21) with

$$c_{ij} \equiv \sum_{r>0} a_i(r)\, b_j(r). \tag{6.3}$$

The above formulation presumes, however, that the agents are free to choose any contract i.e., there are no externalities. Suppose now that $t=(t_1,t_2)$ where t_2 is the price and t_1 is an attribute that defines the type and quality level of the good. Suppose that there are exogenous regulations which prevent the agents from determining prices freely. Let $\kappa_{ij}^{sd}(t_2)=1$ if supplier s of type i and demander d of type j are allowed to realize price t_2, and zero otherwise. The agents are assumed to be perfectly informed about the values of $\kappa_{ij}^{sd}(t_2)$. To the econometrician, however, $\kappa_{ij}^{sd}(t_2)$ is unobservable. Let

$$\theta(t_2) = P\left(\kappa_{ij}^{sd}(t_2)=1\right). \tag{6.4}$$

Note that in (6.4) it is assumed that $\theta(t_2)$ is independent of agent characteristics. This assumption is made for simplicity and can easily be relaxed.

With exogenous constraints on contracts, (6.1) and (6.2) must be modified since these equations only express conditional supply and demand given that contracts are flexible. Let $\tilde{v}_i(t)$, $\tilde{u}_j(t)$ denote the corresponding asymptotic expressions for the conditional supply and demand when we take into account that $\kappa_{ij}^{sd}(t_2)$ is unobservable to the analyst. Then we get

$$\tilde{v}_{ji}(t_1,t_2) = \frac{N_i a_i(t_1,t_2)\,\theta(t_2)}{\tilde{A}_i\sqrt{MN}} \tag{6.5}$$

and

$$\tilde{u}_{ij}(t_1,t_2) = \frac{M_j b_j(t_1,t_2)\,\theta(t_2)}{\tilde{B}_j\sqrt{MN}}, \tag{6.6}$$

where now $\tilde{A}_i$ and $\tilde{B}_j$ are determined by

$$\tilde{A}_i = \bar{a}_{i0} + \sum_k \frac{M_k \tilde{c}_{ik}}{\tilde{B}_k\sqrt{MN}}, \tag{6.7}$$

$$\tilde{B}_j = \bar{b}_{j0} + \sum_k \frac{N_k \tilde{c}_{kj}}{\tilde{A}_k\sqrt{NM}} \tag{6.8}$$

and

$$\tilde{c}_{ij} \equiv \sum_{r_1>0}\sum_{r_2>0} a_i(r_1,r_2)\,b_j(r_1,r_2)\,\theta(r_2). \tag{6.9}$$

The (asymptotic) probabilities of realizing a contract, $\tilde{q}^S_{ij}(t)$ and $\tilde{q}^D_{ji}(t)$, that correspond to (5.25) and (5.26), have the structure

$$\tilde{q}_{ij}^{S}(t) = \frac{a_i(t_1,t_2)\,\tilde{u}_{ij}(t_1,t_2)}{\bar{a}_{i0} + \sum_k \sum_{r_1>0} \sum_{r_2>0} a_i(r_1,r_2)\,\tilde{u}_{ik}(r_1,r_2)} = \frac{a_i(t_1,t_2)\,b_j(t_1,t_2)\,\theta(t_2)\,M_j}{\tilde{A}_i\tilde{B}_j\sqrt{MN}} \tag{6.10}$$

and

$$\tilde{q}_{ji}^{D}(t) = \frac{b_j(t_1,t_2)\,\tilde{v}_{ij}(t_1,t_2)}{b_{j0} + \sum_k \sum_{r_1>0} \sum_{r_2>0} b_j(r_1,r_2)\,\tilde{v}_{jk}(r_1,r_2)} = \frac{a_i(t_1,t_2)\,b_j(t_1,t_2)\,\theta(t_2)\,N_i}{\tilde{A}_i\tilde{B}_j\sqrt{MN}}. \tag{6.11}$$

The respective (asymptotic) probabilities of being self-matched are analogous to (5.27) and (5.28), i.e.,

$$\tilde{q}_{i0}^{S} = \frac{\bar{a}_{i0}}{\tilde{A}_i} \tag{6.12}$$

and

$$\tilde{q}_{j0}^{D} = \frac{b_{j0}}{\tilde{B}_j}. \tag{6.13}$$

Since there are constraints on the set of feasible contracts there will in general be excess supply and excess demand. From (5.27) and (6.12) it follows that a measure of excess supply in category i is given by

$$1 - q_{i0}^{S} - \left(1 - \tilde{q}_{i0}^{D}\right) = \frac{\bar{a}_{i0}}{\tilde{A}_i} - \frac{\bar{a}_{i0}}{A_i}. \tag{6.14}$$

Similarly, excess demand in category j can be measured by

$$1 - q_{j0}^{D} - \left(1 - \tilde{q}_{j0}^{D}\right) = \frac{\bar{b}_{j0}}{\tilde{B}_j} - \frac{\bar{b}_{j0}}{B_j}. \tag{6.15}$$

Above we assumed that only prices are constrained. It is clear that the analysis is completely analogous in the case where constraints on other contract attributes are present.

7. Continuous attributes and latent exogenous constraints on contracts

In this section we assume that the number of agents is infinite and that the attributes of the agents as well as the attributes of the contracts are continuous vector variables. This is of interest in many applications. For example, in the labor market hours and wages are continuous. Also in the goods market prices are continuous.

From the viewpoint of the analyst these variables are thought of as randomly distributed according to a probability mechanism which will be defined below. Let us start by describing the preferences. Let $U^{sd}(x,y,t)$ be the utility of supplier s with observable attributes $x \in K$ of a demander d with attribute $y \in K$ and a contract with attribute $t \in L$, where K and L are compact sets in some Euclidian space. We assume that

$$U^{sd}(x,y,t) = a(x,y,t)\,\varepsilon^{sd}(x,y,t) \tag{7.1}$$

where the interpretation of a(x,y,t) and $\varepsilon^{sd}(x,y,t)$ is completely analogous to the corresponding interpretation in Section 5. The utility of being self-matched is given by

$$U_0^s(x) = a_0(x)\,\varepsilon_0^s(x). \tag{7.2}$$

On the demand side the description is completely similar and is given by

$$V^{ds}(y,x,t) = b(y,x,t)\,\eta^{ds}(y,x,t) \tag{7.3}$$

and

$$V_0^d(y) = b_0(y)\,\eta_0^d(y). \tag{7.4}$$

In the market, only countable subsets of agent- and contract attributes exist. Let $B_s = \{(Y(k),T(k)), k=0,1,2,...\}$, be an enumeration of the equilibrium set of demander and contract attributes that are offered to supplier s with attribute x. The variables (Y(0),T(0)) represent non-market opportunities. We assume that the variables in B_s are generated as independent draws from a conditional probability distribution function, $\tilde{M}(y,t|x)$. For demander d with attribute y the set of feasible attributes in equilibrium, $A_d = \{(X(k),T'(k)), k=0,1,2,...\}$, is also generated as independent draws from a conditional probability distribution function, $\tilde{N}(x,t|y)$. Without loss of generality we assume that X(0)=Y(0)=T(0)=0. For x>0, y>0, t>0 we assume that $\tilde{m}(y,t|x) \equiv \partial^2\tilde{M}(y,t|x)/\partial y\partial t$ and $\tilde{n}(x,t|y) \equiv \partial^2\tilde{N}(x,t|y)/\partial x\partial t$ exist. Furthermore, let $\mu(x)$ and $\lambda(y)$, for $x\in K$, $y\in K$, be the population densities of the X- and Y-attributes, respectively. Finally let $\theta(t)$ be the probability that it is possible to realize a match with contract attribute equal to t given that a supplier and a demander wish to form a match with contract t.

Consider next the distribution of the taste-shifters. For simplicity, let

$$E^s(k) = \varepsilon^{sD(k)}(x, Y(k), T(k)), \quad E^s(0) = \varepsilon_0^s(x)$$

$$E^d(k) = \eta^{dS(k)}(y, X(k), T'(k)), \quad E^d(0) = \eta_0^d(y).$$

where $\{S(k), k=1,2,...\}$ and $\{D(k), k=1,2,...\}$, are the indices of the agents in the conditional choice sets A_d and B_s for demander d and supplier s, respectively.

We assume that $\{(E^s(k), D(k)), k=0,1,...\}$ are generated by a Poisson process on $R_+ \times [0,1]$ with intensity measure

$$\varepsilon^{-2} d\varepsilon \cdot d\delta. \tag{7.5}$$

Similarly, $\{(E^d(k), S(k)), k=0,1,2,...\}$ are also generated by a Poisson process $R_+ \times [0,1]$ with intensity measure

$$\eta^{-2} d\eta \cdot ds. \tag{7.6}$$

We assume that these two processes are independent. Recall that (7.5) means that the probability that there is a point of the first Poisson process for which $(E^s(k) \in (\varepsilon, \varepsilon+\Delta\varepsilon)$, $D(k) \in (\delta, \delta+\Delta\delta))$ is equal to $\varepsilon^{-2}\Delta\varepsilon\Delta\delta + o(\Delta\delta\Delta\varepsilon)$. Furthermore, we assume that $\{(E^s(k), D(k))\}$ and $\{(E^d(k), S(k))\}$ are stochastically independent of B_s and A_d.

Next, let us consider the joint distribution of the potential attributes and taste-shifters. It follows from Proposition 3.8, (page 135) in Resnick (1987) that $\{(Y(k), T(k), E^s(k), D(k)), k=0,1,...\}$ are the points of a Poisson process on $K \times L \times R_+ \times [0,1]$ with intensity measure

$$\tilde{M}(dy, dt \mid x) \cdot \varepsilon^{-2} d\varepsilon \cdot d\delta. \tag{7.7}$$

Similarly, it follows that $\{(X(k), T'(k), E^d(k), S(k)), k=0,1,...\}$ are the points of a Poisson on $K \times L \times R_+ \times [0,1]$ with intensity measure

$$\tilde{N}(dx, dt \mid y) \cdot \eta^{-2} d\eta \cdot ds. \tag{7.8}$$

A theoretical justification for (7.5) to (7.8) is given in Dagsvik (1993a).

Let us now consider the distribution of supply and demand. By the conditional supply probability we mean the probability of preferring a match under contract t with demander d with attribute y given that (y,t) is feasible. Formally, this can be expressed as

$$\tilde{g}^{S}(y,t|x) = P\left\{a(x,y,t)\,\varepsilon^{sd}(x,y,t) > \max_{(Y(k),T(k))\in B_s-\{y,t\}} \left(a(x,Y(k),T(k))\,E^{s}(k)\right) \,\middle|\, (y,t)\in B_s\right\}. \tag{7.9}$$

In (7.9) it is given that a demander and contract attribute with values (y,t) are feasible. Formally, this condition can be accounted for by replacing the intensity measure

$$\tilde{M}(du,dz|x)\cdot\varepsilon^{-2}d\varepsilon\cdot d\delta$$

by

$$M^{*}_{yt}(du,dz|x)\cdot\varepsilon^{-2}d\varepsilon\cdot d\delta,$$

where

$$M^{*}_{yt}(du,dz|x) = \begin{cases} 1 & \text{for } (y,t)\in((u,u+du),(z,z+dz)), \\ \tilde{M}(du,dz|x) & \text{otherwise.}\end{cases} \tag{7.10}$$

According to Appendix 2 the probability that supplier s shall prefer a match with a particular demander $D(k)\in(\delta,\delta+\Delta\delta)$ with attribute y under contract attribute t, given that some demander with attribute y and contract attribute t are feasible, equals

$$\tilde{g}^{S}(y,t|x)\Delta\delta = \frac{a(x,y,t)\Delta\delta}{\tilde{a}_0(x) + a(x,y,t)\Delta\delta + \iint\limits_{KL} a(x,u,z)\,\tilde{m}(u,z|x)\,du\,dz\int_0^1 d\delta} + o(\Delta\delta) \tag{7.11}$$

where

$$\tilde{a}_0(x) = a_0(x)\tilde{M}(0,0|x).$$

Hence

$$\tilde{g}^{S}(y,t|x) = \frac{a(x,y,t)}{\tilde{a}_0(x) + \int\int a(x,u,z)\,\tilde{m}(u,z|x)\,du\,dz}. \tag{7.12}$$

Similarly, the conditional demand probability is given by

$$\tilde{g}^{D}(x,t|y) = \frac{b(y,x,t)}{\tilde{b}_0(y) + \int\int b(y,u,z)\,\tilde{n}(u,z|y)\,du\,dz}. \tag{7.13}$$

where

$$\tilde{b}_0(y) = b_0(y)\tilde{N}(0,0,|y).$$

For simplicity we have suppressed the domain of integration in (7.12) and (7.13).

Next let us consider the equilibrium conditions that determine $\tilde{m}(y,t|x)$ and $\tilde{n}(x,t|y)$. Obviously

$$\tilde{m}(y,t|x) = \tilde{g}^{D}(x,t|y)\lambda(y)\theta(t) \tag{7.14}$$

because

$$\tilde{g}^{D}(x,t|y)\lambda(y)\theta(t)\Delta y\Delta t + o(\Delta t\Delta y)$$

is the probability that there is a demander with attribute in $(y,y+\Delta y)$ and a contract with attribute in $(t,t+\Delta t)$ times the probability that this demander is interested in forming a match with a specific supplier with attribute x under contract t. Similarly

$$\tilde{n}(x,t|y) = \tilde{g}^{S}(y,t|x)\mu(x)\theta(t) \tag{7.15}$$

is the structural form of $\tilde{n}(x,t|y)$. Now let

$$\tilde{A}(x) \equiv \tilde{a}_0(x) + \int\int a(x,u,z)\,\tilde{m}(u,z|x)\,du\,dz \tag{7.16}$$

and

$$\tilde{A}(x) \equiv \tilde{a}_0(x) + \iint a(x,u,z)\,\tilde{m}(u,z|x)\,du\,dz \tag{7.16}$$

and

$$\tilde{B}(y) \equiv \tilde{b}_0(y) + \iint b(y,u,z)\,\tilde{n}(u,z|y)\,du\,dz. \tag{7.17}$$

From (7.12), (7.13), (7.14) and (7.15) it follows that

$$\tilde{A}(x) = \tilde{a}_0(x) + \int \frac{\tilde{c}(x,u)\lambda(u)\,du}{\tilde{B}(u)} \tag{7.18}$$

and

$$\tilde{B}(y) = \tilde{b}_0(y) + \int \frac{\tilde{c}(u,y)\mu(u)\,du}{\tilde{A}(u)}, \tag{7.19}$$

where

$$\tilde{c}(x,y) \equiv \int a(x,y,z)\,b(y,x,z)\,\theta(z)\,dz. \tag{7.20}$$

From (7.12), (7.13), (7.14) and (7.15) we get the next result.

Theorem 3

Under the assumptions stated above the conditional demand and supply probabilities of each type are determined by

$$\tilde{g}^D(x,t|y) = \frac{b(y,x,t)}{\tilde{B}(y)} \tag{7.21}$$

and

$$\tilde{g}^S(y,t|x) = \frac{a(x,y,t)}{\tilde{A}(x)}, \tag{7.22}$$

where $\{\tilde{A}(x)\}$ and $\{\tilde{B}(y)\}$ are determined by (7.18) and (7.19).

Corollary 5

The probability density that a supplier with attribute x shall realize a match with any demander with attribute y and contract t is equal to

$$\tilde{q}^S(y,t|x) = \frac{a(x,y,t)\tilde{m}(y,t|x)}{\tilde{a}_0(x) + \int\int a(x,u,z)\tilde{m}(u,z|x)dudz} = \frac{a(x,y,t)b(y,x,t)\lambda(y)\theta(t)}{\tilde{A}(x)\tilde{B}(y)}. \tag{7.23}$$

On the demand side the probability density of realizing a match is equal to

$$\tilde{q}^D(x,t|y) = \frac{b(y,x,t)\tilde{n}(x,t|y)}{\tilde{b}_0(y) + \int\int b(y,u,z)\tilde{n}(u,z|y)dudz} = \frac{b(y,x,t)a(x,y,t)\mu(x)\theta(t)}{\tilde{A}(x)\tilde{B}(y)}. \tag{7.24}$$

The respective probabilities of being self-matched are given by

$$\tilde{q}_0^S(x) = \frac{\tilde{a}_0(x)}{\tilde{A}(x)} \tag{7.25}$$

and

$$\tilde{q}_0^D(y) = \frac{\tilde{b}_0(y)}{\tilde{B}(y)}. \tag{7.26}$$

Apart from a normalization of $\tilde{m}(y,t|x)$ and $\tilde{n}(x,t|y)$ the structure of (7.23) and (7.24) is analogous to the continuous logit model developed by Ben-Akiva et al. (1985), and Dagsvik and Strøm (1992). Suitably normalized, these authors call $\tilde{m}(y,t|x)$ and $\tilde{n}(x,t|y)$ opportunity densities. Ben-Akiva et al. (1985) consider the opportunity densities as exogenously given. In Dagsvik and Strøm op cit. the opportunity densities are endogenous, but they do not discuss the explicit determination of these densities.

8. Markets for differentiated products with continuous attributes

We shall now consider markets for differentiated products where the demanders (suppliers) only have preferences over variant-attributes. Thus the present setting is to some degree similar to Anderson and Palma (1992), see also Perloff and Salop (1985). The essential differences are the following ones: First, Anderson and Palma assume that suppliers (firms) are uncertain about the demand and therefore maximize expected profit. Second, they assume that all the firms have the same profit functions. In contrast, the present framework assumes that agents operate under perfect certainty - but allows both heterogeneous suppliers and demanders. We also allow for the possibility of latent constraints that restrict the set of feasible variant-attributes. This will be discussed in more detail below. In fact, since the utility of an agent does not depend on attributes of potential partners, the modeling context is not a matching one. We have still chosen to analyze this case here because it is closely related to the analysis in the previous section.

Similarly to the notation in Section 7, let $U^s(x,t)$ be the utility - or profit function of supplier s with observable attribute $x \in K$ for supplying a variant of a differentiated product with characteristics $t \in L$. The vector t may include price and variables that define the quality and particular properties of the variants. Analogously, demander d with observable attribute y has utility function $U^d(y,t)$, where $y \in K$. The utilities of supplier s and demander d for not supplying and demanding any product variant are $U_0^s(x)$ and $U_0^d(y)$, respectively. We assume that

$$U^s(x,t) = a(x,t)\,\varepsilon^s(x,t), \tag{8.1}$$

$$U_0^s(x) = a_0(x)\varepsilon_0^s(x), \tag{8.2}$$

$$V^d(y,t) = b(y,t)\eta^d(y,t) \tag{8.3}$$

and

$$V_0^d(y) = b_0(y)\eta_0^d(y) \tag{8.4}$$

where $\varepsilon^s(x,t)$, $\varepsilon_0^s(x)$, $\eta^d(y,t)$ and $\eta_0^d(y)$ are random tasteshifters. The probability densities of supplier and demander characteristics are $\mu(x)$ and $\lambda(y)$. In the product market only a countable set of product variants are produced. We assume that which demander that trades with which supplier is random to the agents, due to the fact that they are indifferent with respect to the attributes of their potential trading partners. As a consequence, the offers (demands) to a demander (supplier) will depend on the demander (supplier) since the utility functions are agent specific. Let $B_s = \{T(k), k=1,2,...\}$ denote the set of attributes of the variants that are demanded from supplier s (of type x). We assume that this set is generated as independent draws from a conditional probability distribution $\tilde{N}(t|x)$ with density $\tilde{n}(t,x)$. We assume furthermore that each supplier at most produces one unit of the product each period and that each demander cannot buy more than one unit of the product per period. Let $A_d = \{T'(k), k=1,2,...\}$ be the attributes of the variants that are supplied to demander d (of type y). The attributes in A_d are independent draws from a conditional probability distribution $\tilde{M}(t|y)$ with density $\tilde{m}(t|y)$. For simplicity, define

$$E^s(k) = \varepsilon^s(x,T(k)), \quad E^s(0) = \varepsilon_0^s(x)$$

$$E^d(k) = \eta^d(y,T(k)), \quad E^d(0) = \eta_0^d(y).$$

We assume that the tasteshifters, $\{E^s(k), k=1,2,...\}$, are points of a Poisson process on R_+ with intensity measure

$$\varepsilon^{-2}d\varepsilon. \tag{8.5}$$

Similarly $\{E^d(k), k=1,2,...\}$ are the points of a Poisson process on R_+ with intensity measure

$$\eta^{-2}d\eta. \tag{8.6}$$

The variables $E^s(0)$ and $E^d(0)$ are i.i.d. with c.d.f.

$$P\left(E^{s}(0)\le z\right) = P\left(E^{d}(0)\le z\right) = \exp\left(-\frac{1}{z}\right). \tag{8.7}$$

The sets B_s and A_d are assumed stochastically independent for s=1,2,... and d=1,2,... . They are also stochastically independent of $E^s(0)$ and $E^d(0)$. From Proposition (3.8) (page 135) in Resnick (1987) it follows that $\{(T(k),E^s(k)), k=1,2,...\}$ are the points of a Poisson process on $L\times R_+$ with intensity measure

$$\tilde{N}(dt|x)\cdot\varepsilon^{-2}d\varepsilon \tag{8.8}$$

and $\{(T'(k),E^d(k)), k=1,2,...\}$ are the points of a Poisson process on $L\times R_+$ with intensity measure

$$\tilde{M}(dt|y)\cdot\eta^{-2}d\eta. \tag{8.9}$$

Now we are ready to derive conditional supply and demand probabilities.

The conditional supply probability is now defined as the probability of preferring a product with attribute t given that this attribute is feasible. Formally, we can express this as

$$\tilde{g}^{S}(t|x) = P\left\{a(x,t)\varepsilon^{s}(x,t) > \max_{T(k)\in B_s-\{t\}}\left(a(x,T(k))E^{s}(k)\right)|t\in B_s\right\}. \tag{8.10}$$

Under the assumption above the calculation of (8.10) is completely analogous to the calculation of (7.9). Hence, similarly to (7.12) we obtain

$$\tilde{g}^{S}(t|x) = \frac{a(x,t)}{a_0(x)+\int a(x,u)\tilde{m}(u)\,du} \tag{8.11}$$

where

$$\tilde{m}(t) = \int\tilde{m}(t|y)\lambda(y)\,dy \tag{8.12}$$

is the demand density for variants with attribute t.

Similarly, the conditional demand probability is given by

$$\tilde{g}^{D}(t|y) = \frac{b(y,t)}{b_0(y) + \int b(y,u)\tilde{n}(u)\,du} \tag{8.13}$$

where

$$\tilde{n}(t) = \int \tilde{n}(t|x)\mu(x)\,dx \tag{8.14}$$

is the supply density for variants with attribute t. The demand density from demanders with attribute y is given by

$$\tilde{m}(t|y) = \tilde{g}^{D}(t|y)\theta(t) \tag{8.15}$$

and the corresponding supply density equals

$$\tilde{n}(t|x) = \tilde{g}^{S}(t|x)\theta(t) \tag{8.16}$$

where $\theta(t)$ is the probability that product attribute t is feasible. This probability is supposed to account for latent restrictions on production technology as well as environmental regulations, etc. Analogous to (7.16) and (7.17), define

$$\tilde{A}(x) \equiv a_0(x) + \int a(x,u)\tilde{m}(u)\,du \tag{8.17}$$

and

$$\tilde{B}(y) \equiv b_0(y) + \int b(y,u)\tilde{n}(u)\,du. \tag{8.18}$$

From (8.11), (8.13), (8.15) and (8.16) we get

$$\tilde{A}(x) = a_0(x) + \int \frac{\tilde{c}(x,z)\lambda(z)dz}{\tilde{B}(z)} \tag{8.19}$$

and

$$\tilde{B}(y) = b_0(y) + \int \frac{\tilde{c}(z,y)\mu(z)dz}{\tilde{B}(z)} \tag{8.20}$$

where

$$\tilde{c}(x,y) \equiv \int a(x,u)b(y,u)\theta(u)du. \tag{8.21}$$

We summarize the above results in the next theorem.

Theorem 4

Under the assumptions stated in the present section the conditional demand and supply probabilities are determined by

$$\tilde{g}^{D}(t|y) = \frac{b(y,t)}{\tilde{B}(y)} \tag{8.22}$$

and

$$\tilde{g}^{S}(t|x) = \frac{a(x,t)}{\tilde{A}(x)} \tag{8.23}$$

where $\{\tilde{A}(x)\}$ and $\{\tilde{B}(y)\}$ are determined by (8.19) and (8.20).

Finally, we shall consider the probability densities for trading. Recall that which supplier that trade with which demander is random to the agents. Let $\tilde{q}^S(t|x)$ and $\tilde{q}^D(t|y)$ be the respective probability densities of a supplier of type x and a demander of type y for trading a variant with attribute t. As above equilibrium is defined in a probabilistic sense. Since $\tilde{m}(t)\Delta t$ is the probability that there is demand for a (feasible) variant with attribute in $(t,t+\Delta t)$ in the market we must have

$$\tilde{q}^{S}(t|x)\Delta t = \tilde{g}^{S}(t|x)\tilde{m}(t)\Delta t + o(\Delta t). \tag{8.24}$$

Similarly

$$\tilde{q}^{D}(t|y)\Delta t = \tilde{g}^{D}(t|y)\tilde{n}(t)\Delta t + o(\Delta t). \tag{8.25}$$

From (8.12), (8.14), (8.15), (8.16), (8.22), (8.23), (8.24) and (8.25) the next corollary follows.

Corollary 6

The probability density that a supplier with characteristics x shall trade a variant with attribute t is equal to

$$\tilde{q}^{S}(t|x) = \frac{a(x,t)\tilde{m}(t)}{a_0(x) + \int a(x,u)\tilde{m}(u)du} = \frac{\theta(t)a(x,t)}{\tilde{A}(x)}\int\frac{b(z,t)\lambda(z)dz}{\tilde{B}(z)}. \tag{8.26}$$

On the demand side the probability density of trading a variant with attribute t equals

$$\tilde{q}^{D}(t|y) = \frac{b(y,t)\tilde{n}(t)}{b_0(y) + \int b(y,u)\tilde{n}(u)du} = \frac{\theta(t)b(y,t)}{\tilde{B}(y)}\int\frac{a(z,t)\mu(z)dz}{\tilde{A}(z)}. \tag{8.27}$$

The respective probabilities of not trading are given by

$$\tilde{q}^{S}(x) = \frac{a_0(x)}{\tilde{A}(x)} \tag{8.28}$$

and

$$\tilde{q}_0^{D}(y) = \frac{b_0(y)}{\tilde{B}(y)}. \tag{8.29}$$

When we compare (7.21) and (8.24) we realize that when (7.23) is integrated with respect to y we get the same expression as in (8.26). Thus we can conclude (under the present assumptions) that whether or not the agents have preferences over latent attributes of their potential trading partners does not matter for the model structure.

Appendix 1

Proof of Lemma 1

Since $E\tilde{m}^{sd}_{\tau+1} = (M-1)g^D_\tau$, $E\tilde{n}^{ds}_{\tau+1} = (N-1)g^S_\tau$ and $(1+\alpha+x)^{-1}$ is a convex function in x, the left hand side of (2.21) follows by applying Jensen's inequality to (2.13). Consider the right hand side of (2.21). Observe first that $e^{z-1}-z\geq 0$ when $z\in[0,1]$. To realize this we note that $e^{z-1}-z$ is decreasing in z and consequently it's minimum value is zero and it is attained at z=1. Now by applying this inequality to (2.19) we obtain

$$
\begin{aligned}
g^S_{\tau+1} &= \int_0^1 z^\alpha\left(1-g_\tau^D+zg_\tau^D\right)^{M-1}dz \leq \int_0^1 \exp\left((z-1)\left(\alpha+(M-1)g_\tau^D\right)\right)dz \\
&= \frac{1-\exp\left(-\left(\alpha+(M-1)g_\tau^D\right)\right)}{\alpha+(M-1)g_\tau^D} \leq \frac{1}{\alpha+(M-1)g_\tau^D}.
\end{aligned}
\tag{A.1}
$$

Also (2.13) implies that

$$
g^S_{\tau+1} \leq \frac{1}{1+\alpha}. \tag{A.2}
$$

When combining (A.1) and (A.2) we obtain the right hand side of (2.21). The proof of (2.22) is completely analogous.

Q.E.D.

Theorem 4

Eq. (5.20) and (5.21) have a unique solution $(A_1^*,A_2^*,\ldots,A_Z^*,\ B_1^*,B_2^*,\ldots,B_\Delta^*)$ *with* $A_i^* > 0$ *and* $B_j^* > 0$ *provided* $\bar{a}_{i0} > 0$ *and* $\bar{b}_{j0} > 0$ *for all i and j.*

Proof:

Let $x_i = \log A_i$ and let F_i, i=1,2,...,Z, be mappings $F_i : R^Z \to R$ defined by

$$\exp(F_i(\mathbf{x})) = a_{i0} + \sum_k \left(\frac{\bar{M}_k c_{ik}}{b_{k0} + \sum_s \bar{N}_s c_{sk} e^{-x_s}} \right). \tag{A.3}$$

where $\bar{M}_k = M_k/\sqrt{MN}$ and $\bar{N}_k = N_k/\sqrt{MN}$. We notice that we get the expression on the right hand side of (A.3) when inserting (5.21) into (5.20). Thus we wish to prove that the system of equation

$$x_i - F_i(\mathbf{x}), \quad i = 1,2,...,Z, \tag{A.4}$$

has a real and unique root. Note first that since $\bar{b}_{i0} > 0$ is follows from (A.3) that

$$\exp(F_i(\mathbf{x})) < \bar{a}_{i0} + \sum_k \left(\frac{\bar{M}_k c_{ik}}{\bar{b}_{0k}} \right). \tag{A.5}$$

Now define the norm $\|\cdot\|$ by $\|x\| = \max_k |x_k|$, $x \in R^Z$. Since $F_i(x)$ is continuously differentiable the mean value theorem yields

$$F_i(\mathbf{x}) - F_i(\mathbf{y}) = \sum_j \frac{\partial F_i(\mathbf{x}^*)(x_j - y_j)}{\partial x_j} \tag{A.6}$$

for x, y $\in R^Z$ where $\mathbf{x}^*(i) \in R^Z$ is a point on the plane between **x** and **y**. Let

$$d = 1 + \max_i \sum_k \left(\frac{\bar{M}_k c_{ik}}{\bar{b}_{0k} \bar{a}_{i0}} \right).$$

Now partial differentiation of (A.3) yields

$$\exp(F_i(\mathbf{x}))\sum_j \frac{\partial F_i(\mathbf{x})}{\partial x_j} = \sum_k \frac{\bar{M}_k c_{ik}(L_k - b_{k0})}{L_k^2}, \tag{A.7}$$

where

$$L_k = \bar{b}_{k0} + \sum_s \bar{N}_s c_{sk} e^{-x_s} \tag{A.8}$$

and c_{ik} is given by (5.22). From (A.7) and (A.3) it follows immediately that

$$\sum_j \frac{\partial F_i(\mathbf{x})}{\partial x_j} \leq \exp(-F_i(\mathbf{x}))\sum_k \frac{\bar{M}_k c_{ik}}{L_k} = 1 - \bar{a}_{i0}\exp(-F_i(\mathbf{x})). \tag{A.9}$$

From (A.6) and (A.9) we get

$$\|F(\mathbf{x}) - F(\mathbf{y})\| \leq \|\mathbf{x}-\mathbf{y}\| \max_i \left(1 - \frac{\bar{a}_{i0}}{\exp(F_i(\mathbf{x}^*(i)))}\right) \leq \|\mathbf{x}-\mathbf{y}\|\left(1 - \frac{1}{d}\right), \tag{A.10}$$

where $F(\mathbf{x}) = (F_1(\mathbf{x}), F_2(\mathbf{x}), \ldots, F_Z(\mathbf{x}))$. Since $1 < d < \infty$, (A.10) implies that $\mathbf{x} \to F(\mathbf{x})$ is a contraction mapping and consequently it has a unique fixed point.

Q.E.D.

Appendix 2

Derivation of choice probabilities when the choice set is generated by a Poisson process

Here we consider a simplified version of the choice problem discussed in Sections 7 and 8. A version of the derivation below is given by de Haan (1984), but for the readers' convenience it is given here.

The agent faces a countable choice set where alternative z is represented by two variables, T(z) and E(z). The variable T(z) takes values in [0,1] and is an objective attribute that characterizes the alternative while E(z) takes values in R_+ and is interpreted as a tasteshifter that is unobservable to the analyst. The utility function has the form

$$U(z) = v(T(z))\varepsilon(z) \tag{A.11}$$

where $v : [0,1] \to R_+$ is a measurable function (deterministic). The variables (T(z), E(z)), z=1,2,..., are assumed to be realization of a Poisson process on $[0,1] \times R_+$ with intensity measure

$$f(dt) \cdot \varepsilon^{-2} d\varepsilon \tag{A.12}$$

where f is a finite measure. Let $A \subset [0,1]$ be a Borel set and define

$$U_A = \max_{\substack{z \\ T(z)\in A}}(v(T(z))\varepsilon(z)). \tag{A.13}$$

The interpretation of U_A is as the highest utility the agent can attain subject to $T(z) \in A$. We shall now derive the cummulative distribution of U_A. Let

$$B = \{(t,\varepsilon) : v(t)\varepsilon > u,\ t\in A\}$$

and let N(B) be the number of Poisson points within B. By the Poisson law

$$P\big(N(B)=n\big) = \frac{\big(EN(B)\big)^n}{n!}\exp\big(-EN(B)\big) \tag{A.14}$$

where

$$EN(B) = \int_B f(dy)\,\varepsilon^{-2}d\varepsilon = \frac{1}{u}\int_A v(y)f(dy). \tag{A.15}$$

It now follows from (A.14) and (A.15) that

$$\begin{aligned} P\big(U_A \le u\big) &= P\big(\text{There are no points of the Poisson process in B}\big) \\ &= P\big(N(B)=0\big) = \exp\big(-EN(B)\big) = \exp\left(-\frac{1}{u}\int_A v(y)f(dy)\right), \end{aligned} \tag{A.16}$$

which demonstrates that U_A is type I extreme value distributed.

Next we shall derive the probability of choosing an alternative with attribute in A. Let $\bar{A}$ be the complement of A. Since the Poisson realizations are independent it follows that U_A and $U_{\bar{A}}$ are independent. Hence, by straight forward calculus we get

$$P\big(U_A > U_{\bar{A}}\big) = \frac{\int_A v(y)f(dy)}{\int_A v(y)f(dy) + \int_{\bar{A}} v(y)f(dy)}. \tag{A.17}$$

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