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Theoretical Foundations of Lorenz Curve Orderings

by

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Abstract

This paper is concerned with distributions of income and the ordering of related Lorenz curves. By introducing appropriate preference relations over the set of Lorenz curves two alternative theories for Lorenz curve orderings are proposed. Moreover, the Gini coefficient is recognized to be rationalizable under both theories. As a result, a complete axiomatization of the Gini coefficient is obtained. The paper also introduces various criteria of Lorenz dominance and examines their relationship to the welfare functions representing the proposed preference relations over Lorenz curves. The derived dominance results provide, as a special case, an alternative to Rawls (relative) leximin criterion as the most inequality averse criterion for judging between Lorenz curves.

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1. Introduction

The Lorenz curve was introduced by Lorenz [11] as a representation of the concept of inequality with particular reference to the distribution of income. By displaying the deviation of each individual income share from the income share that corresponds to perfect equality, the Lorenz curve captures the essential descriptive features of the concept of inequality. The normative aspects of Lorenz curve orderings have been discussed by Kolm [8, 9, 10] and Atkinson [1] who demonstrated that Lorenz curve orderings of distributions with equal means may correspond to social welfare orderings. The assumption of equal means, however, limits the applicability of their results. Typically, the Pigou-Dalton transfer principle is introduced to examine how social welfare or inequality is affected by transfers of a given amount. However, real world interventions that alter the income distribution are usually not mean preserving changes; taxes and transfer programs, for example, are interventions that decrease and increase the mean level of income. The standard approach for extending the judgements about inequality to distributions with different means is to add the condition of scale invariance to the principle of transfers. This implies that inequality depends only on relative incomes and is compatible with the representation given by the Lorenz curve. Thus, if a social decision-maker adopts the Lorenz curve as a criterion for judging between income distributions then the decision-maker is only concerned about the distributional aspects independent of the level of mean income. See Sen [13] for further discussion.

The purpose of this paper is to extend the normative evaluation of Lorenz curve orderings to cases of variable mean income. Analogous to theories of choice under uncertainty we establish preference orderings of Lorenz curves as a basis for the derivation of measures of inequality. Section 2 presents two alternative set of assumptions concerning social decision-makers' preferences over Lorenz curves defined over income distributions and gives convenient representations of the corresponding preference relations. Furthermore, a complete axiomatization of the Gini coefficient is proposed. Section 3 deals with various criteria for Lorenz dominance. These criteria are concerned with the placement of focus of inequality in a Lorenz curve and prove valuable since we may be able to agree on certain properties of the measure of inequality even if we cannot agree on its precise form. Alternatively, where the Lorenz dominance criteria are satisfied we find that the same conclusions may be drawn from a wide family of measures of inequality. In Section 4 these results are used as basis for examining the properties of two alternative families of inequality measures, the "generalized" Gini family and another that is derived from the results established in Section 2. As a special case we obtain an alternative to Rawl's (relative) leximin as the most inequality averse criterion.

2. Representation results

The normative evaluation of Lorenz curve orderings is traditionally made in terms of absolute welfare and is thus restricted to distributions with equal mean incomes. As is well-known, however, in applied work it is standard practice to employ the Lorenz curve to judge inequality in distributions with different mean incomes. Thus, an important challenge is to extend the normative evaluation of Lorenz curve orderings to general situations without imposing any restrictions on the corresponding distribution functions apart from the assumption of positive support.

In this section we shall demonstrate that the problem of ranking Lorenz curves is formally analogous to the problem of choice under uncertainty. In theories of choice under uncertainty, preference orderings over probability distributions are introduced as basis for deriving utility indexes. Similarly, assuming appropriate preference relations over the set of Lorenz curves enable us to establish a theoretical foundation for Lorenz curve orderings. As a result, a social decision-maker who adopts the Lorenz curve as a criterion for ranking income distributions will select a Lorenz curve according to the maximization of some utility index.

The Lorenz curve L for a cumulative income distribution F with mean μ is defined by

$$L(u) = \frac{1}{\mu}\int_0^u F^{-1}(t)dt, \quad 0 \le u \le 1, \tag{1}$$

where F^{-1} is the left inverse of F.

Now, let $\mathcal{L}$ denote the family of Lorenz curves. Note that L is nondecreasing and convex, and that a convex combination of Lorenz curves is a Lorenz curve and hence a member of $\mathcal{L}$. A social decision-maker's ranking of elements from $\mathcal{L}$ may be represented by a preference relation $\succeq$, which will be assumed to satisfy the following basic axioms,

Axiom 1 (Order). *$\succeq$ is a transitive and complete ordering on $\mathcal{L}$.*

Axiom 2 (Dominance). *Let $L_1, L_2 \in \mathcal{L}$. If $L_1(u) \ge L_2(u)$ for all $u \in [0,1]$ then $L_1 \succeq L_2$.*

Axiom 3 (Continuity). *For each $L \in \mathcal{L}$, the sets $\{L^* \in \mathcal{L} : L \succeq L^*\}$ and $\{L^* \in \mathcal{L} : L^* \succeq L\}$ are closed (w.r.t. L_1-norm).*

Then it follows by Debreu [2] that $\succeq$ can be represented by a continuous and increasing preference functional V from $\mathcal{L}$ to R. Hence V defines an ordering for the society (decision-maker) over the set of all Lorenz curves (over income distributions) and is thus a social welfare function. To avoid any confusion we call V a Lorenz social welfare function.

In order to give the theory of the decision-maker's preferences over alternative Lorenz curves an empirical content it is necessary to impose further restrictions on V. We can obtain convenient and testable representations of $\succeq$ by postulating appropriate independence conditions on the preferences; this is analogous to theories of individual behavior towards risk. Specifically, we shall consider the two following axioms, where L^{-1} is the inverse mapping of L, $L^{-1}(0)=0$ and $L^{-1}(1)=1$.

Axiom 4 (Independence). *Let L_1, L_2 and L_3 be members of $\mathcal{L}$ and let $\alpha \in [0,1]$. Then $L_1 \succeq L_2$ implies $\alpha L_1 + (1-\alpha)L_3 \succeq \alpha L_2 + (1-\alpha)L_3$.*

Axiom 5 (Dual independence). *Let L_1, L_2 and L_3 be members of $\mathcal{L}$ and let $\alpha \in [0,1]$. Then $L_1 \succeq L_2$ implies $(\alpha L_1^{-1} + (1-\alpha)L_3^{-1})^{-1} \succeq (\alpha L_2^{-1} + (1-\alpha)L_3^{-1})^{-1}$.*

Note that Axioms 4 and 5 correspond to the independence axioms of expected utility theory and Yaari's dual theory of choice under uncertainty, respectively (see Yaari [16]). Axioms 4 and 5 require that preferences are invariant with respect to certain changes in the Lorenz curves being compared. If L_1 is weakly preferred to L_2, then Axiom 4 states that any mixture on L_1 is weakly preferred to the corresponding mixture on L_2. The intuition is that identical mixing interventions on the Lorenz curves being compared do not affect the preferences of the decision-maker; the preferences depend solely on how the decision-maker judges the differences between the mixed Lorenz curves. In order to clarify the interpretation of Axiom 4 consider an example with a tax/transfer intervention that alters the shape of the income distributions and leaves the mean incomes unchanged:

Let F_1 and F_2 be income distributions with means μ_1 and μ_2 and Lorenz curves L_1 and L_2. Now suppose that these distributions are affected by the following tax/transfer reform. First, a proportional tax with tax rate $1-\alpha$ is introduced. Second, the collected taxes are in both cases redistributed according to appropriate scale transformations of some distribution function F_3 with mean μ_3. This means that the two sets of collected taxes are redistributed according to the same distribution of relative incomes. It is understood that this redistribution is carried out so as to give equal-sized transfers or transfers that are less progressive than a set of equal-sized transfers. Specifically, this means that $(1-\alpha)(\mu_i/\mu_3)F_3^{-1}(t)$ is the transfer received by the t-fractile unit of the income distribution F_i. At the extreme, when F_3 is a degenerate distribution function, the transfers are equal to the average tax $(1-\alpha)\mu_i$. At the other extreme F_3 will give all the collected tax to the best well-off unit. After this tax/transfer intervention, the inverses of the two income distributions are given by

$$\alpha F_i^{-1}(t) + (1-\alpha)\mu_i \frac{F_3^{-1}(t)}{\mu_3}, \quad i=1,2, \tag{2}$$

where $F_i^{-1}(t)$ is the left inverse of F_i. Now, it follows readily from (2) that this intervention leaves the mean incomes, μ_1 and μ_2, unchanged. Moreover, (2) implies that the Lorenz curve for F_i after the intervention have changed from L_i to

$$\frac{1}{\mu_i}\int_0^u \left[\alpha F_i^{-1}(t) + (1-\alpha)\mu_i \frac{F_3^{-1}(t)}{\mu_3}\right] dt = \alpha L_i(u) + (1-\alpha)L_3(u), \quad i=1,2, \tag{3}$$

Hence, if L_1 is weakly preferred to L_2, then Axiom 4 states that the changes in L_1 and L_2 that follows from the above intervention will not affect the preferences of the decision-maker.

Axiom 5 postulates a similar invariance property on the inverse Lorenz curves to that postulated by Axiom 4 on the Lorenz curves. The essential difference is that Axiom 5 deals with the relationship between given income shares and weighted averages of corresponding population shares, while Axiom 4 deals with the relationship between given population shares and weighted averages of corresponding income shares. Thus, Axiom 5 requires the preferences to be invariant with respect to aggregation of subpopulations across cumulative income shares. That is, if for a specific population the Lorenz curve L_1 is weakly preferred to the Lorenz curve L_2, then mixing this population with any other population with respect to the distributions of their income shares does not affect the preferences of the decision-maker. As an illustration, consider a population divided into a group of poor and a group of rich where each unit's income is equal to the corresponding group mean. In judging between two-points distributions social decision-makers who approve Axiom 4 and disapprove Axiom 5 will be more concerned about the number of poor rather than how poor they are. By contrast, social decision-makers who approve Axiom 5 and disapprove Axiom 4 will emphasize the size of the poor's income share rather than how many they are.

THEOREM 1. *A preference relation* $\succsim$ *on* $\mathcal{L}$ *satisfies Axioms 1-4 if and only if there exists a continuous and non-increasing real function* $p(\cdot)$ *defined on the unit interval, such that for all* $L_1, L_2 \in \mathcal{L}$,

$$L_1 \succsim L_2 \Leftrightarrow \int_0^1 p(u)dL_1(u) \geq \int_0^1 p(u)dL_2(u). \tag{4}$$

Moreover, p is unique up to a positive affine transformation.

Proof. The necessary part of the theorem follows straightforward by noting that

$$\int p(t)\,d(L_1(t)-L_2(t)) = -\int (L_1(t)-L_2(t))\,dp(t).$$

To prove the sufficiency part, note that $\mathcal{L}$ is a subfamily of distribution functions. Furthermore, it follows from Axioms 1-4 that the conditions of Theorem 3 of Fishburn [5] are satisfied and thus that there exists a continuous function $p(\cdot)$ satisfying (4) where $p(\cdot)$ is unique up to a positive affine transformation. It follows from the monotonicity property of Axiom 2 that $p(\cdot)$ is nonincreasing. Q.E.D.

REMARK. By restricting the comparison of Lorenz curves to distributions with equal means Theorem 1 coincides with the representation result of Yaari [16, 17].

Now, by replacing Axiom 4 with Axiom 5 in Theorem 1, we obtain the following alternative representation result.

THEOREM 2. A *preference relation* $\succsim$ *on* $\mathcal{L}$ *satisfies Axioms 1-3 and Axiom 5 if and only if there exists a continuous and nondecreasing real function* $q(\cdot)$ *defined on the unit interval, such that for all* $L_1, L_2 \in \mathcal{L}$,

$$L_1 \succsim L_2 \Leftrightarrow \int_0^1 q(L_1(u))du \geq \int_0^1 q(L_2(u))du. \tag{5}$$

Moreover, q is unique up to a positive affine transformation.

Proof. It follows from (1) that there is a one-to-one correspondence between the Lorenz curve and its inverse, and, moreover, that the inverse Lorenz curve also satisfies the conditions of being a distribution function. Hence, the preference relation $\succeq$ defined on the set of inverse Lorenz curves is equivalent to the preference relation defined on $\mathcal{L}$. Note that $L_1^{-1}(u) \leq L_2^{-1}(u)$ for all $u \in [0,1]$ if and only if $L_1(u) \geq L_2(u)$ for all $u \in [0,1]$. Then, by replacing Axiom 4 with Axiom 5, Theorem 2 follows directly from Theorem 1 where the preference representation is given by

$$\int_0^1 q(t)dL^{-1}(t) = \int_0^1 q(L(u))du. \qquad \text{Q.E.D.}$$

By defining a preference relation on the set of income distributions rather than on the set of Lorenz curves, Yaari [16, 17] proved that preferences satisfying a set of axioms analogous to Axioms 1-3 and 5 can be represented by the functional $\int_0^1 F^{-1}(u)d\tilde{P}(u)$ where $\tilde{P}$ is a continuously differentiable distribution function which characterizes the preferences of the decision-maker. Yaari [17] observed that this functional can be expressed as the product of the mean income and $\int_0^1 \tilde{P}'(u)dL_F(u)$, called the equality rating of the income distribution F with Lorenz curve L_F. As noted by Yaari op cit., the equality rating forms the basis for a particular type of measures of inequality. However, Yaari op cit. did not provide a justification for applying the equality rating as a representation of preferences over income distributions or Lorenz curves. Now, let V_P be a functional, $V_P : \mathcal{L} \rightarrow [0,1]$, defined by

$$V_P(L) = \int_0^1 P'(u)dL(u), \tag{6}$$

where P is a continuously differentiable and concave distribution function, defined on the unit interval. As mentioned above, V_P can be interpreted as a social welfare function on the family of Lorenz curves and thus it is called a Lorenz welfare function. Theorem 1 demonstrates that a social decision-maker who behaves according to Axioms 1-4 will choose among Lorenz curves so as to maximize V_P. Note that V_P has a similar structure as the expectation of $P'(U)$ where U is a random variable with c.d.f. L. Just as the expected utility function represents the preferences of agents in the standard choice theory, V_P represents the preferences of social decision-makers who judge between Lorenz curves. For convenience, and with no loss of generality, we assume $P'(1)=0$. This is a normalization condition which ensures that V_P has the unit interval as its range, taking the maximum value 1 if incomes are equally distributed and the minimum value 0 if one unit holds all income. Thus, $V_P(L)$ measures the Lorenz welfare provided by the Lorenz curve L relative to that attained under complete equality. Hence J_P, defined by

$$J_P(L) = 1 - \int_0^1 P'(u)dL(u), \tag{7}$$

measures the loss of Lorenz welfare or degree of Lorenz inequality of an income distribution with Lorenz curve L, when the social decision-maker has preference function P.

By choosing $P(t) = 2t-t^2$ it follows directly from (7) and Theorem 1 that the Gini coefficient is rationalizable under Axioms 1-4. Note that μV_P corresponds to the utility representation of the "dual theory" of choice under risk proposed by Yaari [16, 17], who also demonstrated that the absolute Gini difference was rationalizable under the "dual theory". Moreover, by establishing appropriate preference functions we can derive attractive alternatives to the Gini coefficient. For example, by choosing the following family of P-functions,

$$P_k(t) = 1 - (1-t)^{k+1}, \quad k \geq 0, \tag{8}$$

we obtain the following family of measures of inequality

$$D_k(L) = 1 - k(k+1)\int_0^1 (1-u)^{k-1}L(u)du, \quad k \geq 0. \tag{9}$$

The family $\{D_k\}$ is the "generalized" Gini family of inequality measures, introduced by Kakwani [7] as an extension of a poverty measure proposed by Sen [15]. For a discussion of the "generalized" Gini family, we refer to Donaldson and Weymark [3, 4] and Yitzhaki [18]. Further discussion of the properties of D_k is left for the next section.

Now, let V_Q^* be a functional, $V_Q^* : \mathcal{L} \rightarrow [0,1]$, defined by

$$V_Q^*(L) = \int_0^1 Q'(L(u))du = \int_0^1 Q'(t)dL^{-1}(t), \tag{10}$$

where Q is a continuous and convex distribution function defined on the unit interval. It follows from Theorem 2 that V_Q^* represents preferences which satisfy Axioms 1-3 and 5. The implication is that social decision-makers whose preferences satisfy Axioms 1-3 and 5 will choose among Lorenz curves so as to maximize V_Q^*. Further restrictions on the preferences can be introduced through the preference function Q. For normalization purposes we impose the condition $Q'(0)=0$. This condition implies that V_Q^* has the unit interval as its range, taking the maximum value 1 if incomes are equally distributed and the minimum value 0 if one unit holds all income. Thus, $V_Q^*(L)$ measures the Lorenz welfare exhibited by L relative to that attained under complete equality. Moreover, J_Q^* defined by

$$J_Q^*(L) = 1 - \int_0^1 Q'(L(u))du \tag{11}$$

measures the loss of Lorenz welfare of income distributions for a social decision-maker whose behavior is consistent with Axioms 1-3 and 5. By choosing $Q(t)=t^2$ in (11) it follows that J_Q^* coincides with the Gini coefficient. Surprisingly, there seems to be no proposals on alternatives to the Gini coefficient which are consistent with Theorem 2. However, by specifying appropriate preference functions in (11) we can derive measures of inequality which are consistent with Theorem 2. For example, by introducing the following family of preference functions

$$Q_k(t) = t^{k+1}, \quad k \geq 0, \tag{12}$$

we obtain the following related family of inequality measures

$$D_k^*(L) = 1 - (k+1)\int_0^1 L^k(u)du, \quad k \geq 0, \tag{13}$$

where D_1^* is the Gini coefficient. The properties of D_k^* will be examined more closely in the next section. Note, however, that measures of inequality which are rationalizable under Axioms 1-3 and Axiom 5 can be viewed as a sum of weighed population shares, where the weights depend on the functional form of the Lorenz curve in question and thereby on the magnitude of the income shares. This property is due to Axiom 5. But because of the restrictions imposed on the preferences by Axiom 4, the weights in J_P, unlike those in J_Q^*, depend on population shares rather than income shares. Hence, these weights do not depend on the magnitudes of the income shares, but merely on the rankings of income shares. Now, let us return to the above discussion of two-points distributions in the context of Axioms 4 and 5. Note that then the effect on J_P-measures from increasing the income share of the poor depends merely on the relative number of poor irrespective of their share of the incomes, while the similar effect on J_Q^*-measures depends both on the poor's share of the population and the incomes. By contrast, the effect on J_Q^*-measures of an increase in the relative number of poor depends merely on the poor's share of the incomes, while the effect on J_P-measures depends both on the poor's share of population and income.

Although several authors have discussed rationales for the absolute Gini difference (see Sen [14] and Yaari [16, 17]), no one has established a rationale for the Gini coefficient as a preference ordering on Lorenz curves. However, as was demonstrated above, the Gini coefficient is rationalizable under the theory provided by Axioms 1-4 and that provided by Axioms 1-3 and 5. Thus, we may conjecture that the Gini coefficient represents the preference relation over the set of Lorenz curves which satisfy Axioms 1-5.

THEOREM 3. *A preference relation $\succsim$ on $\mathcal{L}$ satisfies Axioms 1-5 if and only if $\succsim$ can be represented by the Gini coefficient.*

Proof. From Theorems 1 and 2 it follows that $\succsim$ satisfies Axioms 1-5 if and only if there exist a continuous, nonincreasing real function $p(\cdot)$ and a continuous, nondecreasing real function $q(\cdot)$, both defined on the unit interval, such that for all $L_1,L_2 \in \mathcal{L}$,

$$L_1 \succsim L_2 \Leftrightarrow \int_0^1 p(u)dL_1(u) = \int_0^1 q(L_1(u))du \succsim \int_0^1 p(u)dL_2(u) = \int_0^1 q(L_2(u))du,$$

where $p(\cdot)$ and $q(\cdot)$ are unique up to a positive affine transformation. The equality of the two representations must be true for all Lorenz curves and thus in particular for the Lorenz curve defined by

$$L(u) = \begin{cases} 0 & , \ 0 \le u \le a \\ \dfrac{u-a}{1-a} & , \ a < u \le 1 \end{cases} \tag{14}$$

where $0 \le a < 1$.The corresponding representations are given by

$$\int_0^1 p(u)dL(u) = \frac{1-P(a)}{1-a}, \tag{15}$$

where P is a cumulative distribution function with density p, $0 \le a < 1$, and

$$\int_0^1 q(L(u))du = 1 - a, \quad 0 \le a < 1 \tag{16}$$

where q is a density with $q(0)=0$. Since (15) and (16) both are required to be representations of Axiom 1-5, they must be equal. Hence,

$$P(a) = 1 - (1-a)^2, \quad 0 \le a \le 1 ,$$

which is the P-function for the Gini coefficient. However, the P-function is unique only up to a positive affine transformation. Thus, as a representation of the preference relation satisfying Axioms 1-5, the Gini coefficient is unique up to a positive affine transformation.

Q.E.D.

Theorem 3 provides a complete axiomatization for the Gini coefficient. Thus, application of the Gini coefficient means that both independence axioms are supported jointly. Hence, the preferences of social decision-makers whose ethical norms coincide with the Gini coefficient are invariant with respect to certain types of tax/transfer interventions and with respect to aggregation of subpopulations across income shares.

In order to arrive at ranking criteria that are consistent with weaker conditions than those of Theorem 3 and stronger conditions than those of Theorems 1 and 2 we introduce suitable criteria for Lorenz dominance in the next section. These criteria allow Lorenz curves to be ranked given only limited knowledge of the decision-maker's preferences, and demonstrate that appropriate selection of preference functions for J_P and J_Q^* enable us to vary the emphasis placed on different parts of the income distribution.

3. Lorenz dominance

In expected utility theory it is standard to impose restrictions on the utility function applying various types of stochastic dominance rules. For example, "risk aversion" is equivalent to second-degree stochastic dominance and imposes strict concavity on the utility function. Drawing on results in the theory of choice under uncertainty Atkinson [1] defined inequality aversion as being equivalent to risk aversion. This was motivated by the fact that the Pigou-Dalton transfer principle is identical to the principle of mean preserving spread introduced by Rothschild and Stiglitz [12]. The application of these two principles requires, however, that the distributions in question have equal means. To extend the principle of transfers to deal with distributions with variable means we consider income shares rather than income amounts. In this context it is useful to introduce the principles of relative transfers which, when restricted to distributions with equal means, coincides with the principle of transfers.

DEFINITION 1. *A transfer of an income share which reduces the difference between the income shares of the donor and the recipient always reduces Lorenz inequality.*

It follows readily that the principle of relative transfers is equivalent to the condition of first-degree Lorenz dominance:

DEFINITION 2. *A Lorenz curve L_1 first-degree dominates a Lorenz curve L_2 if and only if*

$$L_1(u) \geq L_2(u) \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for at least one $u \in \langle 0,1 \rangle$.

A social decision-maker who favours first-degree Lorenz dominance, i.e. supports the principle of relative transfers is said to be Lorenz inequality averse. If a social decision-maker acts so as to maximize the welfare function V_P defined by (6) or V_Q^* defined by (10) (or alternatively minimize the loss functions J_P or J_Q^*) an interesting question is which restrictions Lorenz inequality aversion places on the corresponding preference functions? The answer follows from Theorem 4, which gives two alternative characterizations of first-degree Lorenz dominance.

Let $\mathcal{P}_1$ and $\mathfrak{C}_1$ be classes of preference functions related to J_P and J_Q^*, respectively, and defined by

$$\mathcal{P}_1 = \{P: P' \text{ and } P'' \text{ are continuous on } [0,1],\ P'(t)>0 \text{ and } P''(t)<0 \text{ for } t \in \langle 0,1 \rangle, \text{ and } P'(1)=0\}$$

and

$$\mathfrak{C}_1 = \{Q : Q' \text{ and } Q'' \text{ are continuous on } [0,1],\ Q'(t)>0 \text{ and } Q''(t)>0 \text{ for } t\in\langle 0,1\rangle, \text{ and } Q'(0)=0\}.$$

THEOREM 4. *Let L_1 and L_2 be members of $\mathfrak{L}$. Then the following statements are equivalent,*

(i) $J_P(L_1) < J_P(L_2)$ *for all* $P\in \mathcal{P}_1$

(ii) $J_Q^*(L_1) < J_Q^*(L_2)$ *for all* $Q\in \mathfrak{C}_1$

(iii) $L_1(u) \geq L_2(u)$ *for all* $u\in [0,1]$

and the inequality holds strictly for at least one $u\in\langle 0,1\rangle$.

(Proof in Appendix).

Note that Atkinson [1] did not obtain a characterization of first-degree Lorenz dominance except when the judgement about inequality is restricted to distributions with equal means. However, it follows from Atkinson op cit. that his proposals on measures of inequality based on expected utility preserve first-degree Lorenz dominance for strict concave utility functions. By contrast, theorem 4 demonstrates that Lorenz inequality aversion is characterized by the strict concavity of P-functions and strict convexity of Q-functions. Based on these results one might expect that a "more concave" P-function or a "more convex" Q-function would exhibit more Lorenz inequality aversion. To deal with these questions in a more precise way we introduce the following definitions, where L_0 denotes the equality reference of the Lorenz curve ($L_0(u)=u$).

DEFINITION 3a. *Let P_1 and P_2 be members of the class $\mathcal{P}_1$ of social preference functions. Then P_1 is said to be more Lorenz inequality averse than P_2 if and only if $J_{P_1}(L) > J_{P_2}(L)$ for all $L\in \mathfrak{L}-\{L_0\}$.*

DEFINITION 3b. *Let Q_1 and Q_2 be members of the class $\mathfrak{C}_1$ of social preference functions. Then Q_1 is said to be more Lorenz inequality averse than Q_2 if and only if $J^*_{Q_1}(L) > J^*_{Q_2}(L)$ for all $L \in \mathfrak{L}-\{L_0\}$.*

THEOREM 5a. *Let L be a Lorenz curve and let P_1 and P_2 be members of $\mathfrak{P}_1$. Then*

$$J_{P_1}(L) > J_{P_2}(L) \text{ for all } L \in \mathfrak{L}-\{L_0\}$$

if and only if

$$P_1(t) > P_2(t) \text{ for all } t \in \langle 0,1 \rangle.$$

(Proof in Appendix)

THEOREM 5b. *Let L be a Lorenz curve and let Q_1 and Q_2 be members of $\mathfrak{C}_1$. Then*

$$J^*_{Q_1}(L) > J^*_{Q_2}(L) \text{ for all } L \in \mathfrak{L}-\{L_0\}$$

if and only if

$$Q_1(t) < Q_2(t) \text{ for all } t \in \langle 0,1 \rangle.$$

(Proof in Appendix)

Theorem 5a demonstrates that J_{P_1} displays more inequality aversion than J_{P_2} if P_1 lies above P_2, and P_1 and P_2 are concave. Therefore, the cost of inequality in terms of loss of Lorenz welfare is higher when measured by J_{P_1} than by J_{P_2}. Similarly, Theorem 5b shows that $J^*_{Q_1}$ exhibits more inequality aversion than $J^*_{Q_2}$ if Q_1 is lying beneath Q_2 and Q_1 and Q_2 are convex.

Recalling Theorem 4 first-degree Lorenz dominance allows us to partially rank Lorenz curves without knowledge of the precise form of the social preference functions. These results apply to situations where Lorenz curves do not intersect. In order to deal with situations where Lorenz curves intersect, it is necessary to impose further restrictions beyond Lorenz inequality aversion on the preferences. We introduce the following Lorenz dominance criteria:

DEFINITION 4. *A Lorenz curve L_1 second-degree upward dominates a Lorenz curve L_2 if and only if*

$$\int_0^u L_1(t)dt \geq \int_0^u L_2(t)dt \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for at least one u.

DEFINITION 5. *A Lorenz curve L_1 second-degree downward dominates a Lorenz curve L_2 if and only if*

$$\int_u^1 L_1(t)dt \geq \int_u^1 L_2(t)dt \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for at least one u.

Social decision-makers who favour the principle of second-degree upward Lorenz dominance will assign more weight to changes that take place in the lower part of the Lorenz curve than to changes that occur in the upper part of the Lorenz curve. By contrast, the principle of second-degree downward Lorenz dominance emphasizes changes that occur in the upper part of the Lorenz curve.

Now, let $\mathcal{P}_2$ and $\mathcal{C}_2$ be families of preference functions related to J_P and J_Q^*, respectively, and defined by

$$\mathcal{P}_2 = \left\{P : P \in \mathcal{P}_1,\ P''' \text{ is continuous on } [0,1] \text{ and } P'''(t)>0 \text{ for } t \in \langle 0,1 \rangle \right\}$$

and

$$\mathfrak{C}_2 = \left\{Q : Q \in \mathfrak{C}_1,\ Q''' \text{ is continuous on } [0,1] \text{ and } Q'''<0 \text{ for } t \in \langle 0,1 \rangle\right\}.$$

The following result provides two alternative characterizations of second-degree upward Lorenz dominance:

THEOREM 6. *Let L_1 and L_2 be members of $\mathfrak{L}$. Then the following statements are equivalent,*

$$J_P(L_1) < J_P(L_2) \text{ for all } P \in \mathfrak{P}_2, \tag{i}$$

$$J_Q^*(L_1) < J_Q^*(L_2) \text{ for all } Q \in \mathfrak{C}_2, \tag{ii}$$

$$\int_0^u L_1(t)dt \geq \int_0^u L_2(t)dt \text{ for all } u \in [0,1] \tag{iii}$$

and the inequality holds strictly for at least one u.

(Proof in Appendix).

Theorem 6 shows that second-degree upward Lorenz dominance imposes the restrictions of positive third derivatives on the P-functions related to J_P and negative third derivatives on the Q-functions related to J_Q^*. By contrast, preferences which display negative third derivatives on the P-functions or positive third derivatives on the Q-functions, implies that the decision-maker favours second-degree downward Lorenz dominance as demonstrated by Theorem 7.

Let $\tilde{\mathfrak{P}}_2$ and $\tilde{\mathfrak{C}}_2$ be families of preference functions related to J_P and J_Q^*, respectively, and defined by

$$\tilde{\mathfrak{P}}_2 = \left\{P : P \in \mathfrak{P}_1,\ P''' \text{ is continuous on } [0,1] \text{ and } P'''(t)<0 \text{ for } t \in \langle 0,1 \rangle\right\}$$

and

$$\tilde{\mathfrak{C}}_2 = \left\{Q : Q \in \mathfrak{C}_1,\ Q''' \text{ is continuous on } [0,1] \text{ and } Q'''(t)>0 \text{ for } t \in \langle 0,1 \rangle\right\}.$$

THEOREM 7. *Let L_1 and L_2 be members of $\mathcal{L}$. Then the following statements are equivalent,*

$$J_P(L_1) < J_P(L_2) \ \text{for all} \ P \in \tilde{\mathcal{P}}_2 \qquad (i)$$

$$J_Q^*(L_1) < J_Q^*(L_2) \ \text{for all} \ Q \in \tilde{\mathcal{C}}_2 \qquad (ii)$$

$$\int_u^1 L_1(t)dt \geq \int_u^1 L_2(t)dt \ \text{for all} \ u \in [0,1] \qquad (iii)$$

and the inequality holds strictly for at least one u.

(Proof in Appendix)

Theorem 6 and 7 demonstrate that the principles of upward and downward Lorenz dominance divide J_P-measures and J_Q^*-measures into broad categories of inequality measures that differ in the measures' sensivity to changes in the lower or upper part of the Lorenz curve. Members of the families $\{J_P : P \in \mathcal{P}_2\}$ and $\{J_Q^* : Q \in \mathcal{C}_2\}$ give more weight to changes lower down in the Lorenz curve, while the members of the families $\{J_P : P \in \tilde{\mathcal{P}}_2\}$ and $\{J_Q^* : Q \in \tilde{\mathcal{C}}_2\}$ give more weight to changes higher up in the Lorenz curve. Note that $P(t) = 2t-t^2$, the P-function that corresponds to the Gini coefficient, is the only member of $\mathcal{P}_1$ that is included neither in $\mathcal{P}_2$ nor in $\tilde{\mathcal{P}}_2$. The corresponding Q-function is given by $Q(t)=t^2$, the Q-function of the Gini coefficient. Thus, the Gini coefficient is the only member of the families $\{J_P : P \in \mathcal{P}_1\}$ and $\{J_Q^* : Q \in \mathcal{C}_1\}$ that preserves neither second-degree upward Lorenz dominance nor second-degree downward Lorenz dominance. As a preference ordering on $\mathcal{L}$, the Gini coefficient in general, favours neither the lower nor the upper part of the Lorenz curves. Therefore, if we restrict the ranking problem to Lorenz curves with equal Gini coefficients, second-degree upward and downward dominance coincide, in the sense that a Lorenz curve L_1 that second-degree upward dominates a Lorenz curve L_2 is always second-degree downward dominated by L_2. It is clear that L_1 can be attained from L_2 by transferring income shares from the center of L_2 to the tails while keeping the Gini coefficient unchanged. Thus, it is natural to say that the difference represents a Gini-preserving spread, and that decision-makers who focus on inequality among the poor will prefer L_1 to L_2, while decision-makers who focus on inequality among the rich will prefer L_2 to L_1. Note that the principle of Gini-preserving spread is an analogous to the principle of mean preserving spread introduced by Rothschild and Stiglitz [12] as a characterization of comparative risk in the expected utility theory of choice under risk.

The following result, which is a direct implication of Theorems 6 and 7, demonstrates that J_P and J_Q^* satisfy the principle of Gini-preserving spread if and only if the corresponding inequality averse preference functions have positive ($P'''(t)>0$) and negative ($Q'''(t)<0$) derivatives.

COROLLARY 1. *Let L_1 and L_2 be Lorenz curves with equal Gini coefficients. Then the following statements are equivalent.*

$$J_P(L_1) < J_P(L_2) \ \text{for all} \ P \in \mathcal{P}_2, \tag{i}$$

$$J_P(L_1) > J_P(L_2) \ \text{for all} \ P \in \tilde{\mathcal{P}}_2, \tag{ii}$$

$$J_Q^*(L_1) < J_Q^*(L_2) \ \text{for all} \ Q \in \mathcal{Q}_2, \tag{iii}$$

$$J_Q^*(L_1) > J_Q^*(L_2) \ \text{for all} \ Q \in \tilde{\mathcal{Q}}_2, \tag{iv}$$

$$\int_0^u L_1(t)\,dt \geq \int_0^u L_2(t)\,dt \ \text{for all} \ u \in [0,1], \tag{v}$$

$$\int_u^1 L_2(t)\,dt \geq \int_u^1 L_1(t)\,dt \ \text{for all} \ u \in [0,1] \tag{vi}$$

where the inequalities in (v) and (vi) hold strictly for at least one u.

The above dominance principles can be extended by increasing the emphasis on transfers occuring lower down in the Lorenz curve or higher up in the Lorenz curve. To do so we introduce further degrees of upward and downward Lorenz dominance. It is convenient to use the following notation,

$$G_2(u) = \int_0^u L(t)dt, \quad 0 \leq u \leq 1 ,$$
$$G_{i+1}(u) = \int_0^u G_i(t)dt, \quad 0 \leq u \leq 1, \quad i=2,3,\ldots , \tag{17}$$

and

$$\tilde{G}_2(u) = \int_u^1 L(t)dt, \quad 0 \leq u \leq 1$$
$$\tilde{G}_{i+1}(u) = \int_u^1 \tilde{G}_i(t)dt, \quad 0 \leq u \leq 1, \quad i=2,3,\ldots . \tag{18}$$

Now, using integration by parts, we obtain the following alternative expressions for G_{i+1} and $\tilde{G}_{i+1}$, respectively,

$$G_{i+1}(u) = \frac{1}{(i-1)!}\int_0^u (u-t)^{i-1}L(t)\,dt \tag{19}$$

and

$$\tilde{G}_{i+1}(u) = \frac{1}{(i-1)!}\int_u^1 (t-u)^{i-1}L(t)\,dt. \tag{20}$$

Notice from expressions (19) and (20) that as i increases G_{i+1} and $\tilde{G}_{i+1}$ increase the focus on changes in the lower and upper part of the Lorenz curve, respectively.

Now, let $P^{(j)}$ denote the j^{th} derivative of P and let $\mathcal{P}_i$ be families of preference functions defined by

$$\mathcal{P}_i = \{P : P \in \mathcal{P}_1,\ P^{(i)} \text{ is continuous on } [0,1],\ (-1)^{j-1}P^{(j)}(t) > 0 \text{ for } t \in \langle 0,1 \rangle,\ j = 3,4,\ldots,i+1 \text{ and } P^{(j)}(1) = 0,\ j = 2,3,\ldots,i-1\}$$

and

$$\tilde{\mathcal{P}}_i = \{P : P \in \mathcal{P}_1,\ P^{(i)} \text{ is continuous on } [0,1],\ P^{(j)}(t) < 0 \text{ for } t \in \langle 0,1 \rangle,\ j = 3,4,\ldots,i+1 \text{ and } P^{(j)}(1) = 0,\ j = 2,3,\ldots,i-1\},$$

respectively.

As generalizations of Definitions 4 and 5 we introduce the concepts of i^{th}-degree upward and downward Lorenz dominance. Note that subscripts i and j in the notation $G_{i,j}$ and $\tilde{G}_{i,j}$ used below refer to dominance of i^{th} degree for Lorenz curve L_j.

DEFINITION 6. *A Lorenz curve L_1 i^{th}-degree upward dominates a Lorenz curve L_2 if and only if*

$$G_{i,1}(u) \geq G_{i,2}(u) \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for at least one u.

DEFINITION 7. *A Lorenz curve L_1 i^{th}-degree downward dominates a Lorenz curve L_2 if and only if*

$$\tilde{G}_{i,1}(u) \geq \tilde{G}_{i,2}(u) \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for at least one u.

It follows from the definitions (17) and (18) of G and $\tilde{G}$, respectively, that

$$G_{i,1}(u) \geq G_{i,2}(u) \quad \text{for all } u$$

implies

$$G_{i+1,1}(u) \geq G_{i+1,2}(u) \quad \text{for all } u,$$

and that

$$\tilde{G}_{i,1}(u) \geq \tilde{G}_{i,2}(u) \quad \text{for all } u$$

implies

$$\tilde{G}_{i+1,1}(u) \geq \tilde{G}_{i+1,2}(u) \quad \text{for all } u.$$

Thus, the various degrees of upward and downward Lorenz dominance constitute two separate systems of hierarchical dominance criteria, which turn out to be useful for imposing restrictions on preference functions.

The restrictions, imposed by i^{th}-degree upward and downward Lorenz dominance on the decision-makers' preferences, are characterized by the following theorems.

THEOREM 8. *Let L_1 and L_2 be members of $\mathcal{L}$. Then*

$$J_P(L_1) < J_P(L_2) \quad \text{for all } P \in \mathcal{P}_i$$

if and only if

$$G_{i,1}(u) \geq G_{i,2}(u) \quad \text{for all } u \in [0,1]$$

and the inequality holds strictly for at least one u.

(Proof in Appendix).

THEOREM 9. *Let L_1 and L_2 be members of $\mathcal{L}$. Then*

$$J_P(L_1) < J_P(L_2) \quad \text{for all } P \in \tilde{\mathcal{P}}_i$$

if and only if

$$\tilde{G}_{i,1}(u) \geq \tilde{G}_{i,2}(u) \quad \text{for all } u \in [0,1]$$

and the inequality holds strictly for at least one u.

(Proof in Appendix).

These dominance rules increase the number of Lorenz curves which can be strictly ordered by successively narrowing the class of preference functions under consideration. It follows from Theorem 8 that J_P-measures derived from P-functions with derivatives that alternate in sign ($(-1)^{j-1}P^{(j)}(t)>0$, $j=2,3,...$) preserve all degrees of upward Lorenz dominance and, thus, are particularly sensitive to changes that occur in the lower part of the Lorenz curve. By contrast, Theorem 9 shows that J_P-measures derived from P-functions with negative derivatives ($P^{(j)}(t)<0$, $j=2,3,...$) preserve all degrees of downward Lorenz dominance and, thus, are particularly sensitive to changes that occur in the upper part of the Lorenz curve.

Note that, as partial orderings, both upward and downward Lorenz dominance of degree higher than two satisfy Axiom 4 but not Axiom 5. This important property explains why upward and downward Lorenz dominance appear to be useful criteria in judging among J_P-measures rather than among J_Q^*-measures. It suggests that similar dominance rules for the inverse Lorenz curves should be elaborated as basis for evaluating J_Q^*-measures' sensitivity to changes that affect the poor and the rich. Due to a dominance result of Hardy et al. [6] it follows that second-degree Lorenz dominance and inverse Lorenz dominance coincide. However, in contrast to second-degree Lorenz dominance, higher degrees of Lorenz dominance do not in general accord with the corresponding higher degrees of inverse Lorenz dominance. Thus, in order to impose further constraints on the inequality measures J_Q^* it is useful to assume higher degrees of upward and downward inverse Lorenz dominance. To this end it will be convenient to use the following notation,

$$K_2(u) = \int_0^u L^{-1}(t)dt, \quad 0 \le u \le 1$$
$$K_{i+1}(u) = \int_0^u K_i^{-1}(t)dt, \quad 0 \le u \le 1, \quad i=2,3,... \tag{21}$$

and

$$\tilde{K}_2(u) = \int_u^1 L^{-1}(t)dt, \quad 0 \le u \le 1$$
$$\tilde{K}_{i+1}(u) = \int_u^1 \tilde{K}_i(t)dt, \quad 0 \le u \le 1, \quad i=2,3,... \tag{22}$$

Using integration by parts we arrive at the following alternative expressions for K_{i+1} and $\tilde{K}_{i+1}$, respectively,

$$K_{i+1}(u) = \frac{1}{(i-1)!}\int_0^u (u-t)^{i-1} \cdot L^{-1}(t)dt \tag{23}$$

and

$$\tilde{K}_{i+1}(u) = \frac{1}{(i-1)!}\int_u^1 (t-u)^{i-1} \cdot L^{-1}(t)dt. \tag{24}$$

The expressions (23) and (24) demonstrate that K_{i+1} and $\tilde{K}_{i+1}$ increase their focus on changes that concern the poorest and the richest as i increases.

Moreover, let $Q^{(j)}$ denote the y^{th} derivative of Q and let $\mathfrak{C}_i$ and $\tilde{\mathfrak{C}}_i$ be families of preference functions defined by

$$\mathfrak{C}_i = \{Q : Q \in \mathfrak{C}_1,\ Q^{(i)} \text{ is continuous on } [0,1],$$
$$(-1)^j Q^{(j)}(t) > 0 \text{ for } t \in \langle 0,1 \rangle,\ j=3,4,...,i+1$$
$$\text{and } Q^{(j)}(1)=0,\ j=2,3,...,i-1\}$$

and

$$\tilde{\mathfrak{C}}_i = \{Q : Q \in \mathfrak{C}_1,\ Q^{(i)} \text{ is continuous on } [0,1],$$
$$Q^{(j)}(t) > 0 \text{ for } t \in \langle 0,1 \rangle,\ j=3,4,...,i+1$$
$$\text{and } Q^{(j)}(0)=0,\ j=2,3,...,i-1\},$$

respectively.

Now, replacing the Lorenz curves in the definitions of i^{th}-degree Lorenz dominance by their inverses we obtain the definitions of i^{th}-degree upward and downward inverse Lorenz dominance, where the subscripts i and j in the notation $K_{i,j}$ and $\tilde{K}_{i,j}$ refer to dominance of i^{th} degree for Lorenz curve L_j.

DEFINITION 8. *A Lorenz curve L_1 i^{th}-degree upward inverse Lorenz dominates a Lorenz curve L_2 if and only if*

$$K_{i,1}(u) \leq K_{i,2}(u) \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for at least one u.

DEFINITION 9. *A Lorenz curve L_1 i^{th}-degree downward inverse Lorenz dominates a Lorenz curve L_2 if and only if*

$$\tilde{K}_{i,1}(u) \leq \tilde{K}_{i,2}(u) \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for at least one u.

As with upward and downward Lorenz dominance, the various degrees of upward and downward inverse Lorenz dominance also provide two separate systems of hierarchical dominance criteria. The following two theorems identify the restrictions on the preference function of J_Q^* which are consistent with i^{th}-degree upward and downward inverse Lorenz dominance.

THEOREM 10. *Let L_1 and L_2 be members of $\mathcal{L}$. Then*

$$J_Q^*(L_1) < J_Q^*(L_2) \quad \text{for all} \quad Q \in \mathfrak{C}_i$$

if and only if

$$K_{i,1}(u) \leq K_{i,2}(u) \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for at least one u.

(Proof in Appendix).

THEOREM 11. *Let L_1 and L_2 be members of L. Then*

$$J_Q^*(L_1) < J_Q^*(L_2) \text{ for all } Q \in \tilde{\mathfrak{C}}_i$$

if and only if

$$\tilde{K}_{i,1}(u) \leq \tilde{K}_{i,2}(u) \text{ for all } u \in [0,1]$$

and the inequality holds strictly for at least one u.

(Proof in Appendix)

Theorem 10 implies that J_Q^*-measures derived from Q-functions with derivatives that alternate in sign $((-1)^j Q^{(j)}(t)>0,\ j=2,3,...)$ preserve all degrees of upward inverse Lorenz dominance. Alternatively, by restricting to Q-functions with positive derivatives, Theorem 11 demonstrates that the corresponding J_Q^*-measures preserve all degrees of downward inverse Lorenz dominance.

4. Comparison of Measures of Inequality

The dominance results do not in general provide a simple relationship between higher degrees of dominance and higher degrees of inequality aversion. Note, however, that the most inequality averse J_P-measure is obtained as the preference function approaches

$$P_a(t) = \begin{cases} 0, & t=0 \\ 1, & 0<t\leq 1. \end{cases} \tag{25}$$

As P_a is not differentiable, it is not a member of the family $\mathcal{P}_1$ of inequality averse preference functions, but it is recognizable as the upper limit of inequality aversion for members of $\mathcal{P}_1$.

Inserting (25) in (7) yields

$$J_{P_*}(L) = 1 - \frac{F^{-1}(0+)}{\mu}, \tag{26}$$

where μ is the mean income and $F^{-1}(0+)$ is the lowest income. Hence, the inequality measure J_{P_*} corresponds to the Rawlsian leximin criterion; we denote it the Rawlsian relative leximin criterion. Note that in contrast to the Rawlsian (absolute) leximin criterion the Rawlsian relative leximin criterion accords with egalitarianism. Moreover, note that the Rawlsian relative leximin criterion preserves all degrees of upward Lorenz dominance and thus rejects downward Lorenz dominance.

By examining the inequality aversion properties of J_Q^*-measures we find that the upper limit of inequality aversion is attained as the preference function approaches

$$Q_*(t) = \begin{cases} 0, & 0 \leq t < 1 \\ 1, & t = 1 \end{cases}. \tag{27}$$

Inserting (27) in (11) yields

$$J_{Q_*}^*(L) = 1 - \frac{\mu}{F^{-1}(1-)}, \tag{28}$$

where $F^{-1}(1-)$ is the largest income. Thus, J_Q^* is the upper limit of inequality aversion for members of Q_1. Hence, $\mu/F^{-1}(1-)$ is "dual" to the Rawlsian relative leximin criterion in the sense that it preserves all degrees of downward Lorenz dominance while the Rawlsian relative leximin criterion preserves all degrees of upward Lorenz dominance. In contrast to the Rawlsian relative leximin criterion the $J_{Q_*}^*$-criterion focuses on the relative income of the most well-off unit. If decisions are based on the $J_{Q_*}^*$-criterion, the income distributions for which the largest relative income is smaller is preferred, regardless of all other differences. The only transfers which decrease inequality are transfers from the richest unit to anyone else.

The two theories for measuring inequality differ notably with respect to their descriptions of the most inequality averse behavior. For decision-makers who base their decisions on the J_P-measures, the most inequality averse behavior is attained by raising the emphasis on transfers occuring lower down in the Lorenz curve. By contrast, if inequality is assessed in terms of J_Q^*-measures, the most inequality averse behavior is attained by raising the emphasis on transfers occuring higher up in the Lorenz curve. Note that this difference in inequality aversion originates from the difference between Axioms 4 and 5.

Based on the results of Theorems 4-11, we demonstrate how Lorenz dominance results can be applied to evaluate the ranking properties of members of the families $\{D_k\}$ and $\{D_k^*\}$ defined by (9) and (13). Differentiating P_k and Q_k, defined by (8) and (12), we find that

$$P_k^{(j)}(t) = \begin{cases} (-1)^{j-1}\dfrac{(k+1)!}{(k-j+1)!}(1-t)^{k-j+1}, & j=1,2,\ldots,k+1 \\ 0 & , j=k+2,k+3,\ldots \end{cases} \tag{29}$$

and

$$Q_k^{(j)}(t) = \begin{cases} \dfrac{(k+1)!}{(k-j+1)!}t^{k-j+1}, & j=1,2,\ldots,k+1 \\ 0 & , j=k+2,k+3,\ldots \end{cases} \tag{30}$$

Equation (29) implies that $P_k''(t)<0$ for all $t\in\langle 0,1\rangle$ when $k>0$ and thus that $\{D_k\}$ satisfies the principle of relative transfers for $k>0$. Moreover, $P_k'''(t)>0$ for all $t\in\langle 0,1\rangle$ when $k>1$. Hence $\{D_k\}$ satisfies second-degree upward Lorenz dominance for $k>1$. Moreover, the derivatives of P_k alternate in sign up to the $(k+1)^{th}$ derivative and $P_k^{(j)}(1)=0$ for all $j\leq k$. It follows from Theorem 8 that the preference function P_k satisfies upwards Lorenz dominance of degree k and therefore also satisfies upward Lorenz dominance for all degrees lower than k. Finally, by noting that $P_k(t)<P_{k+1}(t)$, $0<t<1$ for $k\geq 0$, it follows from Theorem 5a that D_{k+1} exhibits more inequality aversion than D_k for $k>0$. Therefore, if we restrict the family of preference functions to those defined by (8), a decision-maker who supports an increase in the degree of upward Lorenz dominance increases his degree of inequality aversion. Hence, the cost of inequality is higher when measured by D_{k+1} than by D_k. The most inequality averse behavior occurs as $k \to \infty$, which corresponds to the inequality averse behavior of the Rawlsian relative leximin criterion. At the other extreme, as k=0, the social preference function P_0 exhibits inequality neutrality. By contrast, D_k as $k \to \infty$ satisfies all degrees of upward Lorenz dominance. The stated properties of the D_k-measures are summarized in the following proposition,

PROPOSITION 1. *The family $\{D_k : k>0\}$ of inequality measures defined by (9) has the following properties,*

D_k satisfies the principle of relative transfers for k>0, *(i)*

D_k satisfies the principle of Gini preserving spread for k>1, *(ii)*

D_k preserves upward Lorenz dominance of degree k and all degrees lower than k, *(iii)*

D_{k+1} exhibits more inequality aversion than D_k, *(iv)*

D_k approaches inequality neutrality as $k \to 0$, *(v)*

D_k approaches the Rawlsian relative leximin criterion as $k \to \infty$. *(vi)*

The family $\{D_k^*\}$ of inequality measures turns out to fulfill the principles of relative transfers and second-degree downward Lorenz dominance for k>0 and k>1, respectively. The expression (30) for the derivatives of Q_k demonstrates that Q_k fulfills the conditions of Theorem 11 and hence satisfies down-wards inverse Lorenz dominance of degree k and all degrees lower than k. Furthermore, since $Q_{k+1}(t)<Q_k(t)$, $0<t<1$ for $k \geq 0$, it follows from Theorem 5b that D_{k+1}^* exhibits more inequality aversion than D_k^* for k>0 which means that the cost of inequality is higher when measured by D_{k+1}^* than by D_k^*. Therefore, when we consider the family of Q-functions defined by (12) a decision-maker who favors an increase in the degree of downward inverse Lorenz dominance reveals an increase in the degree of inequality aversion. Note that k=0 represents inequality neutrality and that $D_k^* \to J_{Q_*}^*$ as $k \to \infty$. Thus, in the context of J_Q^*-measures $J_{Q_*}^*$ is recognized as the upper limit in terms of inequality aversion. As $k \to \infty$, D_k^* satisfies all degrees of downward inverse Lorenz dominance. Moreover, since

D_1 and D_1^* coincide with the Gini coefficient, D_k and D_k^* for 0<k<1 exhibit less inequality aversion than the Gini coefficient even if D_k^* for 0<k<1 satisfies the principle of Gini preserving spread. Note that the Gini coefficient gives the same weight to a transfer of an income share between units with a given difference in income shares irrespective if these income shares are lower or if they are higher. The essential properties of the D_k^*-measures are summarized in the next proposition,

PROPOSITION 2. *The family* $\{D_k^* : k>0\}$ *of inequality measures defined by (13) has the following properties,*

D_k^* satisfies the principle of relative transfers for k>0, *(i)*

D_k^* satisfies the principle of Gini preserving spread for 0<k<1, *(ii)*

D_k^* preserves downward inverse Lorenz dominance of degree k and all degrees lower than k, *(iii)*

D_{k+1}^* exhibits more inequality aversion than D_k^*, *(iv)*

D_k^* approaches inequality neutrality as $k \to 0$, *(v)*

D_k approaches $J_{Q_*}^*$ defined by (28) as $k \to \infty$. *(vi)*

APPENDIX

Proofs of Dominance Results

LEMMA 1. *Let H be the family of bounded, continuous and non-negative functions on [0,1] which are positive on ⟨0,1⟩ and let g be an arbitrary bounded and continuous function on [0,1]. Then*

$$\int g(t)h(t)\,dt > 0 \quad \text{for all } h \in H$$

implies

$$g(t) \geq 0 \quad \text{for all } t \in [0,1]$$

and the inequality holds strictly for at least one t ∈ ⟨0,1⟩.

Proof. The proof is by contradiction. Assume that $A \equiv \{t: g(t)<0\} \neq \emptyset$.
Let $g(t) = g_A(t) + g_{\bar{A}}(t)$,

where

$$g_B(t) = \begin{cases} g(t) & \text{if } t \in B \\ 0 & \text{otherwise} \end{cases}$$

and let $M = \min_t |g_A(t)|$ and $N = \max_t g_{\bar{A}}(t)$. Then

$$\int_0^1 g(t)h(t)\,dt = \int g_{\bar{A}}(t)h(t)\,dt + \int g_A(t)h(t)\,dt < N\int_A h(t)\,dt - M\int_A h(t)\,dt.$$

Thus, by choosing h such that

$$N\int_A h(t) < M\int_A h(t)\,dt$$

we get

$$\int_0^1 g(t)h(t)\,dt < 0,$$

which is a contradiction. Q.E.D.

Proof of Theorem 4. From the definition (7) of $J_P(L)$ it follows that

$$J_P(L_2) - J_P(L_1) = -\int_0^1 P''(u)(L_1(u) - L_2(u))\,du.$$

Thus, if (iii) holds then $J_P(L_2) - J_P(L_1) > 0$ for all $P \in \mathcal{P}_1$.

Conversely, by assuming that (i) is true, application of Lemma 1 gives (iii). Hence, the equivalence of (i) and (iii) is proved.

To prove the equivalence of (ii) and (iii) we use that $L_1^{-1}(u) \le L_2^{-1}(u)$ for all $u \in [0,1]$ if and only if $L_1(u) \ge L_2(u)$ for all $u \in [0,1]$. Next, from the definition (11) of J_Q^* and using integration by parts it follows that

$$J_Q^*(L_2) - J_Q^*(L_1) = \int_0^1 Q''(t)(L_2^{-1}(t) - L_1^{-1}(t))\,dt.$$

Hence, if (iii) holds then $J_Q^*(L_2) - J_Q^*(L_1) > 0$ for all $Q \in \mathfrak{C}_1$.

The converse statement follows by straightforward application of Lemma 1. Q.E.D.

Proof of Theorem 5a. Assume that $P_1(t) > P_2(t)$ for all $t \in \langle 0,1 \rangle$. Using integration by parts in the expression (5) for J_P we have

$$J_{P_1}(L) - J_{P_2}(L) = \int (P_1(t) - P_2(t))\,dL'(t) > 0$$

for all $L \in \mathfrak{L}$-L_0.

Conversely, assume that $J_{P_1}(L) > J_{P_2}(L)$ for all $L \in \mathfrak{L}-\{L_0\}$. Then, by choosing

$$L(t) = \begin{cases} 0, & 0 \le t \le u \\ \dfrac{t-u}{1-u}, & u < t \le 1, \end{cases} \qquad u \in \langle 0,1 \rangle,$$

we get

$$0 < J_{P_1}(L) - J_{P_2}(L) = \frac{1}{1-u}\left(P_1(u) - P_2(u)\right) \text{ for all } u \in \langle 0,1 \rangle. \qquad \text{Q.E.D.}$$

Proof of Theorem 5b. Assume that $Q_1(t) < Q_2(t)$ for all $t\in\langle 0,1\rangle$. Then we have, using integration by parts in the expression (9) for J_Q^*, that

$$J_{Q_1}^*(L) - J_{Q_2}^*(L) = \int\big(Q_1(t)-Q_2(t)\big)d\Big(L'\big(L^{-1}(t)\big)\Big)^{-1} > 0,$$

for all $L\in\mathcal{L}-L_0$.

Conversely, assuming $J_{Q_1}^*(L)>J_{Q_2}^*(L)$ for all $L\in\mathcal{L}-\{L_0\}$ and choosing

$$L(t) = \begin{cases} \frac{1}{u}t, & 0\le t\le u \\ & \qquad 0<u<1, \\ 1, & u<t<1 \end{cases}$$

we get

$$0 < J_{Q_1}^*(L) - J_{Q_2}^*(L) = \frac{1}{u}\big(Q_2(u)-Q_1(u)\big) \text{ for all } u\in\langle 0,1\rangle. \qquad \text{Q.E.D.}$$

Proof of Theorem 6. Using integration by parts we have that

$$J_P(L_2) - J_P(L_1) = -P''(1)\int_0^1(L_1(u)-L_2(u))du + \int_0^1 P'''(u)\int_0^u(L_1(t)-L_2(t))dt\,du.$$

Thus, if (iii) holds then $J_P(L_2) > J_P(L_1)$ for all $P\in\mathcal{P}_2$.

To prove the converse statement we restrict to preference functions $P\in\mathcal{P}_2$ for which $P''(1) = 0$. Hence,

$$J_P(L_2) - J_P(L_1) = \int_0^1 P'''(u)\int_0^u(L_1(t)-L_2(t))dt\,du$$

and the desired result it obtained by applying Lemma 1.

Now, turning to statement (ii) and using integration by parts we get

$$J_Q^*(L_2) - J_Q^*(L_1) = Q''(1)\int_0^1\big(L_2^{-1}(t)-L_1^{-1}(t)\big)dt - \int_0^1 Q'''(t)\int_0^u\big(L_2^{-1}(t)-L_1^{-1}(t)\big)dt\,du.$$

Then, since (iii) is equivalent to the statement $\int_0^u L_2^{-1}(t)dt \geq \int_0^u L_1^{-1}(t)dt$ for all $u \in [0,1]$ and holds for at least one u, the proof of the equivalence between (ii) and (iii) follows readily from the proof of the equivalence between (i) and (iii). Q.E.D.

The proof of Theorem 7 is analogous to the proof of Theorem 6 and is based on the expressions

$$J_P(L_2) - J_P(L_1) = -P''(0)\int_u^1 \big(L_1(t) - L_2(t)\big)dt - \int_0^1 P'''(u)\int_u^1 \big(L_1(t) - L_2(t)\big)dt\,du$$

and

$$J_Q^*(L_2) - J_Q^*(L_1) = Q''(0)\int_0^1 \big(L_2^{-1}(t) - L_1^{-1}(t)\big)dt + \int_0^1 Q'''(u)\int_u^1 \big(L_2^{-1}(t) - L_1^{-1}(t)\big)dt\,du,$$

which are obtained by using integration by parts. Thus, by arguments like those in the proof of Theorem 6 the results of Theorem 7 are obtained.

Proof of Theorem 8. To examine the case of i^{th} degree upward Lorenz dominance we integrate $J_P(L_2)$ - $J_P(L_1)$ by parts i times,

$$J_P(L_2) - J_P(L_1) = \sum_{j=2}^{i} (-1)^{j-1} P^{(j)}(1)\big(G_{j,1}(1) - G_{j,2}(1)\big) + (-1)^i \int_0^1 P^{(i+1)}(u)\big(G_{i,1}(u) - G_{i,2}(u)\big)du$$

and use this expression in constructing the proof.

Assume first that

$$G_{i,1}(u) - G_{i,2}(u) \geq 0 \quad \text{for all} \quad u \in [0,1]$$

and > holds for at least one u.

Then $J_P(L_2) > J_P(L_1)$ for all $P \in \wp_i$.

Conversely, assume that

$$J_P(L_2) > J_P(L_1) \quad \text{for all} \quad P \in \wp.$$

Then this statement holds for the subfamily of $\wp_i$ for which $P^{(i)}(1)=0$. For this particular family of preference functions we have that

$$J_P(L_2) - J_P(L_1) = (-1)^i \int_0^1 P^{(i+1)}(u)\big(G_{i,1}(u) - G_{i,2}(u)\big)du.$$

Then, as demonstrated by Lemma 1, the desired result can be obtained by a suitable choice of $P \in \wp_i$ for which $P^{(i)}(1)=0$. Q.E.D.

The proofs of Theorem 9, 10 and 11 can be constructed by following exactly the line of reasoning used in the proof of Theorem 8. The proofs use the following expressions,

$$J_P(L_2) - J_P(L_1) = -\sum_{j=2}^{i} P^{(j)}(0)\big(\tilde{G}_{j,1}(0) - \tilde{G}_{j,2}(0)\big) - \int_0^1 P^{(i+1)}(u)\big(\tilde{G}_{i,1}(u) - \tilde{G}_{i,2}(u)\big)du,$$

$$J_Q^*(L_2) - J_Q^*(L_1) = \sum_{j=2}^{i} (-1)^j Q^{(j)}(1)\big(K_{j,2}(1) - K_{j,1}(1)\big) + (-1)^{i+1} \int_0^1 Q^{(i+1)}(u)\big(K_{i,2}(u) - K_{i,1}(u)\big)du$$

and

$$J_Q^*(L_2) - J_Q^*(L_1) = \sum_{j=2}^{i} Q^{(j)}(0)\big(\tilde{K}_{j,2}(0) - \tilde{K}_{j,1}(0)\big) + \int_0^1 Q^{(i+1)}(u)\big(\tilde{K}_{i,2}(u) - \tilde{K}_{i,1}(u)\big)du,$$

all obtained by using integration by parts i times.

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