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LABOR SUPPLY WITH NON-CONVEX BUDGET SETS, HOURS RESTRICTION AND NON-PECUNIARY JOB-ATTRIBUTES

by

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ABSTRACT

The basic assumption in this paper is that a household labor supply decision can be considered as choice from a set of discrete alternatives, called matches. The matches are characterized by attributes such as hours of work, wages and other aspects of the jobs. This model allows for very general budget specifications, joint decisions of husband and wife as well as restrictions on job opportunities and hours of work.

The model is estimated on Norwegian data from 1979 and 1986. The estimated utility function is found to be rather stable from 1979 to 1986.

Key words: Labor supply, non-convex budget sets, non-pecuniary job-attributes.

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1. INTRODUCTION

This paper presents a new econometric framework for analyzing labor supply. Traditionally, by labor supply it is meant the decisions of whether or not to work and how many hours of work, or from the econometrician's point of view, the hours of work distribution including the participation probability. In the present paper this definition is extended to include the choice among non-pecuniary, qualitative aspects of the jobs.

The principle purpose of our study is to develope an empirical model for labor supply in the presence of complicated budget sets and qualitative job attributes. Similar to most countries in the western world, the Norwegian tax system implies a non-convex budget set. The option of joint and separate taxation for married couples, social security rules and tax allowances turn an otherwise progressive tax structure into a structure which is not uniformly progressive, rather partially regressive. In recent years the tax system has been reformed and considerably simplified in Norway as in many other countries. Still the budget set is kinked and some non-convexities remain. Fixed costs of working contribute to this non-convexity as job-specific wage rates also do (Moffitt (1984)). Therefore, the traditional marginal calculus is not sufficient to simulate optimal behavior. Global rather than local criteria have to be applied.

Most of the labor supply studies have used the counterfactual assumption of a convex and smooth budget set (cf. early contributions such as Rosen (1976), Nakamura and Nakamura (1981), Wales and Woodland (1979) and more recent contributions by Kohlase (1986) and Ransom (1987). Only recently there have been attempts to take the non-convexity properties of the tax structure into account. These attempts are in most cases versions of the approach suggested by Burtless and Hausman (1978) (cf. Arrufat and Zabalza (1986), Blomquist (1983), Hausman (1980), (1981), (1985), Hausman and Ruud (1984) and the special issue of Journal of Human Resources, vol. 25, 3, 1990). In the Hausman procedure consistency of the maximum likelihood method requires that Slutsky restrictions are imposed (Kapteyn et al. (1990) and MaCurdy et al. (1990)) which can be difficult to do even with rather simple functional forms for the utility function. Another problem with the "Hausman-approach" is that in practice it turns out to be difficult to locate a maximum of the likelihood function, even with a convex budget set. A unique extremal point of the likelihood function cannot be guaranteed. In the case of modeling the behavior of married couples - which is the concern of the present paper - the "Hausman-approach" seems to give rather insurmountable calculation problems. This is especially the case if we allow for an exact representation of the budget set and a specification of preferences that allow for non-linear labor supply curves in marginal wage and virtual income.

A crucial assumption in most of the previous studies is that the individual is free to choose whether or not to work and the preferred hours of work. In most western European countries the observed hours of work often show extreme consentration around full time and part time hours. This indicates that important constraints on hours are present (see Dickens and Lundberg (1985)). Even if these peaks at full and part time hours are interpreted as being generated from preferences, the conventional approach with concave preferences will fail to explain the data unless a particular heterogeneity assumption is imposed.

The framework presented here is consistent with the notion of qualitative aspects of jobs as well as fixed hours of work determined by institutional regulations. Specifically, the choice environment is assumed to consist of a latent set of opportunities called matches. A match is defined as a particular combination of working conditions including hours, wages and non-pecuniary characteristics. The set of matches available to the individual is latent and individual specific. It is therefore perceived as random (to the econometrician). While previous empirical labor supply models interpret unobservable heterogeneity as generated from differences in preferences our framework also allows for unobserved heterogeneity in opportunities. Furthermore, the model allows for disequilibrium in which prices and wages are set so that unvoluntairy unemployment occurs.

The empirical part of our paper deals with labor supply of married couples in Norway in 1979 and 1986. We assume that the couple's decisions concerning labor supply are made simultaneously. All previous studies of labor supply with taxes, except for Hausman and Ruud (1984) and Ransom (1987), assume that the wife takes the husband's income as given. In a labor supply study without taxes Ashenfelter and Heckman (1974) have demonstrated that cross elasticities may be significant.

This paper is organized as follows. In the next section we present the individual decision model which includes a characterization of the stochastic properties of the unobservables. In Section 3 equilibrium conditions and the probability distribution function for the labor supply and the realized wage rate is derived for one-person households. In Section 4 the model is extended to two-person households. Data are described in Section 5. The data sets contain detailed information about taxes, transfers and incomes and they are based on filled-in tax returns that are checked and approved by local taxation authorities. Empirical results are presented in the remaining Sections 6-8.

2. THE MODEL

Contrary to the traditional approach in the analysis of labor supply (see Killingsworth (1983) for a review of models) we have adopted a theoretical framework in which the choice environment is assumed to consist of a latent random set of opportunities, called matches. Each match corresponds to a particular combination of skills required to perform certain tasks or activities, and characteristics of the job such as hours of work, wage rate and non-pecuniary attributes.

Our theoretical model is related to the matching models of Tindbergen (1956), Hartog (1978), Heckman and Sedlacek (1985) and Dickens and Lundberg (1985). The implied econometric model is particularly convenient for dealing with general budget constraints, flexible specification of preferences and restrictions on hours of work as well as joint labor supply decisions of husband and wife.

2.1. Preferences and pecuniary budget constraint

We assume that a match, z, can be characterized by a four-dimensional vector variable $(H(z), W(z), T_1(z), T_2(z))$. The attribute $T_1(z)$ is associated with variables that have a direct influence on pecuniary rewards as well as preferences while $T_2(z)$ is assumed only to affect the pecuniary rewards. The latter variable - $T_2(z)$ - thus reflects match-specific requirements which hardly can be by the econometrician. H(z) and W(z) are the hours of work and wage rate respectively, associated with the match. We assume that hours of work and the attributes, $T(z)=(T_1(z),T_2(z))$, are exogenously determined by the firms or by institutional regulations. Thus for a given individual the set of feasible attributes {H(z), T(z), z=1,2,...} is given in the short run. The determination of wage rates, {W(z)}, will be discussed in the next section.

Let U(C,h,z) denote the households utility function where C is annual disposable income, h is yearly hours of work and z indexes the match. For the moment we concentrate our discussion to one-person households. Conditional on match z the budget constraints are defined by

$$h = H(z), \tag{2.1}$$

and

$$pC \equiv pC(z) = f(H(z)W(z),I), \tag{2.2}$$

where f(·) is the function that transforms gross income into after-tax income (consumption) and p is the composite price index. The form of f(·) depends on the tax and social security system. Thus f(·) may have discontinuous jumps. For our analysis below it will be sufficient to assume that f(·) is piecewise continuous. The variable I is non-labor income. Eq. (2.1) means that given the match then hours of work follows.

The utility function is assumed to have the structure

$$U(C,h,z) = v(C,h,T_1(z)) + \varepsilon(z) \tag{2.3}$$

where v(·) is a deterministic function and ε(z) is a random variable. For given z, ε(z) reflects unobserved heterogeneity in tastes across individuals, and for a given individual, ε(z), z=1,2,... account for unobserved taste variations across matches. After the budget constraints (2.1) and

(2.2) are inserted in (2.3) the utility function takes the form

$$U(C(z),H(z),z) = v(C(z),H(z),T_1(z)) + \varepsilon(z). \tag{2.4}$$

In order to distinguish the attributes $\{T(z)\}$ from the interpretation of $\{\varepsilon(z)\}$ we may think of $\{T(z)\}$ as variables that are "objective" in the sense that in principle they can be measured and they have the same value relative to any individual. We might think of the T-attributes, at least $\{T_1(z)\}$, as variables that can be measured through survey questionaires or obtained from official statistics (see Hartog op.cit.). In contrast, the variable $\varepsilon(z)$ is a subjectively perceived, individual specific taste-shifter. It is of course difficult to make a sharp distinction between these variables.

2.2. The distribution of match attributes and taste shifters

Now let us introduce assumptions about the distribution of the attributes and the tasteshifters. Heterogeneity in opportunities implies that the set of feasible matches is perceived as random by the econometrician. The reason why the choice set is random to the observer is because he does not have the same information about the determinants of market opportunities as the individual. Thus while traditional econometric labor supply models only account for unobserved heterogeneity in preferences the approach proposed here focuses on heterogeneity both in opportunities and preferences. The problem is of course to obtain a convenient representation of these types of heterogeneity in a structural model.

As discussed above the market opportunities are represented by $\{H(z), T(z), z=1,2,...\}$. Specifically, we assume that the set of feasible attributes of the matches, and taste values, $\{H(z), T(z), \varepsilon(z)\}$, are random and generated by a Poisson law as follows: The values $\{H(z), T(z), \varepsilon(z)\}$ are assumed to be realizations of a Poisson process on $[0,\bar{h}] \times [0,1]^2 \times R$ with intensity measure

(2.5) $$\lambda(dh,dt\,|\,s)\cdot e^{-\varepsilon}d\varepsilon, \quad t = (t_1,t_2),$$

where $\bar{h}$ is an upper bound on hours and $\lambda(\cdot|s)$ is a finite measure that depends on the worker's experience and level of schooling, s (observable). The reason for conditioning on s will become clear below. Note that s characterizes the qualifications of the agent while $\{T(z)\}$ are match-specific attributes. The interpretation of the Poisson process on $(0,\bar{h}] \times (0,1]^2 \times R$ is analogous to the familiar Poisson process on R. Specifically, the points of this process are independent draws from a probability distribution, and the probability that a match for which

$$(H(z) \in (h,h+dh),\ T(z) \in (t,t+dt),\ \varepsilon(z) \in (\varepsilon,\varepsilon+d\varepsilon))$$

is equal to

$$\lambda(dh,dt|s)e^{-\varepsilon}d\varepsilon + o(dhdtd\varepsilon).$$

Moreover, the mean number of matches within $A \times [\varepsilon_1,\varepsilon_2]$, where

$$A = [h_1,h_2] \times [t_1,t_1'] \times [t_2,t_2'],$$

is equal to

(2.6) $$\Lambda(A\times[\varepsilon_1,\varepsilon_2]|s) \equiv \int_A \int_{\varepsilon_1}^{\varepsilon_2} \lambda(dh,dt_1,dt_2|s)e^{-\varepsilon}d\varepsilon = (e^{-\varepsilon_1}-e^{-\varepsilon_2})\int_A \lambda(dh,dt_1,dt_2|s).$$

Let $N(A\times[\varepsilon_1,\varepsilon_2])$ be the number of matches within $A \times [\varepsilon_1,\varepsilon_2]$. Then by the Poisson law

(2.7) $$P\{N(A\times[\varepsilon_1,\varepsilon_2]) = n\} = \frac{\Lambda^n(A\times[\varepsilon_1,\varepsilon_2]|s)}{n!}\exp(-\Lambda(A\times[\varepsilon_1,\varepsilon_2]|s))$$

which is the wellknown Poisson probability density in higher dimensions. Note that while H(z) and T(z) may be interdependent, $\varepsilon(z)$ is independent of (H(z),T(z)). As we shall discuss below this independence property is not essential for our economic interpretation. It follows from (2.6) that the mean total number of feasible matches, $\Lambda((0,\bar{h}]\times(0,1],R)$, is infinite. This property complicates the discussion on the interpretation of λ. The interpretation is however facilitated by restricting the set of feasible matches to those matches for which $\varepsilon(z)>b$ where b is a constant.

Now define

$$(2.8) \qquad \tilde{G}(h,t\,|\,s) = \frac{\Lambda((0,h]\times(0,t]\times[b,\infty)\,|\,s)}{\Lambda((0,\bar{h}]\times(0,1]\times[b,\infty)\,|\,s)} = \frac{\lambda(h,t|s)e^{-b}}{\lambda(\bar{h},1|s)e^{-b}}$$

where $[0,t] = [0,t_1]\times[0,t_2]$. The interpretation of $\tilde{G}(h,t|s)$ is as the mean number of feasible matches for which $(H(z)\leq h,T(z)\leq t,\varepsilon(z)>b)$ relative to the mean number of feasible matches with $\varepsilon(z)>b$.

By (2.8) it follows that

$$(2.9) \qquad \tilde{G}(h,t\,|\,s) = \frac{\lambda(h,t\,|\,s)}{\lambda(\bar{h},\mathbf{1}\,|\,s)}, \quad \mathbf{1} = (1,1)$$

which means that $\tilde{G}$ is independent of the threshold value b. We shall call $\tilde{G}(\cdot|s)$ the cumulative opportunity distribution (cf. Ben-Akiva et al. (1985)). The opportunity distribution $\tilde{G}(h,t|s)$ can be interpreted as the probability that a match for which $(H(z)\leq h,T(z)\leq t,\varepsilon(z)>b)$ (for any $b > -\infty$) is feasible to the individual. It follows from (2.7) that with probability one the number of feasible matches with $\varepsilon(z)>b$ is finite. Since b does not appear in (2.9), $\tilde{G}(\cdot|s)$ remains unchanged if we let $b\rightarrow-\infty$. Accordingly, it makes sense to interpret $\tilde{G}(h,t|s)$ as the mean number of points in the Poisson process for which $\{H(z)\leq h, T(z)\leq t\}$, relative to the mean numbers of points in the process.

The particular structure of the intensity measure introduced above is chosen for convenience, but it can also be given a choice theoretic justification, cf. Dagsvik (1991).

In the next section it will become clear that $\tilde{G}(\cdot|s)$ and $\lambda(\bar{h},\mathbf{1}|s)$ are needed to express the labor supply distribution. Let us therefore also discuss the interpretation of $\lambda(\bar{h},\mathbf{1}|s)$. To this end it is convenient to introduce a reference state for the sets of market opportunities and it is natural to choose the one where the attributes $\{H(z), T(z), z=1,2,...\}$ are uniformly distributed. Thus in this reference state the intensity measure takes the form

$$m\,dh\,dt_1\,dt_2\,e^{-\varepsilon}d\varepsilon$$

where m is a constant. The corresponding mean number of points for which $\varepsilon(z)>b$, equals

$$m\int_K dh\,dt_1\,dt_2\int_b^\infty e^{-\varepsilon}d\varepsilon = \bar{h}e^{-b}m$$

where $K = [0,\bar{h}] \times [0,1]^2$. Since the notion of reference state is somewhat arbitrary we may - without loss of generality - choose $m=1/\bar{h}$. From (2.6) it then follows that the mean number of feasible attributes for which $\varepsilon(z)>b$ equals $\lambda(\bar{h},1|s)e^{-b}$. Similarly, to (2.9) we may therefore interpret $\theta(s)$ defined by

$$\theta(s) \equiv \frac{\lambda(\bar{h},1|s)e^{-b}}{m\bar{h}e^{-b}} = \frac{\lambda(\bar{h},1|s)}{m\bar{h}} = \lambda(\bar{h},1|s)$$

as the mean number of feasible attributes relative to the mean total number of uniformly distributed attributes. We shall call $\theta(s)$ the opportunity measure of market matches - or simply the opportunity measure.

As demonstrated in appendix III the particular Poisson process formulation above implies that the distribution of the indirect utility function, given that the individual works, is extreme value distributed. It is therefore natural to postulate that the utility of non-participation also is extreme value distributed. Let z=0 index the non-market alternative for which we have $H(0) = T_1(0) = T_2(0) = 0$. This implies that

$$P(\varepsilon(0)\leq y) = \exp(-e^{-y}). \tag{2.10}$$

3. THE MARKET DISTRIBUTION OF WAGES AND HOURS OF WORK

In this section we shall consider the wage formation process and the distribution of wages and hours of work in the market. Our ambition is not to specify a full structural equilibrium model. The sole purpose is to derive an empirical labor supply model which is consistent with different stories about the market adjustment process - whether market clearing is attained or not.

3.1. The probability density of realized wages and hours of work

As already noted wages are match-specific and we shall assume that they depend on match z through the attributes $(H(z),T_1(z),T_2(z))$, i.e., for match z

$$W(z) = \tilde{w}(H(z),T_1(z),T_2(z)), \tag{3.1}$$

where $\tilde{w}(\cdot)$ is a function that specifies how labor is priced out. Recall that $T_2(z)$ is associated with match-specific skills. For given values of the other attributes, $H(z)$ and $T_1(z)$, (3.1) then says that labor is priced out according to these match-specific skills. Given reasonable assumptions about functional forms, (3.1) can be inverted to yield

$$T_2(z) = \tilde{t}_2(W(z),H(z),T_1(z)), \tag{3.2}$$

where $\tilde{t}_2(\cdot)$ is the function that solves $w = \tilde{w}(h,t_1,\tilde{t}_2(w,h,t_1))$. In other words, $\tilde{t}_2$ is the value of the match-specific skills that is consistent with the wage $\tilde{w}$ when $H(z)=h$ and $T_1(z)=t_1$. How the form of the wage-function is determined will be discussed below.

Let $\tilde{g}(h,t|s)$ denote the density of the cumulative distribution $\tilde{G}(h,t|s)$ in (2.9). From (3.1)-(3.2) we can derive the opportunity density, $g(\cdot|s)$, for the attributes $\{H(z), W(z), T_1(z)\}$. Provided $\tilde{t}_2(h,w,t_1)$ is differentiable with respect to w we get

(3.3)
$$g(h,w,t_1|s) = \tilde{g}(h,t_1,\tilde{t}_2(h,w,t_1)|s)\left|\frac{\partial \tilde{t}_2(h,w,t_1)}{\partial w}\right|.$$

The interpretation of (3.3) is as the density of observable match attributes, including wage, conditional on the set of feasible market opportunities.

The individual is assumed to choose a match, $\hat{z}$, that solves the utility maximization problem

(3.4)
$$\max_z\left[v(C(z),H(z),T_1(z)) + \varepsilon(z)\right] = v(C(\hat{z}),H(\hat{z}),T_1(\hat{z})) + \varepsilon(\hat{z}).$$

Recall that $\{\varepsilon(z)\}$ is unknown to the econometrician but known to the maximizing agent who is assumed to have a perfect knowledge of market opportunities while the econometrician only observe the realized values $\{C(\hat{z}),\ W(\hat{z}),\ T_1(\hat{z})\}$. Let $\Phi(\cdot|s)$ be the joint cumulative distribution of $\{H(\hat{z}),\ W(\hat{z}),\ T_1(\hat{z})\}$, i.e.,

(3.5)
$$\Phi(h,w,t_1|s) = P\Big\{\max_z[v(C(z),H(z),T_1(z)) + \varepsilon(z)] = v(C(\hat{z}),H(\hat{z}),T_1(\hat{z})) + \varepsilon(\hat{z})|s\Big\}_{H(z)\le h,\, W(z)\le w,\, T_1(z)\le t_1}.$$

The particular structure of the intensity measure (2.5) implies that the probability density, φ, of Φ has a simple functional form. The derivation of φ is completely analogous to de Haan (1984) and Dagsvik (1991) but since the argument is non-standard we provide a proof in appendix III. The resulting labor supply density has the form

(3.6)
$$\varphi(h,w,t_1|s) = \frac{\theta(s)(\exp(\psi(h,w,t_1)))\, g(h,w,t_1|s)}{\theta(s)\iiint(\exp(\psi(x,y,u)))\, g(x,y,u|s)\,dxdydu + \exp(\psi(0,0,0))}$$

for $h>0$, $w>0$, $t_1>0$ and

(3.7) $$\varphi(0,0,0|s) = \frac{\exp(\psi(0,0,0))}{\theta(s)\iiint(\exp(\psi(x,y,u)))\,g(x,y,u|s)\,dxdydu + \exp(\psi(0,0,0))}$$

where

$$\psi(h,w,t_1) = v(f(hw,I)/p, h, t_1).$$

From the discussion on page 10 we recall that $\theta(s)$ can be interpreted as a measure of the mean size of the set of feasible matches relative to the mean size of the set of uniformly distributed attributes in the reference state.

Eqs. (3.6) and (3.7) express the density of $(H(\hat{z}),W(\hat{z}),T_1(\hat{z}))$ in terms of the mean utility, the opportunity measure and the opportunity density. In other words, the choice density is consistent with the notion of <u>random, unobservable choice sets and preferences</u> where the preferences are represented by ψ and the choice opportunities are represented by $\theta(s)$ and $g(\cdot|s)$.

The model above is similar to the continuous logit type of model developed by Ben-Akiva et al. (1985). They obtain the continuous logit type of model as the limit obtained by letting the choice set of corresponding discrete model tend towards infinity. The advantage with the Poisson process setting is that we obtain the continuous choice model directly. More important, the present approach allows for the interpretation of random choice sets which is appealing since, most likely, individuals are heterogenous with respect to opportunities as well as preferences.

3.2. Equilibrium wage distribution

We now turn to the determination of the wage function; i.e., the form of $\tilde{w}(\cdot)$.

Note first that the density $g(h,w,t_1|s)$ can be decomposed as

(3.8) $$g(h,w,t_1|s) = g_1(h,t_1|s)\,g_2(w|h,t_1,s)$$

where $g_2(w|h,t_1,s)$ is the conditional (opportunity) density of offered wages given matches for which $(H(z)=h,\ T_1(z)=t_1)$ and the workers level of schooling and experience, s. The term $g_1(h,t_1|s)$ is the opportunity density of $\{H(z),T_1(z)\}$. We may think of $g_1(h,t_1|s)$ as the fraction of market matches for which a worker with skills s is qualified for. $g_1(h,t_1|s)$ is assumed to be determined by institutional regulations and firm-specific factors that are beyond the present paper to discuss. For expository reasons it is convenient to express the supply density in (3.6) as a functional of $\theta(s)g_1(\cdot|s)$ and $g_2(\cdot|\cdot,s)$, i.e.,

$$\text{(3.9)} \qquad \varphi(h,w,t_1|s) \equiv \mu(h,w,t_1;\theta(s)g_1(\cdot|s),g_2(\cdot|\cdot,s)).$$

where the functional form of μ is apparent from (3.6).

Let $\gamma(h,w,t_1|s)$ denote the probability density of labor demand, which corresponds to the supply density (3.6). The empirical counterpart of $\gamma(\cdot)$ is the number of jobs - with the specified attributes - offered by firms to workers with education and experience, s, relative to the total number of jobs in the economy.

In the case of flexible wages the density of supply, demand and realized match attributes must be equal, in which case the $\tilde{w}$-function - or equivalently - the density $g_2(w|h,t_1,s)$ is determined by

$$\text{(3.10)} \qquad \mu(h,w,t_1;\theta(s)g_1(\cdot|s),g_2(\cdot|\cdot,s)) = \gamma(h,w,t_1|s),$$

In fact, a definition of equilibrium similar to (3.10) is given by Tinbergen (1956). The model (3.6)-(3.7) is thus embedded in an equilibrium setting where labor is priced out according to match-specific skills (eq. (3.1)) and the wage opportunity density $g_2(w|h,t_1,s)$ is determined by (3.10) so that all markets clear.

As mentioned above the tasteshifters $\{\varepsilon(z)\}$ are not correlated with the match attributes. However, the distributional properties of the tasteshifters are reflected in the functional form of μ. If the sample is small, this means that the empirical distribution of supply does not necessarily coincide with the demand distribution. Thus the present equilibrium concept only provides market clearing in the sense that supply equals demand in a large sample.

3.3. Disequilibrium

The Poisson process framework also allows for a different wage formation process than the one adopted above. In the disequilibrium literature it is acknowledged that there often are severe market imperfections that may prevent markets from clearing. These market imperfections are related to the fact that contracts are costly to dissolve, learning is costly and takes time and unions control to a great extent the wage formation process and they do not always adjust their negotiation policy so as to fully accommodate for excess supply and demand.

Consider now the case when wages are rigid, i.e., the wage function $\tilde{w}(\cdot)$ is not determined so as to yield equilibrium. For simplicity we shall assume that the demand side is not rationed, i.e., the demanders are always able to realize their demand. Let $g_2^*(w|h,t_1,s)$ denote the corresponding conditional wage opportunity density. The supply density in this case thus takes the form

$$\mu(h,w,t_1;\theta(s)g_1(\cdot|s),g_2^*(\cdot|\cdot,s)) \tag{3.11}$$

which may differ from $\gamma(h,w,t_1|s)$ when g_2^* differs from g_2.

The distribution of the tasteshifters $\{\varepsilon(z)\}$ and the form of $v(\cdot)$ is assumed to be invariant with respect to different wage setting policies and consequently the form of $\mu(\cdot)$ will remain invariant with respect to changes in $\theta(s)$, $g_1(\cdot|s)$ and $g_2(\cdot|\cdot,s)$. Thus the parameters of $\mu(\cdot)$, or rather $\psi(\cdot)$, are deep structural parameters.

Since the supply now differs from the demand, the set of feasible matches no longer is represented by $\theta(s)g_1(\cdot|s)$. Specifically, some workers now will face quantity constraints in addition to the qualification requirements represented by $\theta(s)g_1(\cdot|s)$. Let $\theta^*(s)g_1^*(\cdot|s)$ represent the actual constrained distribution of feasible matches. Evidently, $\theta^*(s)g_1^*(\cdot|s)$ is determined from

(3.12)
$$\mu(h, w, t_1; \theta^*(s) g_1^*(\cdot|s), g_2^*(\cdot|\cdot, s)) = \gamma(h, w, t_1 | s).$$

The left hand side of (3.12) expresses the "constrained" supply density - or the density of realized attributes, $\{H(\hat{z}), W(\hat{z}), T_1(\hat{z})\}$ when the workers are facing choice sets of feasible matches that differ from the ones generated by $\theta(s)g_1(\cdot|s)$.

Let us summarize the difference between the stories in subsections 3.2 and 3.3. In subsection 3.2 each worker faces a distribution of matches for which he is qualified and the wage function $\tilde{w}(\cdot)$, or equivalently, the conditional wage opportunity density $g_2(\cdot|h,t_1,s)$ in (3.10) is set such that markets clear. In subsection 3.3, however, the wage opportunity density $g_2^*(\cdot|h,t_1,s)$ is exogenously given which implies that the supply density differs from the demand density. Consequently, some workers will be rationed and the actual set of feasible market matches is represented by $\theta^*(s)g_1^*(\cdot|s)$, which may differ from $\theta(s)g_1(\cdot|s)$.

Consider finally the relationship between the unemployment rate, and the densities of supplied and realized match attributes. Specifically, let $\rho(h,w,t_1|s)$ be defined as

(3.13)
$$\rho(h, w, t_1 | s) = \frac{\mu(h, w, t_1; \theta(s) g_1(\cdot|s), g_2^*(\cdot|\cdot, s)) - \mu(h, w, t_1; \theta^*(s) g_1^*(\cdot|s), g_2^*(\cdot|\cdot, s))}{\mu(h, w, t_1; \theta(s) g_1(\cdot|s), g_2^*(\cdot|\cdot, s))}.$$

The interpretation of (3.13) is as the unemployment rate for jobs with attributes $(H(z)=h,\ W(z)=w,\ T_1(z)=t_1)$, similarly to the standard definition of the unemployment rate. From (3.13) we obtain

(3.14)
$$\mu(h, w, t_1; \theta^*(s) g_1^*(\cdot|s), g_2^*(\cdot|\cdot, s)) = \mu(h, w, t_1; \theta(s) g_1(\cdot|s), g_2^*(\cdot|\cdot, s))(1 - \rho(h, w, t_1 | s)).$$

Eq. (3.14) is of interest for identification and estimation purposes. Provided data on unemployment, for each category of attribute combination, is available, then the parameters of the supply density $\mu(h,w,t_1;\theta(s)g_1(\cdot|s),g_2^*(\cdot|\cdot,s))$ rather than the constrained density $\mu(h,w,t_1;\theta^*(s)g_1^*(\cdot|s),g_2^*(\cdot|\cdot,s))$ can be estimated. A related expression for the labor supply densities is given by Blundell et al. (1987).

3.4. Further simplifications

For the sake of reducing the problems related to identification and estimation of structural parameters it is of interest to consider the structure of the model that follows when some further assumptions are maintained. Assume now that the opportunity density of hours of work and observable match attributes $\{T_1(z)\}$ does not depend on the workers skills s and that the wage W(z) only depend on z through T(z), i.e.,

(3.15) $$W(z) = \tilde{w}(T(z)).$$

Assumption (3.15) does not necessary mean that the match-specific wages and hours are uncorrelated. In fact, since T(z) and H(z) may be correlated across matches it follows that W(z) and H(z) also may be correlated. Assumption (3.15) simply states that T(z) is a sufficient statistics in the wage function $\tilde{w}(\cdot)$. Then it follows that the opportunity density of $\{H(z),W(z),T_1(z)\}$ can be written as

(3.16) $$g(h,w,t_1|s) = g_1(h,t_1)\,g_2(w|t_1,s)$$

where

(3.17) $$g_2(w|t_1,s) = \tilde{g}_2(\tilde{t}_2(w,t_1)|t_1,s)\left|\frac{\partial \tilde{t}_2(w,t_1)}{\partial w}\right|$$

and $\tilde{t}_2(w,t_1)$ is determined by

(3.18) $$w = \tilde{w}(t_1,\tilde{t}_2(w,t_1)).$$

From (3.16) and (3.6) it follows that the conditional density of $H(\hat{z})$ given $(W(\hat{z}),T_1(\hat{z}))$, and given that the individual works takes the form

$$(3.19) \qquad \varphi(h|w,t_1,s) \equiv \frac{\varphi(h,w,t_1|s)}{\int_{x>0}\varphi(x,w,t_1|s)dx} = \frac{(\exp(\psi(h,w,t_1)))g_1(h,t_1)}{\int(\exp(\psi(x,w,t_1)))g_1(x,t_1)dx}.$$

Similarly, the conditional density of $W(\hat{z})$ given $(H(\hat{z}),T_1(\hat{z}))$ and given that the individual works takes the form

$$(3.20) \qquad \varphi(w|h,t_1,s) = \frac{\varphi(h,w,t_1|s)}{\int\varphi(h,y,t_1|s)dy} = \frac{(\exp(\psi(h,w,t_1)))g_2(w|t_1,s)}{\int(\exp(\psi(h,y,t_1)))g_2(y|t_1,s)dy}.$$

From (3.20) we note that $\varphi(w|h,t_1,s)$ does not depend on the opportunity density $g_1(\cdot)$. Moreover, formula (3.20) illustrates the well-known selectivity bias problem in that the density of realized wages in general is different from the density of offered (feasible) wages, $g_2(w|t_1,s)$. Evidently, an analogous expression to (3.20) holds in the general case where $\varphi(\cdot|s)$ is given by (3.6). The particular assumptions maintained in the present subsection illustrate, however, an additional consequence of selectivity, namely that even if the offered wage density $g_2(w|t_1,s)$ does not depend on hours, the realized wage density, conditional on hours, depends on hours. An analogous observation can be made regarding the conditional offered and realized densities of hours in (3.19).

Finally, we will consider the special case that emerges when the preferences do not depend on $\{T_1(z)\}$, i.e.,

$$(3.21) \qquad v(C,h,t_1) \equiv v(C,h)$$

and in addition the offered wage distribution is independent of $\{T_1(z)\}$;

(3.22)
$$g_2(w|t_1,s) \equiv g_2(w|s).$$

Then (3.6) and (3.7) reduce to

(3.23)
$$\varphi(h,w|s) = \frac{\theta(s)(\exp(\psi(h,w)))g_1(h)g_2(w|s)}{\theta(s)\int\int(\exp(\psi(x,y)))g_1(x)g_2(y|s)dxdy + \exp(\psi(0,0))}$$

and

(3.24)
$$\varphi(0,0|s) = \frac{\exp(\psi(0,0))}{\theta(s)\int\int(\exp(\psi(x,y)))g_1(x)g_2(y|s)dxdy + \exp(\psi(0,0))}$$

where

(3.25)
$$g_1(h) = \int g_1(h,t_1)dt_1$$

and

(3.26)
$$\psi(h,w) = v(f(hw,I)/p,h).$$

In the empirical part of the paper we estimate a model version for two-person households that is analogous to (3.23) and (3.24). Moreover, unemployment will not be accounted for. The justification for these simplifications of the model are:

- It seems reasonable that hours of work are determined by institutional regulations and the nature of the jobs.
- We have no reliable observations of T_1-variables.
- The unemployment rate in Norway was less than 2 per cent both in 1979 and 1986 and thus close to full employment by international standards.

4. EXTENSION OF THE MODEL TO TWO-PERSON HOUSEHOLDS (MARRIED COUPLES)

The decision problem of a married couple is to determine jointly the labor supply of the wife and husband as well as the level of consumption of the household, subject to the budget and hours constraints.

Let $U(C(h_F,h_M,z)$ denote the household's utility function where h_F and h_M denote the wife's and husband's hours of work, respectively. C is total consumption of the household and $z = (z_F,z_M)$ indexes the matches of the wife, z_F, and husband, z_M, respectively.

The constraints are given by

$$(4.1) \qquad (h_F,h_M) = (H_F(z),H_M(z)),$$

$$(4.2) \qquad pC(z) = f(H_F(z)W_F(z),H_M(z)W_M(z),I)$$

where $H_F(z)$, $W_F(z)$, $H_M(z)$ and $W_M(z)$ are the match-specific hours of work and wages for the wife and for the husband, respectively. C is household consumption and I denotes capital income. $f(\cdot)$ is the function that transforms gross income into consumption. p is the price-index. As above $(T_F(z),T_M(z)) = (T_{1F}(z),T_{2F}(z),T_{1M}(z),T_{2M}(z))$ represents the match-specific attributes relative to wife and husband.

Under assumptions that are straight forward extensions of the assumptions of the preceding section we can write

$$(4.3) \qquad U(C(z),H_F(z),H_M(z),z)) = v(C(z),H_F(z),H_M(z),T_{1F}(z),T_{1M}(z)) + \varepsilon(z)$$

where $\{H_F(z),H_M(z),T_F(z),T_M(z),\varepsilon(z)\}$ is an enumeration of the points of a Poisson process with intensity measure

$$\lambda(dh_F,dh_M,dt_F,dt_M|s)e^{-\varepsilon}d\varepsilon, \quad t_F = (t_{1F},t_{2F}), \quad t_M = (t_{1M},t_{2M}), \quad s = (s_F,s_M).$$

We define $\theta_{11}(s)$ as the mean number of market matches that is feasible for the husband and for the wife. Furthermore, $\theta_{01}(s_F)$ is the mean number of market matches that is feasible for the female. $\theta_{10}(s_M)$ is defined analogously (by replacing wife and husband).

Let $G(w_F,w_M,h_F,h_M,t_{1F},t_{1M}|s)$ be the equilibrium opportunity distribution, i.e., the probability that a market match for which

$$(W_F(z)\leq w_F,\ W_M(z)\leq w_M,\ H_F(z)\leq h_F,\ H_M(z)\leq h_M,\ T_{1F}(z)\leq t_{1F},\ T_{1M}(z)\leq t_{1M})$$

is feasible. The density of $G(\cdot|s)$ corresponds to (3.3) in the case of single person households. Let $G_M(w_M,h_M,t_{1M}|s_M)$ be the probability that a match $(W_M(z)\leq w_M, H_M(z)\leq w_M, T_{1M}(z)\leq t_{1M})$ is feasible for the husband and let $G_F(w_F,h_F,t_{1F}|s_F)$ be the analogous opportunity distribution for the wife.

Assumptions that are completely analogous to those of Section 3 yield the densities of realized hours and wages for the couple.

We only state the choice probabilities in the case where the preferences are independent of $\{T_{1F}(z),T_{1M}(z)\}$, offered hours for the husband and the wife are independent, offered hours are independently distributed of other attributes and the wage opportunity density is independent of $\{H_F(z),H_M(z),T_{1F}(z),T_{1F}(z)\}$. This is completely analogous to the case considered in (3.23)-(3.24). We then get

(4.4)

$$\varphi(h_F,h_M,w_F,w_M|s) = \frac{\theta_{11}(s)(\exp(\psi(h_F,h_M,w_F,w_M)))g_{1F}(h_F)g_{1M}(h_M)g_2(w_F,w_M|s)}{D}$$

for $h_F>0$, $h_M>0$, $w_F>0$, $w_M>0$, where

(4.5) $$\psi(h_F,h_M,w_F,w_M) = v(f(h_F w_F,h_M w_M)/p,h_F,h_M)$$

and

(4.6)

$$D = \theta_{11}(s)\iiiint(\exp(\psi(x_1,x_2,y_1,y_2)))\,g_{1F}(x_1)\,g_{1M}(x_2)\,g_2(y_1,y_2|s)\,dx_1\,dx_2\,dy_1\,dy_2$$
$$+\ \theta_{10}(s_F)\iint(\exp(\psi(0,x_2,0,y_2,0)))\,g_{1F}(x_2)\,g_{2F}(y_2|s_F)\,dx_2\,dy_2$$
$$+\ \theta_{01}(s_M)\iint(\exp(\psi(x_1,0,y_1,0,0)))\,g_{1M}(x_1)\,g_{2M}(y_1|s_M)\,dx_1\,dy_1\ +\ \exp(\psi(0,0)).$$

Here $g_{2F}(w_F|s_F)$ and $g_{2M}(w_M|s_M)$ denote the respective conditional marginal opportunity (offered) wage densities, and $g_{1F}(h_F)$, $g_{1M}(h_M)$, the marginal opportunity hours densities. The densities that correspond to the respective corner solutions are analogous to (3.24).

Note that, a priori, the offered wage rates for wife and husband are allowed to be dependent. This is to account for correlation in unobserved qualification variables due to the fact that men with high education tend to marry women with high education.

5. DATA

The data are obtained from two different data sources with information about couples in Norway that are married (not cohabitating) through 1979 and 1986. The first source is based on a questionnaire and contains data on hours worked (by intervals), wage rates and socio-demographic variables such as the number and age of children and education level. The other source is based on filled in and approved tax reports and yields detailed information about reported incomes, legal deductions, taxes paid and transfer payments received. The two sets of data are linked on the basis of personal identification numbers. The Central Bureau of Statistics has been responsible for collecting and preparing the data sets. The data based on the tax reports have been used to check the answers on the wage rates and hours worked given in the questionaire. For around 90 per cent of those working in 1979 as well as in 1986 the reported wage rate has been used. Hours worked per year are obtained by dividing the reported labor income per year by the reported wage rate (or the predicted rate in about 10 per cent of the cases).

The sample selection rules are as follows. Only couples where the age of the husband is less than 66 years and the age of the wife is between 27 (25 in 1986) and 66 years are included. Those couples for which one or both spouses have entrepreneurial income that exceeds wage income are excluded. Couples for which the wife or husband have reported hours of work above 3600 hours per year are excluded. When the reported female wage rate is below 15 or above 56 NOK (1979) it is predicted by a wage equation. The same procedure is followed when the male wage rate is below 24 or above 74 NOK (1979). Not working is defined as working less than or equal to 60 hours per year. An analogous procedure is followed for 1986.

In Table I-79 we report sample statistics for some selected variables in 1979.

(Table I-79)

(Table I-86)

From Table I we realize that wage rates for females are on average 75 per cent of the wage rates for males. Female hours of work are on average only 50 per cent of the male hours of work. Full-time and part-time hours of work-intervals are defined as follows: Full-time hours for males is [2120,2200], for females, [2000,2080], and part-time for females is [1020,1260]. These definitions are based on observed full-time and part-time peaks in these intervals. Table I-86 gives the corresponding sample statistics for 1986. The consumer price index is used as the composite price index with 1979 as the reference year. The level in 1986 is 1.83.

6. EMPIRICAL SPECIFICATION AND ESTIMATION RESULTS

In the present section we report estimation results. As noted in Sections 2 and 3 we have estimated the model conditional on household characteristics that are supposed to influence the preferences and the qualifications. The variables that are supposed to influence preferences are age and number of children in the household less than 6 and above 6 years. Variables that measure qualifications are length of schooling and experience defined as age minus length of schooling minus 6.

In the version estimated here we have not taken into account the heterogeneity on the demand side with respect to job types, represented in the model by $\{T_1(z)\}$. Since we believe that the qualitative aspects of the jobs is very important this means that the quality of the present empirical study is reduced.

In order to estimate the model we need to specify functional forms for $v(C,h_F,h_M)$, $g_{1F}(h)$, $g_{1M}(h)$, $g_2(w_F,w_M|s)$, $\theta_{11}(s)$, $\theta_{10}(s_F)$ and $\theta_{01}(s_M)$. Recall that we do not estimate the true structural parameters of an equilibrium wage density and as a consequence we are therefore not able to apply the estimated model to perform policy experiments that would require estimates of a full structural equilibrium model. We have chosen $v(C,h_F,h_M)$ to be a Box-Cox type function, separable in leisure and consumption. This specification allows for fairly flexible income and substitution elasticities (given separability). Thus

(6.1)

$$v(C,h_F,h_M) = \alpha_2\left(\frac{(10^{-4}C)^{\alpha_1}-1}{\alpha_1}\right) + \left(\frac{L_M^{\alpha_3}-1}{\alpha_3}\right)(\alpha_4 + \alpha_5 \log A_M + \alpha_6(\log A_M)^2)$$
$$+ \left(\frac{L_F^{\alpha_7}-1}{\alpha_7}\right)(\alpha_8 + \alpha_9 \log A_F + \alpha_{10}(\log A_F)^2 + \alpha_{11}CU6 + \alpha_{12}CO6) + \alpha_{13}L_F L_M$$

where A_F,A_M are the age of the wife and the husband, respectively, CU6 and CO6 are number of children less than 6 and above 6 years, L_K is leisure for gender k=M,F, defined as

$$L_K = 1 - h_K/8760,$$

and α_j, j = 1,2,...,13, are unknown parameters.

If $\alpha_1<1$, $\alpha_3<1$, $\alpha_7<1$, $\alpha_2>0$,

$$\alpha_4 + \alpha_5 \log A_M + \alpha_6(\log A_M)^2 > 0,$$

and

$$\alpha_8 + \alpha_9 \log A_F + \alpha_{10}(\log A_F)^2 + \alpha_{11} CU6 + \alpha_{12} CO6 > 0$$

then $v(C,h_F,h_M)$ is increasing in C, decreasing in (h_F,h_M) and strictly concave in (C,h_F,h_M).

The densities of offered hours, $g_{1F}(h_k)$, k=F,M, are assumed uniform except for a peak at full-time hours for males and peaks at full-time and part-time hours for females. The offered wage densities are specified as a joint lognormal density with means

$$E \log W_k(z) = \beta_{0k} + \beta_{1k} s_{1k} + \beta_{2k} s_{2k} + \beta_{3k} s_{2k}^2 \tag{6.2}$$

k=F,M, where s_{1k} denote years of schooling, gender k, and s_{2k} = experience = A_k-s_{1k}-6. It should be noted that $\{W_k(z)\}$ are nominal wages and hence the estimate of β_{0k} will reflect the nominal changes from 1979 to 1986. Moreover, the opportunity measures $\theta_{10}(s_F)$, $\theta_{01}(s_M)$ and $\theta_{11}(s)$ are parametrized as

$$\log\left(\frac{\theta_{10}(s_F)}{\theta_{11}(s)}\right) = \alpha_{14} + \alpha_{15} s_{1F}, \tag{6.3}$$

$$\log\left(\frac{\theta_{01}(s_M)}{\theta_{11}(s)}\right) = \alpha_{16} \tag{6.4}$$

and

$$-\log(\theta_{11}(s)) = \alpha_{14} + \alpha_{15} s_{1F} + \alpha_{16} + \alpha_{17}. \tag{6.5}$$

The parameters to be estimated are α_j, j=1,2,...,17, β_{jk}, j=1,2,3, the covariances in the wage density, and coefficients that correspond to full-time and part-time hours (three coefficients).

Above we assumed that the opportunity distributions for hours were uniform except for full-time and part-time peaks. Unless this or analogous assumptions are made it is not possible to separate some of the structural coefficients in the mean utility function from the parameters of the opportunity densities for hours.

It is of interest to note that since the logarithm of the opportunity density of hours and the utility function enter symmetrically into (4.4) it would be possible to interpret the peaks as stemming from preferences in which case the offered hours would be generated by a uniform distribution. In fact, if preferences and the opportunity density of hours are kept fixed we can perform policy simulations with respect to changes in demographic variables, taxes and wage rates based on the estimated model that are consistent with either interpretation.

The estimation is based on a procedure suggested by McFadden (1978), cf. Ben-Akiva and Lerman (1985), which yields results that are close to the full information maximum likelihood method. We are not able to use the exact likelihood function to estimate the model because the evaluation of the integrals in (4.6) would be too costly and cumbersome. The estimation procedure applied replaces the continuous four-tuple integral in the denominators of the densities by a sum over 30, (alternatively 70), random points, where each term is adjusted by appropriate weights. In other words, the continuous logit model is replaced by a discrete logit version. McFadden has demonstrated that this method yields consistent and asymptotically normal parameter estimates. We found McFadden estimation procedure to be remarkably efficient. Our experience suggests that it is enough to replace the choice set by 10 random points (draws in R_+^4) to obtain good results. When the number of draws increases to 30 then the estimated standard errors seem to be close to the corresponding ones obtained by the full maximum likelihood procedure.

The estimation results are reported in Table II and III.

(Table II)

(Table III)

Note that most parameters are rather precisely determined (apart from the cross leisure term which is set equal to zero in 1986) and they have the theoretically expected signs.

If we compare the estimates from 1979 with those from 1986 we find that the estimates of α_2, α_7 and α_{11} have changed slightly in the sense that the respective estimated differences have 95 per cent confidence intervals that not include zero. The most important differences is found to be in the parameter associated with small children. It is reasonable to interpret this result as a consequence of the increased availability in childcare opportunities from 1979 to 1986.

The estimates imply that the mean utility function is an increasing and strictly concave function in consumption and leisure. The males marginal mean utility of leisure attains a minimum at the age of 41.9 years and in the case of females, at the age of 35 years (1979) and at the age of 38.5 and 33.8 in 1986. The wife's education turns out to affect the fraction of feasible market matches such that a higher educated woman has more job opportunities than a less educated one. (Implied by $\hat{\alpha}_{15}<0$.)

Table III demonstrates that the selectivity bias is rather small; i.e., the OLS estimates (only 1979 estimates are reported) are close to the maximum likelihood estimates. Preliminary estimation results showed that the offered male and the female wage rates were close to be independent. For computational convenience the joint wage density therefore is assumed to factor, i.e., $g_2(w_F,w_M|s) = g_{2F}(w_F|s_F)g_{2M}(w_M|s_M)$. The mean wage rate is found to be a concave function of experience with a peak at 32.3 years of experience for men and at 30.8 years for women (1979). The corresponding figures for 1986 are 34.6 and 36.8. Moreover, it seems that education has a stronger impact on female wage- than on male wage rates.

It is interesting to note that the parameter estimates $\hat{\alpha}_{14}$ and $\hat{\alpha}_{16}$ associated with the female and male opportunity measures have decreased considerably from 1979 to 1986. This means that the corresponding opportunity measures have increased in this period. This finding accords well with stylized facts about the Norwegian economy. According to these facts the Norwegian economy experienced a boost in consumption and labor demand in the mid 1980s. Thus the parameter estimates of the model are consistent with a story where the increasing level of employment in the period 1979-1986 is due mainly to increased labor demand. In conventional labor supply models, without opportunity measures, the increase in labor supply would be interpreted as resulting from a shift in preferences. Thus our empirical results demonstrate the importance of allowing for an econometric specification that accounts for aspects of the jobs other than wages and hours of work.

It can be demonstrated that the following simulation procedure of the model is consistent and rather efficient.

First draw whether a market match is feasible for one or both adults of the households according to probabilities $\theta_{11}(s)d$, $\theta_{10}(s)d$ and $\theta_{01}(s)d$ where $(1+\theta_{01}(s)+\theta_{10}(s)+\theta_{11}(s))d=1$. Second, draw n points (say),

$$\{H_F(z), H_M(z), W_F(z), W_M(z), \varepsilon(z)\}, \quad z = 1,2,...,n.$$

Here $\{H_F(z)\}$ and $\{H_M(z)\}$ are drawn from uniform distributions with full- and part-time peaks, $\{W_F(z)\}$ and $\{W_M(z)\}$ are drawn from lognormal distributions according to the wage equations and $\{\varepsilon(z)\}$ are drawn from the extreme value distribution, $\exp(-e^{-\varepsilon})$. Third, find the realized hours and wages $(H_F(\hat{z}),H_M(\hat{z}),W_F(\hat{z}),W_M(\hat{z}))$ by maximizing

$$\psi(H_F(z), H_M(z), W_F(z), W_M(z)) + \varepsilon(z)$$

with respect to z=1,2,...,n. Repeat this procedure for every household in the sample. When n is large this procedure yields results that are close to an "exact" simulation of the model.

The simulation procedure we have followed in the present paper is a refinement of the one described above and it is unbiased for finite n and more efficient. This refined procedure will be described and analyzed elsewhere.

Figures 1 and 2 give the observed and simulated distributions for hours of work in 1979. These figures demonstrate that the model is able to reproduce the observed distributions quite well. (Figures for 1986 give a similar conclusion.) For females the fit is remarkably good while for males the model is less able to reproduce hours of work at the right hand tail. The explanation of these high observed hours of work may (in addition to possible measurement errors) be overtime and/or particular jobs with high workload.

[Figure 1]

[Figure 2]

7. SPECIFICATION TESTS

The structure of the econometric model implies that the conditional density given that the realized hours and wages fall within a given region, $K \in R_+^4$, has the same structure as the unconditional one. This property is essential for the McFadden estimation procedure. Formally, the conditional density is defined by

$$(7.1) \qquad \varphi(h_F, h_M, w_F, w_M | s, K) = \frac{\varphi(h_F, h_M, w_F, w_M | s)}{\int_K \varphi(x_1, x_2, y_1, y_2 | s) dx_1 dx_2 dy_1 dy_2}.$$

Thus since (7.1) has the "same structure" as the unconditional density it is easy to carry out estimation on the basis of subsamples (K) through the corresponding conditional likelihood. If the theoretical model is correctly specified the estimates based on different subsamples should not be significantly different.

The procedure developed by Hausman and McFadden (1984) (HM) is designed to test this kind of hypothesis. We have carried out 14 estimation runs in which different subsamples have been selected each time from the 1979 sample. In each case we have computed the Chi-square statistics, Q_K, given by HM, that corresponds to the difference between the estimates based on the respective subsample indexed by K and the estimate based on the whole sample. These subsamples have been selected as follows: The first 7 ones were selected by removing those couples with hours of work within [3000,3600], [2400,3000], [1800,2400], [1200,1800], [600,1200], [60,600], [0,60], respectively. The next 7 subsamples were obtained by removing couples for which only the <u>female</u> hours of work were within the above intervals, respectively. According to HM, Q_K has two degrees of freedom. To determine the joint significance level for all 14 cases we have applied the Bonferoni method. By the Bonferoni method the 0.05 critical value is about 11.3. All 14 values of Q_K are found to be less than 11.3.

8. WAGE AND INCOME ELASTICITIES

In labor supply studies it is common to report individual elasticities for mean sample values, or for subsamples of individuals who are grouped according to some socio-demographic characteristics. This is meaningful when the error terms are assumed to be independent of C and h. This is not the case in our model since, for given $H_F(z)=h_F$, $H_M(z)=h_M$, $w_F(z)=w_F$, $w_M(z)=w_M$, the utility function is given by

$$\psi(h_F, h_M, w_F, w_M) + \varepsilon(\bar{z}_F(h_F, w_F), \bar{z}_M(h_M, w_M)) \tag{8.1}$$

where $\bar{z}_j(h_j,w_j)$, j=F,M, denotes the match with hours and wage equal to h_j and w_j.

The mean utility, $\psi(\cdot)$, is the utility concept that comes closest to the one used by others in the calculation of elasticities. With these reservations in mind we have calculated local elasticities based on ψ for the "mean sample household" given that the couple works.

To calculate these elasticities one has to assume that the labor supply of the mean sample household can be simulated by maximizing the deterministic part of the utility function under the constraint of a linearized version of the budget constraint.

Of course, this assumption is at odds with the assumptions underlying the model outlined above. But it is done here to present results that to some extent can be compared with the results reported by others. Therefore, let m_j denote the marginal wage rate (gross wage rate after marginal tax rate) for spouse j around the optimal point of adjustment for the mean sample household. And let $\tilde{I}$ denote the virtual income which means that the linearized budget constraints can be written

$$C = \sum_j m_j h_j + \tilde{I}. \tag{8.2}$$

Maximizing $v(C,h_F,h_M)$ in (6.1) w.r.t. h_j, given (8.2), with m_j and $\tilde{I}$ considered as given yields the following first order conditions, (8.3) $$m_j = -\frac{\partial v/\partial h_j}{\partial v/\partial C}; \quad j = M,F.$$

From (8.2) and (8.3) "optimal values" for h_j follows. Applying the estimated utility function and mean sample values for m_j and $\tilde{I}$ for a household where both spouses work (m_M=19.01, m_F=19.80 and $\tilde{I}$=30729) give the result (1979) that {h_F=1307, h_M=2289, C=101000}.

The female's labor supply is close to the observed mean sample value (1312) and the male's is also pretty close (2218), which also imply that the simulated consumption does not deviate too much from the observed value (96 500), (1979).

From (8.2), (8.3) and from the estimated utility function one can derive exact formulas for labor supply elasticities. We have computed

- uncompensated marginal wage elasticities; hours h_j with respect to marginal wage rates m_j (Cournot elasticities)
- virtual income elasticities; hours h_j w.r.t. $\tilde{I}$
- compensated or utility constant elasticities (Slutsky elasticities)
- consumption constant elasticities (Frisch elasticities).

In the derivation of the formulaes below we have set the cross leisure term α_{13} equal to 0.

The elasticity formulas for the females are given in Appendix II. The elasticities for the males are completely analogous.

These elasticities are denoted mean utility elasticities and they are reported in Table IV for the selected sample means.

(Table IV-79)

(Table IV-86)

We see from Table IV that the own-wage elasticities are rather high. Females seem to be more wage-responsive than males and the Slutsky-elasticities show that substitution effects are strong, especially for women. For females all wage elasticities have numerically been reduced from 1979 to 1986.

Another set of elasticities arise when we consider how the mean in the distribution of labor supply is affected by changes in say, wage levels. These elasticities are denoted aggregate ones since they take into account unobserved and observed heterogeneity in the population as well as the non-convexity of the budget constraint and hours restrictions.

(Table V)

The aggregate elasticities are calculated as follows: The model is used to simulate (stochastic simulations) the labor supply for each household (wife and husband) under the current regime and when the mean wage rate is increased by 10 per cent. The aggregate elasticity of, for example, female labor supply is obtained by calculating the relative change in the mean (mean over all females) female labor supply that results from this wage increase. These elasticities are reported in Table V.

The results show that female participation is slightly more elastic than hours supplied, conditional on working in 1979, while the opposite is true in 1986. For men the differences in elasticities are quite smaller. Moreover, hours supplied by men, conditional on working, is almost inelastic.

The elasticities in the second line of Table V can be compared with the Cournot elasticities given in Table IV. We observe that the cross terms are of the same sign, and they are not so very different in size. Since the aggregate elasticities are the ones that are consistent with the model, we observe that the mean utility elasticities overestimate the own-wage response among women to a great extent.

The last line of Table V is equal to the sum of the first and second line, since for each individual the unconditional expectation equals the product of the probability of working and the expected hours worked, conditional on working. The total supply elasticities imply that a 10 per cent increase in the mean of the male wage rates will increase expected hours supplied by men by 4.5 per cent and reduce expected hours supplied by women by 5.1 per cent (1979). The corresponding estimates for 1986 are 3.1 and 2.4.

Women are more responsive than men since a 10 per cent increase of the female wage rates will increase expected hours supplied by 18.2 per cent. The corresponding negative cross-effect for males labor supply is rather weak, 1.5 per cent (1979). Th estimates for 1986 are numerically quite lower, 9.2 and 0.8 respectively.

The impact of an overall wage increase of 10 per cent can be found by adding own-wage and cross-wage elasticities. The positive impact of an overall wage increase on female labor supply is substantially lower than a partial increase, a fact that should be kept in mind when cross-section estimates are compared with estimates based on partial time-series studies in which, typically, male and female wage rates grow almost at the same rate.

CONCLUSION

The difficulties with the current econometric models of labor supply are their inadequacy to deal with; (i) Exact representation of complicated budget sets, for example separate taxation for some income combinations and joint taxation for other combinations, (ii) latent constraints on hours of work and qualitative job attributes, (iii) general specifications of the utility function.

The methodology presented in the present paper has proved to be practical for dealing with (i) through (iii). While the Hausman methodology is operational for simple labor supply curves with piecewise linear non-convex budget sets it is hard to apply this procedure in general.

A particular empirical version of the model has been estimated on samples of Norwegian married couples where neither spouse has main income from selfemployment. The empirical specification is based on a Box-Cox type utility function which is separable in consumption and leisure, globally concave and allows for flexible patterns of substitution and income effects.

The empirical results demonstrate that the model reproduces the (unconditional) distribution for labor supply of males and females quite well and the parameters of the utility function are found to be rather stable from 1979 to 1986 except for the effect of small children which has been reduced. Thus, the observed increase in employment and hours worked in this period is not explained by a shift in preferences but due to an increase in market opportunities. This finding accords well with the fact that the Norwegian economy experienced a boost in consumption and labor demand in the mid 1980s.

The wage elasticities of the mean labor supply turn out to be rather high for females. From 1979 to 1986 all wage elasticities have been reduced which is explained by the increased working loads.

Table I-79. Sample Values - Married Couples (778 households)

	Averages	Standard deviations	Min. values	Max. values
Hours worked per year by wife	919	859	0	3 368
Hours worked per year by husband	2 059	740	0	3 572
Female gross wage rate, NOK per hour (among those who work)	31.30	6.10	15.50	55.80
Male gross wage rate, NOK per hour (among those who work)	41.60	9.4	24.00	73.90
Female gross labor income, NOK per year	30 021	29 914	0	152 497
Male gross labor income, NOK per year	84 911	35 701	0	185 988
Consumption (disposable income)	88 739	26 191	20 554	222 325
Wife's education in years	10.5	1.7	9.0	17.5
Husband's educations in years	11.4	2.5	9.0	18.0
Age of the wife	43.6	11.3	27	66
Age of the husband	46.1	11.5	25	66
Number of children below 6	0.36	0.66	0	4
Number of children 7-20	1.01	1.55	0	6
Female participation rate, per cent	70.3	-	-	-
Male participation rate, per cent	92.8	-	-	-

Table I-86. Sample Values - Married Couples (1 518 households)*

	Averages	Standard deviations	Min. values	Max. values
Hours worked per year by wife	1 133	812	0	3 459
Hours worked per year by husband	1 958	749	0	3 551
Female gross wage rate, NOK per hour (among those who work)	35.52	9.29	14.20	92.90
Male gross wage rate, NOK per hour (among those who work)	46.45	12.57	20.77	108.20
Female gross labor income, NOK per year	39 962	30 538	0	163 101
Male gross labor income, NOK per year	91 284	42 855	0	262 273
Consumption (disposable income)	109 545	32 254	20 296	217 911
Wife's education in years	10.4	2.4	7.0	18.0
Husband's educations in years	11.2	2.9	7.0	20.0
Age of the wife	41.4	11.1	25	66
Age of the husband	45.2	11.7	25	66
Number of children below 6	0.41	0.70	0	4
Number of children 7-20	0.60	0.86	0	6
Female participation rate, per cent	81.8	-	-	-
Male participation rate, per cent	94.5	-	-	-

* Wage levels and incomes are in 1979 prices; i.e. nominal 1986-values are deflated by 1.83.

Table II. Estimates of the parameters of the utility function and of the opportunity density*)

Variables	Coefficients	Estimates 1979		Estimates 1986	
Consumption	α_1	0.916	(22.4)	0.951	(16.4)
	α_2	1.997	(9.7)	1.269	(5.6)
Male leisure	α_3	-4.782	(6.7)	-4.312	(6.8)
	α_4	76.657	(2.4)	100.598	(3.0)
	α_5	-38.532	(2.3)	-53.091	(3.0)
	α_6	5.159	(2.3)	7.270	(3.0)
Female leisure	α_7	-0.961	(2.8)	-2.240	(5.5)
	α_8	258.717	(2.7)	237.438	(3.9)
	α_9	-136.624	(2.7)	-130.174	(3.9)
	α_{10}	19.208	(2.8)	18.492	(4.1)
	α_{11}	6.573	(8.1)	3.397	(6.4)
	α_{12}	1.967	(5.5)	1.648	(4.8)
Leisure interaction term	α_{13}	-3.869	(0.8)	0	
Female opportunity measure	α_{14}	1.164	(1.6)	0.063	(0.1)
	α_{15}	-0.208	(3.0)	-0.203	(3.7)
Male opportunity measure	α_{16}	2.414	(7.4)	-3.296	(4.5)
Interaction	α_{17}	1.841	(4.6)	1.289	(4.5)
Full-time peak, males	α_{18}	0.765	(5.8)	1.062	(11.2)
Full-time peak, females	α_{19}	0.868	(4.2)	0.710	(5.8)
Part-time peak, females	α_{20}	0.407	(3.0)	0.425	(2.5)

*) t-values in parenthesis.

Table III. Wage opportunity density: Simultaneous ML estimation procedure versus OLS*)

	Males			Females		
	OLS	Simultanous ML		OLS	Simultaneous ML	
	1979	1979	1986	1979	1979	1986
Intercept	3.036	2.738	3.545	2.657	2.679	3.359
	(84.8)	(44.4)	(56.2)	(53.5)	(36.6)	(47.7)
Education	0.036	0.042	0.036	0.051	0.051	0.043
	(13.7)	(14.5)	(11.4)	(13.1)	(11.5)	(10.5)
Experience	0.018	0.020	0.018	0.018	0.008	0.014
	(9.7)	(6.1)	(5.3)	(8.8)	(2.3)	(4.0)
(Experience squared). 10^{-2}	-0.036	-0.031	-0.026	-0.030	-0.013	-0.019
	(8.1)	(5.7)	(4.6)	(7.5)	(1.9)	(3.0)
Standard error		0.178	0.207		0.171	0.206
		(39.0)	(39.7)		(34.0)	(38.5)
R^2	0.23			0.23		

*) t-values in parenthesis.

Table IV-79. Mean utility/mean sample elasticities. Elasticities of hours with respect to marginal wage levels and virtual income for those couples who work. Here C=101000, m_M=19.00, m_F=19.80, h_M=2290, h_F=1300.

Type of elasticity	Males		Females	
	Own	Cross	Own	Cross
Cournot	0.46	-0.04	2.69	-0.15
Slutsky	0.48	-0.03	2.75	-0.05
Virtual income	-0.01		-0.07	
Frisch	0.49	0.00	2.92	0.00

Table IV-86. Mean utility/mean sample elasticities. Elasticities of hours with respect to marginal wage levels and virtual income for those couples who work. Here C=124500, m_M=25.50, m_F=23.10, h_M=2300, h_F=1400.*

Type of elasticity	Males		Females	
	Own	Cross	Own	Cross
Cournot	0.51	-0.02	1.57	-0.06
Slutsky	0.52	-0.01	1.59	-0.02
Virtual income	-0.01		-0.02	
Frisch	0.53	0.00	1.62	0.00

* Income and wage rates are in 1979 prices.

Table V. Aggregate labor supply elasticities

Type of elasticity	Year	Male elasticities		Female elasti- cities	
		Own wage elast.	Cross elast.	Own wage elast.	Cross elast.
Elasticities of the probability of participation	1979 1986	0.29 0.17	-0.08 -0.03	0.83 0.37	-0.25 -0.12
Elasticities of conditional ex- pectation of total supply of hours	1979 1986	0.16 0.11	-0.07 -0.05	0.99 0.54	-0.26 -0.12
Elasticities of unconditional expectation of total supply of hours	1979 1986	0.45 0.31	-0.15 -0.08	1.82 0.92	-0.51 -0.24

APPENDIX I

Table AI: Effective total marginale tax rates for a married woman age 35, one child under 13 years of age, her wage rate is NOK 40 per hour and her husband's income is NOK 75 000 per year, near average sample values. Norway 1979

Gross income NOK per year	Hours worked per year	Effective marginal tax rates per cent	Remarks
0 - 2 000	0 - 50	0	
2 000 - 9 000	50 - 225	23.0	
9 000 - 9 500	225 - 238	48.0	
9 500 - 10 000	238 - 250	63.4	
10 000 - 10 833	250 - 270	61.9	Joint taxation minimizes taxes
10 833 - 11 250	270 - 281	66.6	paid by the husband
11 250 - 16 000	281 - 400	46.7	
16 000 - 17 500	400 - 438	42.3	
17 500 - 22 777	438 - 570	44.1	
22 777 - 24 313	570 - 607	49.1	
24 313 - 31 000	607 - 775	29.7	
31 000 - 38 800	775 - 970	32.4	Separate taxation minimizes
38 800 - 47 800	970 - 1 195	38.4	taxes
47 800 - 64 800	1 195 - 1 620	43.4	
64 800 - 75 000	1 620 - 1 877	48.4	
75 000 - 75 941	1 877 - 1 899	43.6	Separate taxation, but capital
75 941 - 87 705	1 899 - 2 192	49.0	income and capital expenses
87 705 - 99 470	2 192 - 2 487	54.3	are allocated to the wife's
99 470 - 119 470	2 487 - 2 987	58.8	income and expences
119 470 - 154 764	2 987 - 3 869	63.3	
154 764 - 182 400	3 869 - 4 560	66.8	
182 400 - 213 588	4 560 - 5 339	61.8	Hours worked per year not
213 588 - 331 235	5 339 - 8 280	65.4	feasible
331 235 - 350 400	8 280 - 8 760	67.2	

APPENDIX II. Mean utility elasticities, females

Cournot elasticities:

$$\frac{\partial \log h_F}{\partial \log m_F} = \frac{8760 L_F}{N(1-\alpha_7)h_F}\left(1 - \frac{(1-\alpha_1)h_F m_F}{C} + \frac{8760(1-\alpha_1)L_M m_M}{C(1-\alpha_3)}\right),$$

$$\frac{\partial \log h_F}{\partial \log m_M} = -\frac{8760 L_F(1-\alpha_1)}{N(1-\alpha_7)h_F C}\left(h_M m_M + \frac{8760 L_M m_M}{1-\alpha_3}\right).$$

Slutsky elasticities:

$$\left.\frac{\partial \log h_F}{\partial \log m_F}\right|_{\text{const. utility}} = \frac{8760 L_F}{N(1-\alpha_7)h_F}\left(1 + \frac{(1-\alpha_1)8760 L_M m_M}{(1-\alpha_3)C}\right),$$

$$\left.\frac{\partial \log h_F}{\partial \log m_M}\right|_{\text{const. utility}} = -\frac{8760^2 L_F L_M (1-\alpha_1) m_M}{N(1-\alpha_3)(1-\alpha_7)h_F C}.$$

Virtual income elasticity:

$$\frac{\partial \log h_F}{\partial \log \tilde{I}} = -\frac{8760 L_F (1-\alpha_1)\tilde{I}}{N(1-\alpha_7)h_F C}.$$

Frisch elasticities:

$$\left.\frac{\partial \log h_F}{\partial \log m_F}\right|_{\text{const.C}} = \frac{8760 L_F}{(1-\alpha_7)h_F}, \quad \left.\frac{\partial \log h_F}{\partial \log m_M}\right|_{\text{const.C}} = 0$$

where

$$N = \left(1 + \frac{(1-\alpha_1)}{C}\left(\frac{8760 L_M m_M}{1-\alpha_7} + \frac{8760 L_F m_F}{1-\alpha_3}\right)\right).$$

Strict concavity implies positive direct Slutsky and Frisch elasticities.

With a zero cross-leisure term in the utility function the cross Frisch elasticities equal zero. The cross Slutsky-elasticities as well as the cross Cournot are negative. The model allows for backward bending supply curves, that is, negative direct Cournot elasticities. A necessary condition for negative direct Cournot elasticity for females is

$$1 - (1-\alpha_1)\frac{m_F h_F}{C} + \frac{1-\alpha_1}{1-\alpha_3} \cdot \frac{m_M L_m \cdot 8760}{C} < 0.$$

We observe that a sufficient, but not necessary, condition for positive direct Cournot-elasticities is positive α_1; i.e., $0 \leq \alpha_1 \leq 1$. Since we have estimated α_1 to be positive, both direct Cournot elasticities are always positive.

APPENDIX III. Derivation of choice probabilities

Consider the following choice problem. For expository simplicity we shall discuss the case with a single "objective" attribute and a tasteshifter associated with each alternative. The general case with several attributes is completely analogous. The agent's choice set is a realization of a Poisson process on [0,1]×R where each point is a choice alternative. Thus to each point z there are two (continuous) attributes (T(z), ε(z)) where T(z) ∈ [0,1] and ε(z) ∈ R. The intensity measure of the Poisson process is

$$\lambda(dt)\cdot e^{-\varepsilon}d\varepsilon = \lambda'(t)dt\cdot e^{-\varepsilon}d\varepsilon$$

where $\lambda'(t)$ is the derivative of $\lambda(t)$. The agents utility function is defined by

(A.1) $$U(z) = v(T(z)) + \varepsilon(z)$$

where $t \to v(\cdot)$ is a (measurable) function. Let A be a Borel set $A \subset [0,1]$ and define

(A.2) $$U_A = \max_{T(z)\in A} (v(T(z)) + \varepsilon(z)).$$

The interpretation of U_A is as the highest utility the agent can attain subject to $T(z)\in A$. We shall now derive the c.d.f. for U_A. Let

$$B = \{(t,\varepsilon) : v(t) + \varepsilon > u,\ \ t\in A\}$$

and let N(B) be the number of Poisson points within B. By the Poisson law

(A.3) $$P(N(B)=n) = \frac{\Lambda(B)^n}{n!}\exp(-\Lambda(B))$$

where $\Lambda(B) = EN(B)$ and is given by

(A.4)
$$\Lambda(B) = \int_B \lambda'(y)e^{-\varepsilon}\,dy\,d\varepsilon$$
$$= \int_{y\in A}\int_{v(y)+\varepsilon>u} \lambda'(y)e^{-\varepsilon}\,d\varepsilon\,dy = e^{-u}\int_A e^{v(y)}\lambda'(y)\,dy.$$

Now it follows from (A.3) that

(A.5)
$$P(U_A \le u) = P(\text{There are no points of the Poisson process in B})$$
$$= P(N(B)=0) = \exp(-\Lambda(B)) = \exp(-e^{-u}\int_A e^{v(y)}\lambda'(y)\,dy).$$

Eq. (A.5) proves that U_A is extreme value distributed with

(A.6)
$$EU_A = \log\int_A e^{v(y)}\lambda'(y)\,dy + 0.5772\ldots\,.$$

(cf. Ben-Akiva and Lerman, 1985).

Let $\bar{A}$ be the complement of A. Since the Poisson realizations are independently distributed it follows that U_A and $U_{\bar{A}}$ are independent and extreme value distributed.

Let $\hat{z}$ be the index of the alternative that maximizes utility, i.e.,

(A.7)
$$v(T(\hat{z})) + \varepsilon(\hat{z}) = \max_z(v(T(z)) + \varepsilon(z)).$$

Obviously

(A.8)
$$P(T(\hat{z})\in A) = P(U_A = \max_z(v(T(z)) + \varepsilon(z)))$$
$$= P(U_A > U_{\bar{A}}) = \frac{\exp(EU_A)}{\exp(EU_A) + \exp(EU_{\bar{A}})}.$$

The last equality follows from standard results in discrete choice analysis (cf. Ben-Akiva and Lerman, op.cit.). With A = [0,t] we get from (A.8) that

(A.9)
$$P(T(\hat{z}) \le t) = \frac{\int_0^t e^{v(y)} \lambda'(y)\,dy}{\int_0^1 e^{v(y)} \lambda'(y)\,dy}$$

which has probability density

(A.10)
$$\frac{dP(T(\hat{z}) \le t)}{dt} = \frac{e^{v(t)} \lambda'(t)}{\int_0^1 e^{v(y)} \lambda'(y)\,dy}.$$

The proof of (3.6) is completely analogous to the argument presented above.

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