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Tax reforms when utility is composed of additive functions

by

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Abstract

The paper discusses how a tax reform should be designed when utility can be written as additive, separable in an additive sub function, or nested additive. General characterization results can be simplified considerably in these cases. When utility is additive, an (income) inelastic good with a low tax rate is a candidate for higher taxation if it is compared with a high-tax good. If it is compared with a low-tax good, it is a candidate for lower taxation. Labor income is also a candidate for lower taxation. Similar, but more qualified conclusions are reached when utility is separable in an additive sub function, or it is nested additive.

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1 Introduction

This paper considers revenue neutral marginal tax reforms in which one distorting tax is increased by a small amount and another is decreased. It answers the following question: How should a tax reform be designed when the utility function is composed of additive functions[1], and taxes are imposed for efficiency reasons only?

One reason for being interested in this question is that some quite popular empirical demand systems are derived from additive utility, like the Linear Expenditure System (LES), the Constant Elasticity of Substitution (CES) system and the Cobb Douglas system. Intertemporal utility is usually additive, as is utility over differentiated products, and sub utility functions that define aggregate commodities. It is of interest to describe how tax reforms work when some component of utility is additive.

Much applied work on tax reform is based on some form of additive utility, like for instance the applied general equilibrium analyses of tax reform. The papers often assume nested additive utility (i.e. utility consists of a hierarchical structure of additive functions). Moreover, applied work often concentrates on efficiency aspects of tax reform, like the literature on the "Marginal Cost of Public Funds". (See Ballard and Fullerton (1992) for references to this literature). Explicit results on efficient tax reform under additive utility may be of interest when discussing the results of empirical analyses of tax reform. Finally, results on efficient tax reform under additive utility may serve as benchmarks for more complicated preferences, and considerations of distribution.

[1] For practical reasons, I use utility language instead of preference language in this paper. The results do not depend on the cardinalisation of utility, however.

Except general characterization results, little is known about the design of tax reforms in general[2]. As Dixit (1985, p. 347) has put it, "it is clear that by restricting the permissible changes suitably, we can create all sorts of paradoxical results". When utility is additive, Engel curves are identical and linear, and there is an optimal poll tax, Deaton (1987) shows that any movement in the direction of more uniform tax rates (which in this case is optimal) will increase welfare. An empirical regularity from the applied general equilibrium literature on the Marginal Cost of Funds in particular is that (price) inelastic goods with low tax rates are candidates for higher taxation. (See Ballard and Fullerton (1992) and the references there).

This paper suggests when a tax should increase or decrease, and outlines the consequences of repeated tax reform. The paper shows that when utility is purely additive, the tax on a high-tax income inelastic good should increase when compared with an elastic good with the same tax rate. Similarly, the tax on a low-tax good should increase when compared with a high-tax good. Thus, subsidies should usually decrease. These are changes that will move the system toward the second best optimum. However, if an inelastic good with a low tax is compared with an elastic good with an equally low tax, it is the tax on the elastic good that should increase. For low tax rates, the optimum conditions work in "reverse". The paper also shows that the tax on labor is a candidate to decrease unless this decrease is financed by increasing a high tax rate on an elastic good. This is because a tax on labor formally is equivalent to a subsidy (on leisure).

The direction of tax reform is more intricate when only parts of utility are additive, or

[2] Auerbach (1985, section 8) and Dixit (1985, section 4) survey the tax reform literature.

utility is nested additive. In the case of nested additive utility, the most obvious candidates for a tax increase are inelastic goods with a tax rate lower than the average tax rate of its group if the average of the group is lower than the total average. The last condition means that aggregates are treated similarly to individual goods under purely additive utility. The reason is the assumption of top level additive utility. In the cases of partially additive or nested additive utility, it is not clear that repeated use of the tax reform rule will lead to the second best optimum.

This paper is organized as follows: Section 2 gives notation. Section 3 treats the case of purely additive utility. Labor supply is given special consideration. Section 4 analyzes weakly separable utility where at least one sub utility function is additive. Section 5 discusses tax reform in the case of two level nests of additive utility. Two of the most popular empirical systems, the CES—CES and the LES—CES systems, are given special consideration. Section 6 concludes by indicating how the results of the paper can be extended to producer taxation provided the product and utility functions both are additive, and how endogenous producer prices and distributional considerations can be incorporated. Proofs are in the appendix.

2 Notation

Let $x = (x_1 \ldots x_n)$ be the vector of consumer goods. Let $p = (p_1 \ldots p_n)$ be the vector of consumer prices. Let $t = (t_1 \ldots t_n)$ be the vector of taxes. One tax is zero. We need not specify which one. The relation $p = t + q$ associates consumer prices with taxes and producer prices. q is the vector of constant producer prices. There is one consumer whose utility is indicated by U. Government revenue is $R = t'x$.

The lump sum income of the consumer is y. It simplifies the discussion of the inequalities characterizing tax reform to assume that the lump sum income is positive. We write $\frac{\partial x}{\partial y}$ for the vector of derivatives of x w.r.t. the argument y, $\frac{\partial x}{\partial p_j}$ for the vector of derivatives of x w.r.t. p_j etc. s_k is the vector of Slutsky derivatives w.r.t. p_k.

We define the expenditure elasticity of good k as $e_k = \frac{\partial x_k}{\partial y}\frac{y}{x_k}$. Similarly, the expenditure elasticity of good k belonging to group G is $e_{Gk} = \frac{\partial x_k}{\partial y_G}\frac{y_G}{x_k}$ where y_G is expenditure on goods in group G. y_G is also assumed to be positive.

Corresponding to the tax t_j on good j there is the tax rate $\tau_j = t_j/p_j$, which will be important in this analysis. Another important notion is *the general tax rate* $T = t'\frac{\partial x}{\partial y}$, which is the sum of taxes weighed by the expenditure derivatives. Alternatively, it is the sum of tax rates weighed by marginal propensities to spend. It can be interpreted as an *average* tax rate of the economy, as the following example shows: imagine that all goods face the same tax rate τ. T will then be equal to τ. The general tax rate is however a *marginal* tax rate in the sense that the weights are the marginal propensities to spend. It is possible that total revenue from taxation is positive, but T is negative.

Another noteworthy point about T is that it includes the untaxed good. If all goods except the untaxed good face the tax rate τ, T will be strictly smaller than τ. This fact will play a role below.

Similarly to T, we define the general tax rate within group G as $T_G = t'_G\frac{\partial x_G}{\partial y_G}$.

3 Additive utility

We consider additive utility, i.e. the utility function can be written $U = \sum_i u_i(x_i)$.

We have the following result:

Proposition 1 *When utility can be written as an additive function, the tax reform rule is: increase t_k and decrease t_l if and only if*

$$e_k(\tau_k - T) < e_l(\tau_l - T) \tag{1}$$

Proposition 1 says that to increase the tax on good k, the difference between the tax rate on good k and the general tax rate, weighed by the expenditure elasticity on good k, should be smaller than the similar expression for good l.

In general, goods with negative or small positive own and cross Slutsky elasticities are the candidates for tax increases in a tax reform. Why are the expenditure elasticities of interest? The reason is that under additive utility, a small expenditure elasticity implies that all Slutsky elasticities are small as well (see the proof of proposition 1). The tax reform rule for additive utility is therefore no more than an application of a quite well known property of the Slutsky terms under additive utility.

To interpret the proposition further, we can distinguish three cases: the case where both tax rates in question are greater than the general tax rate, the case where one of them is greater, the other lower, and the case where both are lower than the general tax rate.

Consider first the case where both tax rates are greater than the general tax rate, and assume further that the two tax-rates in question are equal before reform. Proposition 1 says to increase the tax on good k if the expenditure elasticity on good k is smaller than that on good l. (Note that expenditure elasticities are positive under additive utility). I.e. the tax rate on the most inelastic good should increase. By repeatedly applying this formula, inelastic goods will face high tax rates, and elastic goods will face low tax rates until the point where the potential for reform is exhausted. Thus the *optimal* tax rates vary inversely with the expenditure elasticities (Atkinson and Stiglitz (1980)).

Consider next the case where the tax rate on good k is below the general tax rate, while the tax rate on good l is higher than this average. It follows unambiguously that the tax rate on good k should increase. That is, it doesn't matter if good k is more elastic than good l. The reason is simply that the negative number on the left-hand side is always smaller than the positive number on the right-hand side of equation (1). Since there will always be at least one tax-rate higher than the general tax rate, it follows that by applying the reform rule repeatedly, all tax rates will eventually end up being higher than the general tax rate (this is possible because the untaxed good pulls the general tax rate down).

As subsidies are likely to be below the general tax rate, it also follows that if a subsidy is matched with a (sufficiently high) tax-rate, the subsidy should be reduced.

Consider finally the case where both tax rates in question are lower than the general tax rate. Proposition 1 then gives the paradoxical advice that one should *decrease* the tax rate one the most inelastic good. This will clearly move the tax rate on the most inelastic good away from the optimum. On the other hand, the other tax rate changes in the right direction. Of course, one of them two has to change away from the optimum in this situation.

By repeated use of the tax reform rule among subsidies and other tax rates below the average, there will appear a pattern in which the most inelastic goods are the most heavily subsidized. The "optimal" subsidy is inversely related to the expenditure elasticities. The optimum rule therefore applies in "in reverse", so to speak when we compare goods with subsidies and low tax rates. Of course, as soon as a subsidy is matched with a higher than general tax-rate, the subsidy should be reduced as described above.

If all expenditure elasticities are equal, as in the popular CES and Cobb Douglas systems, we know by the rule that optimal taxes are inversely related to the expenditure elasticities that all tax rates are equal in the optimum. Proposition 1 provides an interesting extension of this optimum condition. We should increase the tax on good k if the tax rate on good k is lower than that on good l. In other words, anything that makes tax rates *more* equal will improve welfare.

As shown in the appendix, saying that a tax should increase is equivalent to saying that "Marginal Cost of Funds" (MCF) of this tax is low. The discussion of this section has therefore shown that subsidies and low tax rates most likely will have a low MCF (in an additive system): It will be cheaper to finance a public project by reducing a subsidy or increasing a low tax rate than by other means. A high tax rate will tend to have a high MCF unless the good is sufficiently inelastic.

3.1 Endowment of leisure

The tax reform rule is different if we allow for endowments of goods. A prominent example of an endowment is in the leisure/labor choice, with labor supply the negative amount of leisure demanded from the marketplace. Of course, if leisure is the only good with a non zero endowment we can make it the untaxed good and proceed with the analysis of the last section. Calculating the tax reform formula for leisure may nevertheless be of interest, either because another good than leisure is the numeraire, or because there are endowments of more than one good, or to better understand estimates of the marginal cost of funds related to taxation of labor income.

To study the problem let

$$x_l = W - L$$

With x_l having the interpretation of leisure, W is total time endowment, and L is labor supply. Define $t_L = -t_l$ as the tax on labor, $\tau_L = -\tau_l$ as the labor tax rate and $e_L = \frac{\partial L}{\partial y} y / L$ as the labor elasticity with respect to full income y, which now includes the value of the time endowment. We obtain the following proposition

Proposition 2 *When utility can be written as an additive function over goods and leisure, the tax reform rule is: increase t_k and decrease t_L if and only if*

$$\tau_k < \frac{-e_L}{e_k}\tau_L + \left[1 + \frac{-e_L}{e_k}\right] T$$

Recall that e_L is negative because leisure is a normal good. Proposition 2 says that the tax on a good should increase (and the tax on labor decrease) if the tax rate on the good is smaller than the tax on labor weighed by the ratio of expenditure elasticities, plus the general tax rate weighed by one plus the ratio of expenditure elasticities. To put it differently, we should reduce the labor income tax if the commodity tax is "not too high" to begin with. This means we have a case for reducing the labor income tax to a level well below that on commodities, or in other words: The system of additive utility favors indirect taxation over direct taxation.

Leisure is an exceptional good in at least two ways. One is that the price that the consumer "pays" for leisure is lower than the price the producer "receives". Leisure is therefore subsidized. We recall from the discussion above that subsidies are likely to be reduced in an additive system. Reducing a subsidy on leisure toward zero is equivalent to reducing the labor income tax toward zero. This effect pulls in the direction of a low tax on labor income.

The other exceptional thing about leisure is that the increase in the "tax" on leisure (a lower tax on labor) is accompanied by a tax increase on some good. Two increases go together, instead of an increase and a decrease. If the cost of increasing the alternative tax is high, which will be the case if that good is elastic and its tax rate is high, reducing the labor income tax may not improve welfare after all. This explains why the subsidy on leisure only should decrease if it is offset by increasing a "not too high" tax. When goods are subsidised, we recall that comparing with a high tax rate strengthens the case for reducing the subsidy.

This discussion has assumed that the general tax rate is positive. That is more questionable when the system includes labor than when it just includes commodities. The presence of labor means that one tax normally is negative, that on consumed leisure. As a result, the general tax rate is lower than it otherwise would have been. If the marginal propensity to spend on leisure is high enough, the general tax rate may be negative. If it is negative, the conclusion that the commodity tax usually should increase is reversed: It should decrease (and the labor income tax increase) if it is "not too low" to begin with. If the general tax rate is exactly zero, the question of tax reform boils down to which tax rate weighed by expenditure elasticities is the largest.

It is unusual to model leisure as part of an additive system that involves several consumption goods. However, it is quite common in applied analyses to model the choice between leisure and an aggregate consumption good by means of an additive function. The consumption good may be a quantity index of several goods. The analysis of this section will in such a setting apply to the choice between a labor income tax and eg. a VAT on all consumption goods (except the numeraire).

If the general tax rate is positive, we should from this discussion expect the marginal cost of funds related to labor income taxation to be higher than that related to commodity taxation. This is borne out by some empirical studies that attempt to measure the marginal cost of funds by means of additive utility functions. For instance, Ballard et. al. (1985) find much smaller marginal costs of funds when increasing most sales taxes, than when increasing the labor (and capital) income tax.

4 Partially additive utility

When there are additive nests within a general utility function, we can derive a proposition similar to proposition 1 for goods within a nest. This is proposition 3. We model utility as weakly separable here and study tax reform within one additive sub function. The remaining sub functions have one or more elements each. At one extreme, there is one good outside the additive system. At another extreme, there is a "flexible" demand system over (a few) aggregates, each of which are defined by additive sub utility functions. We define $\mathcal{T} = \sum_H \alpha_H T_H + T_G$ as the group tax rate $T_G = t'_G \frac{\partial x_G}{\partial y_G}$ plus a weighed sum of tax rates of all groups. The weights however sum to zero. We call this term the *modifying factor*, and $\mathcal{T}$ a *modified* group tax-rate.

Proposition 3 *Suppose utility can be written $U = U(u_1, \ldots u_N)$, and suppose $u_G = \sum_{i \in G} u_{Gi}(x_i)$. The tax reform rule is: increase t_k and decrease t_l, $k, l \in G$, if and only if*

$$e_{Gk}(\tau_k - \mathcal{T}) < e_{Gl}(\tau_l - \mathcal{T})$$

Proposition 3 says that when there is an additive node within a general utility function, a tax reform on goods within this node is governed by the following rule: Increase the tax on good k if the difference, weighed by the expenditure elasticity, between the tax rate on this good and the modified general tax rate is greater than the similar expression for good l. The only differences between this rule and the rule in a completely additive system is the presence of the modifying factor in the modified general tax rate, and the fact that the general tax rate and expenditure elasticities are defined relative to a group.

Hence when the tax-rate of two goods are equal and higher than the modified general tax rate, the tax on the most (income) inelastic good should increase. When the modified general tax rate lies between the tax rates of the two goods, the lower tax rate should increase. When both tax rates lie below the modified general tax rate, the tax rate on the most elastic good should increase.

The modifying factor is a weighed sum of the group tax rates. The weights are proportional to terms λ_{GH} that can be interpreted as "the intergroup substitution terms when each group is defined as a Hicks aggregate with fixed relative prices within the groups" (Deaton and Muellbauer, 1980 p.129). These terms, and hence the weights in the modified general tax rate express the substitution possibilities between the aggregates. Positive weights are put on goods that are positively affected by price increases of the additive aggregate, i.e. on alternatives to this aggregate. Negative weights are put on complements to the aggregate. The modifying factor will therefore be positive if alternatives to goods in group G are heavily taxed. There is consequently a greater chance that tax rates in group G are below the modified general tax rate.

It follows that it is not obvious that any tax rate in group G is higher than the modified tax rate. But if one is, we know, similar to the case of purely additive utility, that all tax rates in group G will end up being higher than the modified general tax rate if proposition 3 is used repeatedly. This means that the general tax rate of group G in the end will be higher when alternatives to group G are heavily taxed. The intuition is that this will restore the balance between goods in group G and its alternatives: Taxing the alternatives only will lead to too much consumption of goods in group G. Taxing goods in group G only will lead to too much consumption of its alternatives. Taxing both the group G and alternatives will to some extent cancel the two impulses. A low tax on alternatives to G also implies a high tax on G, for the same reason.

If all tax rates in group G are below the modified general tax rate to begin with, the outcome of a repeated use of proposition 3 may depend on the order in which taxes are changed. In a circumstance where relatively high tax rates in group G are decreased first, all taxes in group G will continue below the modified general tax rate, but they will be revised so that the most inelastic goods in group G have the lowest tax rates. (If the modifying factor is too high, the order doesn't matter). The algorithm represented by a repeated use of proposition 3 will in this case find a local optimum, i.e. one in which the impulse to and from other groups is approximately constant, but the rearrangement within group G gives some improvement.

The discussion is simplified considerably if the expenditure elasticities in the additive node are all unity. Given the widespread use of CES and Cobb Douglas functions, this may be the more usual case in applied work. If all expenditure elasticities are unity, the tax reform rule simply is to increase the lower of two tax rates until the point where the two are equal. Other taxes in or outside group G do not matter. Thus, the consequences of unitary elasticities are similar to unitary elasticities in purely additive utility.

5 Nested additive utility

Nested additive utility is utility that can be written $U = \sum_G f_G(u_G)$, and $u_G = \sum_{i \in G} u_{Gi}(x_i)$ $\forall G$. The construction may arise in intertemporal modeling, or as a way to structure atemporal utility, or both. There is a simple rule for tax reform here too. First we need to define the *constant of substitution*

$$\mu = \frac{s_{kl}}{\frac{\partial x_k}{\partial y}\frac{\partial x_l}{\partial y}}$$

The similar constant within group G is

$$\mu_G = \frac{s^G_{kl}}{\frac{\partial x_k}{\partial y_G}\frac{\partial x_l}{\partial y_G}} \qquad k, l \in G$$

where s_{kl}^G is the cross substitution term within group G for given sub utility u_G. We have the following proposition

Proposition 4 *Suppose utility can be written* $U = \sum_G f_G(u_G)$, *and* $u_K = \sum_{i \in K} u_{Ki}(x_i)$ $K = G, H$. *The tax reform rule is: increase* t_k $k \in G$ *and decrease* t_l $l \in H$ *if and only if*

$$e_k[\tau_k - T + \nu_G(\tau_k - T_G)] < e_l[\tau_l - T + \nu_H(\tau_l - T_H)]$$

where $\nu_K = \frac{\mu_K}{\mu \frac{\partial y_K}{\partial y}}$ $K = G, H$.

Remark: When all groups are additive, proposition 4 holds for any two groups, that is for all groups.

Remark: In proposition 4, it is not necessary that $G \neq H$. And one good per group is sufficient.

Proposition 4 says that the starting point for evaluating whether the tax on good k in group G should increase at the expense of the tax on good l in group H is to compare the weighed difference between the tax rate of good k and the group tax rate, *plus* the difference between the group tax rate and the general tax-rate. If the sum of these two differences, weighed by the expenditure elasticity of good k, is smaller than the similar expression for good l, then the tax on good k should increase.

The weight related to the difference between the tax rate of a good and the general tax rate of the group that it is in, consists of two items that are readily observable, and one item that sometimes is. The two readily observable terms are μ and μ_G, which follow from the cross substitution terms at the top and bottom levels, respectively. The derivative

$\partial y_G/\partial y$ is readily observable when the system fits the conditions for two stage budgeting (see Gorman (1959)). Most applied examples of nested additive utility do fit the conditions for two stage budgeting.

To interpret the formula, note that two things happen if we increase the tax on a particular good. One is that the tax rate of the particular good is increased. The other is that the tax rate of the group is increased. The tax reform formula of proposition 4 balances these two effects. The case for increasing the tax on a particular good is best if the tax rate of the good is lower than the group tax rate, and the group tax rate is lower than the general tax rate. If the tax rate of the good is lower than the group tax rate, but the group tax rate is higher than the general tax rate, the two effects pull in different directions. If the tax rate of the good is higher than the group tax rate and the group tax rate is higher than the general tax rate, it is likely that the tax should be decreased.

The group tax rate also has an independent effect. If the tax rates on two goods (in groups G and H) are equal, and the expenditure elasticities are equal, and the tax rate of group H is zero, one should increase the tax on the good in group G if the weight ν_G is greater than unity. This will be the case if the constant of substitution at the top level is larger than the constant of substitution at the bottom, or the marginal propensity to spend on group G is smaller than unity, or both[3]. If the weight is smaller than unity, one should decrease the tax on the good in group G. If the weight is exactly unity, the tax rate of group G is irrelevant.

The unclear effect of the group G tax rate is explained by its two roles in the formula. A high group tax rate makes it more likely that a tax should increase because it increases the difference between the group tax rate and the particular tax rate. A high group tax rate on the other hand makes it less likely that a tax should increase because it decreases the difference between the general tax rate and the group tax rate. Which of these aspects is the strongest, depends on whether the weight is greater or smaller than unity. When the weight is unity, the two effects cancel. Then proposition 4 looks exactly equal to proposition 1.

5.1 The CES—CES and LES—CES combinations

We would like to compute the tax reform formula when utility has the CES—CES or LES—CES forms. These are popular combinations of nested additive utility in applied work. It turns out that the tax reform rule is simplified in these systems compared with

[3] An alternative way of writing the weight may be helpful here. We can rewrite the weight in terms of the absolute value of the 'elasticity of marginal utility' with respect to expenditure. It can be shown that

$$\phi = \frac{y}{\mu}$$

where ϕ is the absolute value of the elasticity of marginal utility. (In an expected utility framework, ϕ is the coefficient of relative risk aversion). ϕ_G is defined similarly to ϕ. Now we can write

$$\nu_G = \frac{\phi}{\phi_G e_y}$$

where e_y is the elasticity of group expenditure y_G with respect to total expenditure. The weight is greater than unity if the elasticity of marginal utility at the top level is greater than the elasticity of marginal utility at the bottom level, or the elasticity of group expenditure is greater than unity, or both.

the case of general nested additive utility.

Consider the CES—CES system

$$\begin{aligned} U &= \sum_G a_G u_G^c \\ u_G &= \left(\sum_{j \in G} a_j x_j^{c_G}\right)^{1/c_G} \end{aligned}$$

It can be shown that $\sigma = \frac{1}{1-c}$ is the (Allen) elasticity of substitution of the function U, and $\sigma_G = \frac{1}{1-c_G}$ is the (Allen) elasticity of substitution of the function u_G. We have the following proposition:

Proposition 5 *When utility has the CES—CES form, the tax reform rule is: Increase t_k $k \in G$ and decrease t_l $l \in H$ if and only if*

$$T_G + \frac{\sigma_G}{\sigma}(\tau_k - T_G) < T_H + \frac{\sigma_H}{\sigma}(\tau_l - T_H)$$

Proposition 5 says that the tax on a good should increase if the weighed difference between the tax rate on the good and the group tax rate, plus the group tax rate, is smaller than the similar expression for the other good. The weight is the ratio of the group elasticity of substitution to the 'top' elasticity of substitution.

If not for the weighed term, proposition 5 would have said to increase the lower of two group tax rates. This is because the top level additive function has the CES form. Similarly to the one level CES, the tax rates on appropriate aggregates should change toward each other if we view the top level in isolation.

When utility is nested CES, this is only a partial effect however. Another partial effect is the weighed term, which says that tax rates on individual goods should change toward their respective group tax levels. This effect can also be recognized from one level CES functions.

Just as with general additive utility, the effects may go in opposite directions. That is, by increasing a tax one may close the gap to the group tax rate, which is "good", but increases in the group tax rate may be "bad". In such cases the weight of the two effects becomes important.

The weight is the ratio of the group elasticity of substitution to the top elasticity of substitution. If the lower level function is more elastic (the group elasticity of substitution is larger), the weight is larger than unity. This means that it is more important to correct the low level deviation from uniform tax rates. If the top level function is more elastic, the weight is less than unity, and it is more important to correct the top level deviation from uniform tax rates.

By repeated iterations on proposition 5 among taxes in group G, all tax rates in group G will end up equal. Repeating for all groups, all tax rates will be equal to their group tax rates except one group. That is the group that contains the numeraire, where the group tax rate will be smaller than the individual non zero tax rates. Comparing groups, all group tax rates except the one that contains the numeraire will end up equal. The one with the numeraire will be smaller than the rest. The difference will be equal to the weighed difference between the individual tax rates of this group and the group tax rate. It turns out that the individual tax rates of the group that contains the numeraire should only equal the other tax rates if the weight is exactly unity. Whether it is, depends on the group elasticity of substitution of the group where the numeraire is placed.

Although the iteration procedure just outlined will lead to the second best optimum, the previous discussion of proposition 5 suggested that it may improve welfare eg. to increase a tax rate that is higher than the group tax rate if the group tax rate is too low. Or we may decrease one that is already too low if the group tax rate is too high, etc. These examples show that even in the simple nested CES formulation, tax reforms may not direct us to the optimum. This is contrary to the one level CES function.

When we have a nested Cobb Douglas function however, all elasticities of substitution are equal to unity. Hence they cancel in proposition 5, and all reforms that make two tax rates more equal will improve welfare. As an example, we may have an intertemporal system that is Cobb Douglas in time and space. Proposition 5 says that the tax on a low-tax good in year t should increase and the tax on a high-tax good in year $t+s$ should decrease to the point where the two are equal. This is irrespective of whatever differences there may be in tax levels (group tax rates) at time t and $t+s$.

Another example of a double-additive system is the LES—CES system

$$\begin{aligned} U &= \sum_G a_G \ln(u_G - b_G) \\ u_G &= \left(\sum_{j \in G} a_j x_j^{c_G} \right)^{1/c_G} \end{aligned}$$

We obtain the tax reform formula

Proposition 6 *When utility has the LES—CES form, the tax reform rule is: increase t_k and decrease t_l if and only if*

$$(1 - b_G/u_G)(T_G - T) + \sigma_G(\tau_k - T_G) < (1 - b_H/u_H)(T_H - T) + \sigma_H(\tau_l - T_H)$$

Proposition 6 says that the tax on a good should increase if the weighed difference between the group tax rate and the general tax rate plus the weighed difference between the tax rate of the good and the group tax rate is smaller than the similar differences for the other good. The weights are one less relative committed expenditure on the group on the one hand, and the group elasticity of substitution on the other. We call these the LES-weight and CES-weight respectively.

As in the earlier discussion of nested additive utility, there are two terms in the condition guiding tax reform. One is the difference between the group tax rate and the general tax rate. This term is weighed by the LES-weight, as it would have been in a one level LES system[4]. If this term was the only term, optimal group tax rates would be inversely related to the LES-weights and thus to the expenditure elasticities for the CES-aggregates.

The other term is again the difference between the individual tax rate and the group tax rate, weighed by the group elasticity of substitution. The reform formula proposition 6 balance the two terms. The LES-weight is always smaller than unity.

As an example of how proposition 6 works, consider the following stylized example: Imagine a CES-aggregate for electricity and heating fuel consumption within a LES system. Assume the group tax rate to be lower than the general tax rate. Assume a low elasticity of substitution between different forms of energy like electricity and heating fuel. Imagine another CES-aggregate, for alcohol and tobacco, which has a higher group tax rate. A tax reform (not accounting for external effects) in which a high-taxed item within energy

[4]Defining E_G as the expenditure elasticity for the CES-aggregate of group G, we may write the LES-weight of group G as $1 - b_G/u_G = E_G PU/y$.

is increased at the expense of an equally high-taxed good within alcohol and tobacco will improve welfare if the elasticity of substitution between different forms of energy is low enough. The reason is that when the elasticity of substitution within energy is low, the CES-weight is lower than the LES-weight, and it counts more to increase the group tax level of energy than to narrow the difference between the group tax level and the individual tax.

Imagine that the estimate of the elasticity of substitution within energy is revised to a quite high figure. Now it will improve welfare to *decrease* the tax on the energy item and increase the tax on the item within alcohol and tobacco. The reason is that with the higher elasticity of substitution, the CES-weight is greater and it counts more to bring the tax in line with its group average.

6 Conclusions

This paper has shown that when utility is composed of additive functions, the rules for tax reform are significantly simplified and explicit compared with the characterization results that mostly are available in the general case. Starting from uniform expenditure elasticities, we have found that low tax rates should increase, and high tax rates should decrease. The labor income tax in particular, should decrease. Starting from uniform (and not too low) tax rates, we have found that income inelastic goods are candidates for a tax increase, and elastic goods are candidates for a tax decrease. The group tax rates and other parameters however also have influence on whether a tax should increase or decrease in this case. We have suggested where the tax reform rules point in the direction of optimal taxation, and where they do not.

In a model of a market, one sometimes assumes that the production function is additive homogeneous (i.e. CES or Cobb Douglas), and the utility function is additive. In such cases we may use the results of section 5 to analyze the impact of reform in taxation of inputs. The only difference from the model of section 5 is public revenue. With X the vector of consumption goods, and x the vector of inputs, the revenue function will be $R = t'_X X + t'_x x$. From this we must construct equivalent taxes $R = t'x$ to use in the analysis. Even if the utility function is not additive, we can use proposition 3 to analyze tax reforms within one industry.

The assumption of exogenous producer prices is clearly violated in most tax reforms. In that case it is convenient to work with the difference between the market price for the consumer and a shadow price. Call this concept a "shadow tax". It can be shown that all the results of this paper hold in a setting of endogenous producer prices if shadow taxes are substituted for actual taxes. For instance, under additive utility and unitary expenditure elasticities, the tax on good k should increase if the shadow tax rate on good k is larger than that on good l.

In a setting that includes distribution, the results of this paper are subject to the qualification that the two goods involved have the same "distributional characteristic". This is generally not true. If consumers have identical CES or Cobb Douglas utility however, one can easily show that the distributional characteristics of all goods are equal, and the tax reform rules of this paper hold without qualification.

Appendix

Consider a revenue preserving tax reform involving infinitesimal changes in t_k and t_l. The tax reform satisfies the condition

$$\frac{\partial R}{\partial t_k} dt_k + \frac{\partial R}{\partial t_l} dt_l = 0 \tag{2}$$

We will use the word "decrease" to describe what happens to t_l when we increase t_k, as this is likely to be the normal case. Our theory does not depend on t_l being decreased, however (compare proposition 3).

A reform improves welfare if and only if

$$\frac{\partial U}{\partial t_k} dt_k + \frac{\partial U}{\partial t_l} dt_l > 0 \tag{3}$$

Equations (2) and (3) can be combined to give

$$\beta_k < \beta_l \tag{4}$$

$$\beta_h = -\frac{\partial U/\partial t_h}{\partial R/\partial t_h} \qquad h = k, l$$

where β_h/λ is the "Marginal Cost of Funds" related to increasing t_h. λ is the marginal utility of money.

Condition (4) is equivalent to

$$\begin{aligned}
-\frac{-\lambda x_k}{x_k + t'\frac{\partial x}{\partial t_k}} &< -\frac{-\lambda x_l}{x_l + t'\frac{\partial x}{\partial t_l}} \\
\frac{\lambda}{1 - T + \frac{t' s_k}{x_k}} &< \frac{\lambda}{1 - T + \frac{t' s_l}{x_l}} \\
\frac{t' s_k}{x_k} &> \frac{t' s_l}{x_l}
\end{aligned} \tag{5}$$

In terms of the Slutsky elasticities $a_{ij} = p_i s_{ij}/x_j$ and tax rates $\tau_j = t_j/p_j$, equation (5) is $\tau' a_k > \tau' a_l$. Whether or not it improves welfare to increase t_k and decrease t_l therefore depends on the sum of tax rates weighed by the slutsky elasticities of goods k and l.

Proof of proposition 1: The point of departure is equation (5). When utility is additive, the Slutsky derivatives can be written (Deaton and Muellbauer (1980); Frisch (1959))

$$s_{kl} = \mu \frac{\partial x_k}{\partial y} \frac{\partial x_l}{\partial y} \tag{6}$$

$$s_{kk} = \mu \frac{\partial x_k}{\partial y} \left(\frac{\partial x_k}{\partial y} - \frac{1}{p_k} \right) \tag{7}$$

Semi-definiteness requires that μ is positive. We therefore have

$$\begin{aligned} \frac{t' s_k}{x_k} &> \frac{t' s_l}{x_l} \\ \frac{\mu}{y} \frac{y}{x_k} \frac{\partial x_k}{\partial y} (T - \tau_k) &> \frac{\mu}{y} \frac{y}{x_l} \frac{\partial x_l}{\partial y} (T - \tau_l) \\ e_k(\tau_k - T) &< e_l(\tau_l - T) \end{aligned}$$

Q.E.D.

Proof of proposition 2: The point of departure is equation (5), properly modified: Increasing t_k improves welfare if and only if

$$\frac{t' s_k}{x_k} > \frac{t' s_l}{-L}$$

where the denominator on the right-hand side is $(-L)$, the (negative) *market* demand for leisure. We divide by the market demand because the effect on utility of a marginal change in t_L is proportional to market demand, and because market demand enters the public revenue function.

Using the formula for the Slutsky matrix in additive functions, we obtain

$$
\begin{aligned}
e_k \frac{\mu}{y}[T-\tau_k] &> \frac{\partial x_l}{\partial y}\frac{\mu}{-L}[T+\tau_L] \\
e_k \frac{\mu}{y}[T-\tau_k] &> -\frac{\partial L}{\partial y}\frac{\mu}{-L}[T+\tau_L] \\
e_k \frac{\mu}{y}[T-\tau_k] &> e_L \frac{\mu}{y}[T+\tau_L] \\
T-\tau_k &> \frac{e_L}{e_k}[T+\tau_L] \\
\tau_k - T &< \frac{-e_L}{e_k}[T+\tau_L] \\
\tau_k &< \frac{-e_L}{e_k}\tau_L + \left[1+\frac{-e_L}{e_k}\right]T
\end{aligned}
$$

Q.E.D.

Proof of proposition 3: Again, equation (5) is the point of departure. There are three types of entries in the Slutsky vector, the substitution terms w.r.t. goods outside of group G, the substitution terms w.r.t. other goods in group G, and the direct term.

Begin with the terms w.r.t. goods outside of group G. Because the utility function is separable,

$$
x_i = x_{Gi}(p_G, y_G) \quad i \in G \quad G = 1 \ldots N
$$

where p_G is the vector of prices of x_G, and y_G is expenditure on the x_G-vector. When demand has this structure, the substitution terms w.r.t. goods in group H is

$$
s_{km} = \lambda_{GH} \frac{\partial x_k}{\partial y_G}\frac{\partial x_m}{\partial y_H} \quad k \in G \;\; m \in H \;\; G \neq H
$$

where

$$\lambda_{GH} = \left.\frac{\partial y_H/\partial p_k}{\partial x_k/\partial y_G}\right|_{U=const} = \left.\frac{\partial y_G/\partial p_m}{\partial x_m/\partial y_H}\right|_{U=const}$$

see Deaton and Muellbauer (1980). We may also, for later reference, write λ_{GH} in terms of the group expenditure functions $y_H = e_H(p_H, u_H)$:

$$\lambda_{GH} = \left.\frac{\frac{\partial e_H}{\partial u_H}\frac{\partial u_H}{\partial p_k}}{\frac{\partial x_k}{\partial y_G}}\right|_{U=const}$$

Next, we confront the substitution terms within group G. We find it convenient to make the substitution

$$x_i(p_G, y_G) = x_i(p_G, e_G(p_G, u_G)) \qquad i \in G$$

where $e_G(\cdot)$ is the expenditure function of group G. The substitution term of good k w.r.t. price p_l when both are elements of G is

$$s_{kl} = \left.\frac{\partial x_k}{\partial p_l} + \frac{\partial x_k}{\partial y_G}\frac{\partial e_G}{\partial p_l} + \frac{\partial x_k}{\partial y_G}\frac{\partial e_G}{\partial u_G}\frac{\partial u_G}{\partial p_l}\right|_{U=const}$$

The first two terms are the Slutsky equation for constant group utility u_G, so we can write

$$s_{kl} = s_{kl}^G + \frac{\partial x_k}{\partial y_G}\frac{\partial e_G}{\partial u_G}\frac{\partial u_G}{\partial p_l} \tag{8}$$

This equation is good both for the cross substitution terms within group G and for the own price substitution term.

Consider the cross substitution terms within group G. We know that s_{kl}^G is symmetric to s_{lk}^G (since $e_G(p_G, u_G)$ is a "real" expenditure function) so by symmetry we have

$$\frac{\partial x_k}{\partial y_G}\frac{\partial e_G}{\partial u_G}\frac{\partial u_G}{\partial p_l} = \frac{\partial x_l}{\partial y_G}\frac{\partial e_G}{\partial u_G}\frac{\partial u_G}{\partial p_k}$$

Rearranging, we have

$$\left.\frac{\frac{\partial e_G}{\partial u_G}\frac{\partial u_G}{\partial p_l}}{\frac{\partial x_l}{\partial y_G}}\right|_{U=const} = \left.\frac{\frac{\partial e_G}{\partial u_G}\frac{\partial u_G}{\partial p_k}}{\frac{\partial x_k}{\partial y_G}}\right|_{U=const} = \lambda_{GG}$$

The left hand side is the same for all substitution terms that involve price p_l. The right hand side is the same for all substitution terms that involve price p_k. Thus the term is the same for all substitution terms involving any two goods in group G, and can be denoted by eg. λ_{GG}. Note that the definition of λ_{GG} is similar to that of λ_{GH}.

Since the cross substitution terms for constant sub-utility is given by equation (6) for group G, we may write the cross substitution term as

$$s_{kl} = \mu_G \frac{\partial x_k}{\partial y_G}\frac{\partial x_l}{\partial y_G} + \lambda_{GG}\frac{\partial x_k}{\partial y_G}\frac{\partial x_l}{\partial y_G} = (\mu_G + \lambda_{GG})\frac{\partial x_k}{\partial y_G}\frac{\partial x_l}{\partial y_G}$$

Consider finally the own price substitution term. One way to find this term is by using the homogenity property of the full system, but we find it more useful to use the similar property within group G. We add all Slutsky terms in group G using equation (8) and the homogenity property to find

$$\sum_{l\in G} p_l s_{kl} = \sum_{l\in G} p_l s_{kl}^G + \frac{\partial e_G}{\partial u_G}\frac{\partial u_G}{\partial p_k}\sum_{l\in G} p_l \frac{\partial x_l}{\partial y_G} = 0 + \lambda_{GG}\frac{\partial x_k}{\partial y_G}$$

Armed with this result it is straight-forward to find the direct substitution term:

$$\begin{aligned}
s_{kk} &= (-\sum_{\substack{l\neq k\\ l,k\in G}} p_l s_{kl} + \lambda_{GG}\frac{\partial x_k}{\partial y_G})/p_k \\
&= (-(\mu_G + \lambda_{GG})\frac{\partial x_k}{\partial y_G}\sum_{\substack{l\neq k\\ l,k\in G}} p_l \frac{\partial x_l}{\partial y_G} + \lambda_{GG}\frac{\partial x_k}{\partial y_G})/p_k \\
&= (-(\mu_G + \lambda_{GG})\frac{\partial x_k}{\partial y_G}(1 - p_k\frac{\partial x_k}{\partial y_G}) + \lambda_{GG}\frac{\partial x_k}{\partial y_G})/p_k \\
&= \frac{\partial x_k}{\partial y_G}\frac{\partial x_k}{\partial y_G}(\mu_G + \lambda_{GG}) - \mu_G\frac{\partial x_k}{\partial y_G}/p_k
\end{aligned}$$

Collecting all Slutsky terms we obtain

$$
\begin{aligned}
\frac{t's_k}{x_k} &= \frac{e_{Gk}}{y_G}\left(\sum_{H\neq G}\lambda_{GH}T_H + (\mu_G+\lambda_{GG})T_G - \mu_G\tau_k\right) \\
&= e_{Gk}\frac{\mu_G}{y_G}\left(\sum_{H=1}^{N}\frac{\lambda_{GH}}{\mu_G}T_H + T_G - \tau_k\right) \\
&= e_{Gk}\frac{\mu_G}{y_G}(\mathcal{T}-\tau_k) \quad \forall\, i\in G
\end{aligned} \tag{9}
$$

where

$$\mathcal{T} = \sum_H \frac{\lambda_{GH}}{\mu_G}T_H + T_G = \sum_H \alpha_H T_H + T_G$$

This implies the tax reform criterion

$$
\begin{aligned}
e_{Gk}\frac{\mu_G}{y_G}(\mathcal{T}-\tau_k) &> e_{Gl}\frac{\mu_G}{y_G}(\mathcal{T}-\tau_l) \\
e_{Gk}(\tau_k-\mathcal{T}) &< e_{Gl}(\tau_l-\mathcal{T}) \quad k,l\in G
\end{aligned}
$$

Q.E.D.

Remark: By inserting p for t in equation (9), we see that $\sum_H \lambda_{GH} = 0$.

Remark: If there is one untaxed good outside the additive system, we have

$$\mathcal{T} = \left(\frac{\lambda_{GG}}{\mu_G}+1\right)T_G$$

Proof of proposition 4: In the proof of proposition 3 we obtained that $\frac{t's_k}{x_k} = e_{Gk}\frac{\mu_G}{y_G}(\mathcal{T}-\tau_k)$ $k\in G$ if u_G is additively separable.

When top level utility is additive as well, we can show that

$$\lambda_{GH} = \mu\frac{\partial y_G}{\partial y}\frac{\partial y_H}{\partial y} \quad G\neq H$$

see Deaton and Muellbauer (1980, pp. 141). The constant μ may or may not be equal to the same constant in section 3, but it has a similar interpretation.

Using the fact that $\sum_H \lambda_{GH} = 0$, see the remarks to proposition 3, we find λ_{GG} as

$$\begin{aligned}
\lambda_{GG} &= -\sum_{H \neq G} \lambda_{GH} = -\mu \frac{\partial y_G}{\partial y} \sum_{H \neq G} \frac{\partial y_H}{\partial y} \\
&= -\mu \frac{\partial y_G}{\partial y}(1 - \frac{\partial y_G}{\partial y}) \\
&= \mu \frac{\partial y_G}{\partial y} \frac{\partial y_G}{\partial y} - \mu \frac{\partial y_G}{\partial y}
\end{aligned}$$

These results imply that $\mathcal{T}$ can be written

$$\begin{aligned}
\mathcal{T} &= \frac{1}{\mu_G}(\sum_{H \neq G} \lambda_{GH} T_H + \lambda_{GG}) + T_G \\
&= \frac{1}{\mu_G} \frac{\partial y_G}{\partial y} \left(\mu T - \mu T_G + \frac{\mu_G}{\frac{\partial y_G}{\partial y}} T_G \right) \\
&= \frac{\mu}{\mu_G} \frac{\partial y_G}{\partial y} \left(T + (\frac{\mu_G}{\mu \frac{\partial y_G}{\partial y}} - 1) T_G \right) \\
&= \frac{1}{\nu_G}(T - T_G + \nu_G T_G)
\end{aligned}$$

where as before, $T = t' \frac{\partial x}{\partial y}$ and $\nu_G = \frac{\mu_G}{\mu \frac{\partial y_G}{\partial y}}$. Now we insert in the formula for $t's_k / x_k$:

$$\begin{aligned}
\frac{t's_k}{x_k} &= e_{Gk} \frac{\mu_G}{y_G} (\mathcal{T} - \tau_k) \\
&= e_{Gk} \frac{\mu_G}{y_G} \left(\frac{1}{\nu_G}(T - T_G + \nu_G T_G) - \tau_k \right) \\
&= e_k \frac{\mu}{y} (T - T_G + \nu_G (T_G - \tau_k))
\end{aligned}$$

Since both functions u_G and u_H are additive, this relation must hold for both groups, and we obtain the condition for a welfare improving tax reform

$$\frac{t's_k}{x_k} > \frac{t's_l}{x_l} \qquad k \in G \ \ l \in H$$

$$
\begin{aligned}
e_k \frac{\mu}{y}\left[T - T_G + \nu_G(T_G - \tau_k)\right] &> e_l \frac{\mu}{y}\left[T - T_H + \nu_H(T_H - \tau_l)\right] \\
e_k[T_G - T + \nu_G(\tau_k - T_G)] &< e_l[T_H - T + \nu_H(\tau_l - T_H)]
\end{aligned}
$$

Q.E.D.

Proof of proposition 5: In proposition 4, we obtain

$$
\mu = \frac{s_{ij}}{\frac{\partial x_i}{\partial y}\frac{\partial x_j}{\partial y}} = \sigma y
$$

and similarly $\mu_G = \sigma_G y_G$. This gives us

$$
\begin{aligned}
\nu_G &\doteq \frac{\sigma_G y_G}{\frac{\partial y_G}{\partial y}\sigma y} \\
&= \frac{\sigma_G}{\sigma}
\end{aligned}
$$

Inserting in the formula of proposition 4, we obtain

$$
\begin{aligned}
e_k[T_G - T + \frac{\sigma_G}{\sigma}(\tau_k - T_G)] &< e_l[T_H - T + \frac{\sigma_H}{\sigma}(\tau_l - T_H)] \\
T_G + \frac{\sigma_G}{\sigma}(\tau_k - T_G) &< T_H + \frac{\sigma_H}{\sigma}(\tau_l - T_H)
\end{aligned}
$$

Q.E.D.

Proof of proposition 6: In proposition 4, we obtain

$$
\mu = PU \quad P = \prod_G p_G^{a_G}
$$

$$
\mu_G = \sigma_G y_G
$$

We have

$$
\nu_G = \frac{\sigma_G y_G}{\frac{\partial y_G}{\partial y} PU}
$$

$$
\begin{aligned}
&= \frac{\sigma_G y_G}{\frac{\partial y_G}{\partial y}(y - \sum_G b_G p_G)} \\
&= \frac{\sigma_G y_G}{y_G - p_G b_G} \\
&= \frac{\sigma_G}{1 - b_G/u_G}
\end{aligned}
$$

A tax reform therefore improves welfare if and only if

$$
\begin{aligned}
& e_k(T_G - T + \frac{\sigma_G}{1 - b_G/u_G}(\tau_k - T_G)) \\
& \qquad < \quad e_l(T_H - T + \frac{\sigma_H}{1 - b_H/u_H}(\tau_l - T_H)) \\
& e_k \frac{1}{1 - b_G/u_G}((1 - b_G/u_G)(T_G - T) + \sigma_G(\tau_k - T_G)) \\
& \qquad < \quad e_l \frac{1}{1 - b_H/u_H}((1 - b_H/u_H)(T_H - T) + \sigma_H(\tau_l - T_H)) \\
& e_{Gk} \frac{y}{PU}((1 - b_G/u_G)(T_G - T) + \sigma_G(\tau_k - T_G)) \\
& \qquad < \quad e_{Hl} \frac{y}{PU}((1 - b_H/u_H)(T_H - T) + \sigma_H(\tau_l - T_H))
\end{aligned}
$$

$$
(1 - b_G/u_G)(T_G - T) + \sigma_G(\tau_k - T_G) \quad < \quad (1 - b_H/u_H)(T_H - T) + \sigma_H(\tau_l - T_H)
$$

Q.E.D.

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