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An Applied General Equilibrium Assessment of the Marginal Cost of Public Funds in Norway

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Abstract

The purpose of this paper is to estimate the marginal cost of public funds (MCF) in Norway. An econometrically specified large scale applied general equilibrium model of the Norwegian economy is employed to derive the results. The paper considers three types of financing; poll tax, wage income tax and VAT; and seven types of government expenditure. Results indicate that the MCF is above unity in Norway. The wage income tax is more expensive than the poll tax, while the VAT is in between.

It is claimed that this particular ranking follows primarily from the preference specification. The female labour supply response is the primary reason why the MCF of a wage income tax is that much higher than the poll tax.

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1 Introduction

The purpose of this paper is to estimate the marginal cost of public funds in Norway. This concept is equal to the marginal social cost of raising revenue for a public project that does not affect private supply and demand schedules[1].

The MCF has many applications in public finance. One is in cost benefit analysis. An appropriate cost benefit analysis of a public project must recognize that public funds has a "price tag" on its own. Another is in tax reform. Knowledge of the MCF's related to different taxes is sufficient to design a successful tax reform in which a tax with a high MCF is lowered, and a tax with a low MCF is increased.

A third is in the economics of the size of the public sector. High values of MCF cet.par. increase the possibility for reductions in the size of the public sector to improve welfare. High values of MCF are therefore usually taken to support the view that the public sector is "too big", or that transfer programs are "too large"[2].

A final use of the MCF concept is in the efficiency/equity trade off. Each specification of the welfare function imply a different MCF (or set of MCF's). When measuring the MCF, one usually specifies a "benthamite" welfare function, which measures the "efficiency cost" of public projects. Increasing the inequality aversion yields measures of MCF that integrate efficiency and equity considerations. For efforts along these lines, see Ahmad and Stern (1984),(1990) and Ballard (1988).

There is a fairly substantial literature that measures the MCF and related concepts[3], but

[1]See Mayshar (1990) or Vennemo (1991). In parts of the literature, the same term has been used in the broader sense of raising revenue for any kind of public project, including transfer projects.

[2]Note that the optimum quantity of a public good generally is higher in second best (allowing a poll tax) than in first best. The reason is basically that the price of private goods is higher in second best because of indirect taxation, inducing consumers to substitute into the public good. This is shown by Wilson (1991).

[3]Wildasin (1984), Stuart (1985) and Ballard, Shoven and Whalley (1985), Browning (1987) and Jorgenson and Yun (1990) for the US; Hansson (1984) and Hansson (1985) for Sweden.

the empirical basis of some of the papers can be questioned. This paper is an attempt to measure the MCF by an econometrically specified large scale applied general equilibrium model. A multi-industry (27), multi-household (14) model of the Norwegian economy is used in the computations.

An attempt to assess the MCF for the US by the aid of a large scale CGE model is Ballard, Shoven and Whalley (1985). Being calibrated, their model has a weaker empirical basis than assumed for this study[4]. Moreover, the US and Norwegian systems of taxation are quite different. The overall level of taxation is higher in Norway, Sweden and other European countries than in the US. Norway relies more on indirect taxation than does the US, and transfer payments are higher. These differences indicate that second best type interactions of taxation are different in different countries. It is of interest whether extending the analysis to a high tax Scandinavian country will influence results.

Earlier attempts to measure the MCF of a Scandinavian country are Hansson (1984) and Hansson and Stuart (1985). They however focus on the relation between the formal and informal sectors of the economy, using a two sector model in which the formal economy is one of the sectors, and fairly rough data. This paper analyzes the formal economy, and therefore evaluates the MCF from a different perspective. Moreover, more emphasis is put on the empirical basis of the computations.

The paper is organized as follows: Section 2 describes the CGE model used. Section 3 interprets the results of the experiments. I compute the MCF of three forms of tax financing, and seven types of public expenditure. Section 4 concludes.

[4]Jorgenson and Yun (1990) estimate the social cost of a transfer project employing the econometric approach in an aggregate, dynamic model.

2 Model structure

The CGE model is an extended and modified version of the MSG-4E model (Longva, Lorentsen and Olsen (1985))[5]. Holtsmark et.al. (1991) give a detailed documentation.

Figure 1 gives a diagrammatic summary of the model. There are around 2 million households, and 14 *types* of households in the model. They are distinguished by socioeconomic status. Labour supply of each type is specified by the function

$$l^h = d\left[1 - a^h \left(\frac{q}{\Pi_i p_i^{\beta_i}}\right)^{b^h}\right] \tag{1}$$

where l^h is labour supply of household h, d is total time endowment, a^h is a positive constant, b^h is a negative constant, β_i is the marginal expenditure share of good i, q is the net of tax wage rate and p_i is the price of good i.

The parameters a^h and b^h are aggregated estimates from a micro simulation model of labour supply developed and estimated by Dagsvik and Strøm (1990). The parameters differ between men and women. Moreover, in the case of women, the parameter a^h depends on the number and age of children in the household. The mapping from male and female labour supply to households is generated using data from Norwegian Income Statistics (CBS (1989)).

A feature of the labour supply equation that will have effect on the calculations, is that the income effect on labour supply is zero. This property of aggregate data can be found in 80 per cent of households in the micro data set, see Aaberge, Dagsvik and Strøm (1990). There is however a negative income effect on labour supply among the 10 per cent poorest households, and a positive effect among the 10 per cent richest households. These effects cancel in the aggregate labour supply equation.

[5]MSG stands for "Multi Sectoral Growth". The MSG model has for many years been used in Norwegian long-term planning. In this paper I however use the model as a purely static model, as I focus on the year one solution.

The labour supply elasticity is around 0.3 for men and around 2 (on average) for women in the base year. These are higher values than normally assumed in the CGE literature[6], but not higher than in some other econometric studies of labour supply (see Killingsworth (1983) for a survey).

Private consumption is distributed on 14 consumption goods according to a linear expenditure system for each type of household:

$$c_j^h = \gamma_j^h + \beta_j(r^h + ql^h - \sum_{j=1}^{n} p_j\gamma_j^h)/p_j \tag{2}$$

where r^h is transfer and capital income of household type h. The parameters β_j and γ_j^h are estimated using the latent variable method described in Aasness, Biørn and Skjerpen (1988). The γ-terms depend on the number of children and adults in each type of household.

Household utilities are measured in terms of money metric utility functions derived in the appendix. The functions are evaluated in base year (1987) prices. Welfare is measured as the sum of utilities over all households, ie.

$$W = \sum_{h=1}^{14} n^h V^h \tag{3}$$

where n^h is the number of households of type h.

Each consumption *good*, investment *good* etc. of the model is a fixed coefficient mix of up to 40 Armington *composites*, which in turn consists of a domestic and imported *variety*. Exports are generally functions of relative prices, and of world market demand indicators. The market elasticities are estimated on a 1968-87 data set (Lindquist (1990)).

For the price elasticities of export demand, I have somewhat arbitrarily imposed values that are 10 times their estimated values. The results would otherwise be dominated by the

[6]Ballard, Shoven and Whalley (1985) for instance assume 0.15 for the uncompensated elasticity in their "central case".

"tax exporting" effect — the effect that part of the burden of financing public expenditure is shifted on to foreigners. A price elasticity of 2 for instance, which is high according to many econometric studies, will yield an MCF of around 0.5 if the small open economy yields 1 (see Vennemo (1991)). One can of course argue that such is the "correct" model of the Norwegian economy, but then a more efficient way to increase welfare would be to impose tariffs of 100 per cent (see Vennemo (1990)). The implicit assumption when running large price elasticities of trade is that prices are determined according to an approximate small open economy model.

The domestic variety of each Armington composite in the model is produced according to a fixed coefficient mix in one or more of the 27 sectors of production. 17 are endogenous private industries. 7 are public sectors, and 3 (related to energy production and ocean transport) are exogenous private industries.

Production behaviour is modelled in dual terms by Generalized Leontief (GL) cost functions. The output measure is "gross production". Output is produced according to a constant returns to scale technology. Two stage budgeting is assumed. At the "top" level there are four input factors, labour, real capital, material inputs and energy. At the "bottom" level, demand for energy is divided into electricity and fuels according to a GL subfunction. The parameters of substitution is estimated on national accounts data by Bye and Frenger (1985). All factors are assumed to be freely moveable and malleable between the 17 endogenous private industries.

The exchange rate is the numeraire of the model. The total capital stock and the current account are exogenous. I ignore capital accumulation and assume a savings response of zero to changes in the rate of return. In an open economy such as the Norwegian, it is not clear whether the rate of return — and savings — "really" should change in response to public projects. It is the effect of changes in present value consumption on savings that should be of interest. This effect is left for studies of the MCF by means of dynamic optimization models.

The model incorporates a (by CGE standards) detailed account of the structure of indirect taxation. The price equation for a consumption good looks like the following (dropping subscript j for the consumption good):

$$p = \sum_{i=1}^{40} \lambda_i b_i (1 + t_{mi}) \tag{4}$$

λ_i is the consumption composite-to-good coefficient, including ad valorem and unit excise taxes on composite i in the context of consumption good j. t_{mi} is the VAT rate. b_i is the pre-tax price of composite i (including output taxes or subsidies, and tariffs).

Seven types of government activity are modelled. These are Defence, Local and Central Education/Research, Local and Central Healthcare/Veterinary Services, and Local and Central "Other". There is one government budget constraint, which is exogenous.

3 The marginal cost of public funds

This section present results of the simulations. I consider all seven branches of government separately. For each branch, I examine three types of financing: A poll tax, wage tax and VAT financing. In all cases, a baseline equilibrium is perturbed by a marginal increase in government expenditure on goods and services. The change in welfare, which is negative because households derive no utility from public goods, is then compared to the change in government expenditure. The resulting ratio is the empirical measure of the MCF. I consider an increase of NOK (NOrwegian Krone) 100 million in public purchases of goods and services. This amounts to 0.07 per cent of GDP.

3.1 Poll tax financing

One reason for discussing poll tax financing is that the poll tax is the only tax that just has income effects. It is thus a reference from which to go on to assess other taxes. Under certain assumptions the cost of most other forms of financing can be written as a combination of poll tax financing and a term describing substitution effects.

Second, poll taxes or taxes that are close to poll taxes are a feature of most tax systems, especially at the local level. In Norway, fees for garbage collection, fees for sewage treatment, the TV license, fixed tariffs in the utility and telephone bills are examples of taxes that for practical purposes can be considered poll taxes. Parts of the social security system, like child benefits, can be considered approximate negative lump sum taxes at the central level.

The results of the simulations in the case of poll tax financing are indicated in table 1. Since the poll tax is non-distorting, many economists have the intuitive feeling that the MCF of poll tax financing is unity. Table 1 shows that this is not correct, however. MCF range from 1.07 to 1.24, with an average around 1.10 – 1.15.

Table 1: Marginal cost of public funds. Financing by poll tax

Defence	1.13
Central Education and Research	1.07
Central Healthcare and Veterinary Services etc.	1.17
Other Central Services	1.12
Local Education and Research	1.13
Local Healthcare and Veterinary Services	1.24
Other Local Services	1.16

It can be shown the MCF of poll tax financing is equal to[7]

$$MCF = \frac{1}{1-\sum_{i=1}^{n}\tilde{t}_i\beta_i}\frac{\nu_j}{p_{gj}} \tag{5}$$

where $\tilde{t}_i$ is the *shadow* tax rate on consumption good i, ν_j is the shadow price of government expenditure on good j, and p_{gj} is the actual price of good j paid by the government. A shadow

[7]See Vennemo (1991) for a derivation of this equation. Note that since the income effect in labour supply is zero, only the income effects in commodity demands enter the formula.

tax is the (percentage) difference between shadow price and actual price. In the second best optimum, shadow taxes are equal to actual taxes.

Equation (5) reveals why it is incorrect to assume that the MCF of a poll tax is unity: We do not start out from a first best economy when we increase the poll tax. The existence of non-zero tax rates on goods and factors makes even poll taxation costly. The argument would however be correct if we had a first best economy to begin with. Tax-rates (both formal and shadow) would then be zero. From equation (5), financing a public project through by a poll tax yields an MCF of unity when tax rates are zero.

Equation (5) also reveals that it is the repercussions of the lump sum tax on the public budget that generates MCF values above unity in table 1. A lump sum tax removes income from the household. Thus it reduces its purchases of goods and services. This reduces government revenue from *indirect* taxation. To compensate, the government must increase lump sum taxation a second, and subsequent times. The total increase in lump sum taxation, the amount of income removed from the consumer, exceeds the cost of the project itself.

The repercussions on the public budget for the case of defence spending is illustrated in table 2. We note the loss of 17 million NOK in revenue from indirect taxation, that comes on top of the outlay of 100 million NOK on the project itself. After accounting for other changes in government revenue, the required increase in lump sum taxation is 113 million NOK. Since consumer prices and utility of income are approximately constant (because of the economy is close to the small open case), this translates into a welfare loss of 113 million for a 100 million project, ie. an MCF of 1.13, as in the first row of table 1.

It may seem paradoxical that it is the substitution of a poll tax for indirect taxation that constitutes the excess cost of the project. According to ordinary intuition, such substitution should rather yield a gain than a cost. Part of the answer is of course that no indirect tax

Table 2: Change in the public budget. Million NOK.

"New" goods and services	-100
Wage costs	-2
Financial income	-2
Indirect taxes	-17
Wage taxes of household	4
Producer labour taxes	4
Amount to be covered by poll tax	113

rates are reduced, just the revenue from indirect taxation. The substitution is not of the same kind as if one lowered a tax rate and increased the poll tax to compensate. A more complete answer, focusing on the real economic side of the issue, is that for resources of the amount p to be transferred to the government sector, the private sector has its income/welfare reduced by $p + t$, because it pays taxes t to the government. Dividing $p + t$ by p yields a number larger than one, ie. the MCF is larger than one.

Equation (5) reveals that it is important for the estimate of the MCF how close the shadow taxes are to actual taxes. In a closed economy, shadow taxes may well be negative when actual taxes are positive (see eg. the calculations in Smith (1987)). In the model of this paper, shadow taxes of commodities are as a general bias close to actual taxes (Vennemo (1991)).

The difference between the MCFs of different public projects is explained by differences in the term ν_j / p_{gj} of equation (5). It turns out that healthcare has the highest MCF, while central research and education has the lowest. The MCF of healthcare is in fact 16 per cent higher than that of research and eduction. This is to say that the shadow price of research and education is relatively lower than the shadow price of healthcare. There is no single reason for this difference. The government does for instance not pay less tax when purchasing goods and services for healthcare than for research and education. Rather, it is the general equilibrium repercussions starting from the fact that producers of goods and services for healthcare are different from producers of goods and services for research and education, that yield the difference. This in turn suggests that even ignoring the cost of public revenue aspect, a general equilibrium model is helpful in estimating the shadow price of a government expenditure item.

In summary, the general equilibrium calculations of the MCF seems reasonable given the model. The two crucial assumptions behind the results are first, that the income effect on labour supply is zero. If it was negative, as is commonly assumed (but recall that it is not supported by Norwegian data), MCF would most certainly have been lower. The other assumption is the "small open economy" hypothesis that structures the relation between shadow taxes and formal taxes on consumption commodities. If smaller price elasticities of trade were allowed for, we would have experienced tax exporting, and shadow taxes would have been lower than formal taxes in terms of formula (5). The MCF would have been lower.

3.2 Wage income tax financing

The MCF from financing increased public expenditure through the wage income tax is indicated in table 3. The numbers have an average of around $1.80 - 1.90$. As in the case of poll tax

Table 3: Marginal cost of public funds. Financing by wage income tax

Defence	1.82
Central Education and Research	1.75
Central Healthcare and Veterinary Services etc.	1.95
Other Central Services	1.82
Local Education and Research	1.86
Local Healthcare and Veterinary Services	2.06
Other Local Services	1.89

financing, central education and research yields the lowest MCF (now 1.75), while local healthcare and veterinary services yields the highest (2.06). There is in other words a fairly extreme degree of waste associated with wage income tax financing, according to these numbers.

It can be shown that the MCF of a wage income tax is equal to[8]

$$MCF = \frac{1}{1 - \sum_{j=1}^{n} \tilde{t}_j \beta_j - h} \frac{\nu_j}{p_{gj}} \tag{6}$$

where

$$h = (\sum_{j=1}^{n} \tilde{t}_j \beta_j + \tilde{t}_l) \sum_{h=1}^{k} b^h (l^h - d)/L \tag{7}$$

Based on eq. (6), it is clear that MCF will be larger when financed by a wage income tax, than when financed by a poll tax. The difference is indicated by the h term, which is positive (for positive shadow tax rates), since both b^h and $(l^h - d)$ are negative. The specific form of the MCF is due to the assumption of additive separability between leisure and consumption goods, which rules out complementarity between leisure and any of the consumption goods. That is, effects like "increased leisure demand increases demand for high taxed gasoline", is ruled out. If complementarity of this kind was allowed for, and it was strong enough, a wage income tax would in certain situations be less costly than a poll tax. Once again, it is the repercussions on the public budget that summarize the MCF.

Equation (6) can be used as a check on the general equilibrium computations of table 3. I assume that men work around 1600 hours a year on average, and women around 1200 hours, and that two thirds of the work force are men (these are all "reasonable" numbers for Norway). I put d at 8760 hours as in Aaberge, Dagsvik and Strøm (1990) (and in the model). I further assume that shadow taxes are equal to actual taxes, and that the only commodity tax is a flat VAT rate of 16.67 per cent (which even the government pays), while the wage income tax rate is 25.9 per cent (average in Norway). I come up with a value of MCF of around 1.88.

This value of MCF is a good approximation to the full equilibrium estimate. Since most of

[8]See Vennemo (1991).

the data put into the formula were stylized facts about the economy, the crucial factor is the behavioural assumption b^h.

This parameter is related to the labour supply elasticity. In the literature, the value of the labour supply elasticity is often argued to influence the cost of taxation significantly. Substituting for b^h, we can rewrite the h term in terms of the labour supply elasticity as follows:

$$h = (\sum_{j=1}^{n} \tilde{t}_j \beta_j + \tilde{t}_l) \sum_{h=1}^{k} \bar{l}^h l^h / L \tag{8}$$

It is obvious that parametric increases in the labour supply elasticities will increase the MCF.

Instead of performing empirical sensitivity analysis w.r.t. the elasticity of labour supply, I use equations (6) and (8) to perform analytical sensitivity analyses. If I assume the labour supply elasticity of women to be equal to that of men, ie. 0.3, I can "guesstimate" MCF at around 1.33. This is of course much less drastic than the values reported in table 3. The sensitivity of female labour supply to the wage therefore seems a crucial parameter in the model. It has been suggested that the large female wage elasticity is difficult to distinguish from the shift in female attitudes to paid work that occurred in the 70's and 80's. It may therefore be a relationship with little "autonomy"[9].

3.3 VAT financing

Results for the case of VAT financing is shown in table 4. It shows that VAT financing is cheaper than wage income tax financing, and more expensive than poll tax financing. The MCF ranges from 1.33 to 1.47, with an average around 1.35-1.40.

It is not very surprising that VAT financing is in the middle, compared to the other two. Consider an *ideal* VAT system, ie. household purchases of all goods and services are subject to a single VAT rate, and no other agents (except possibly the government) pay VAT on their

[9]The female wage elasticity in Norway has been estimated at 0.2 on time series data, see Lindquist, Sannes and Stølen (1990).

Table 4: Marginal cost of public funds. Financing by VAT

Defence	1.36
Central Education and Research	1.33
Central Healthcare and Veterinary Services etc.	1.47
Other Central Services	1.33
Local Education and Research	1.37
Local Healthcare and Veterinary Services	1.47
Other Local Services	1.39

purchases. In such a system one can divide through by the VAT-rate in the household budget constraint, which implies that VAT financing is equal to financing by a decrease in the real value of transfers, together with an increase in the taxation of real wage income.

Purchases of housing services and some service industry output are however excepted from VAT in Norway, and these have substantial budget shares. In industries whose output is excepted from VAT, the VAT paid by producers on their material input is not refunded. The VAT is therefore a tax on producer purchases of material inputs in these industries. As such it has (partially speaking) undesirable efficiency properties that come on top of the costs of an ideal VAT. These effects are captured by this model — recall that it allows for substitutability between material inputs and other factors of production. The results of table 4 however suggest that this extra cost is not empirically decisive. Otherwise the MCF of VAT financing would have been closer to, or even higher than that of labour income tax financing.

The modelling of the female labour supply influence the MCF of VAT financing because VAT financing in part is like a labour income tax. If the female labour supply elasticity was lower, the MCF of VAT financing would be lower as well.

4 Concluding comments

The paper allows us to draw some tentative conclusions regarding the MCF in Norway. I have argued that taxes that for practical purposes are poll taxes, are possible sources of revenue for the local branches of government. Using a poll tax to finance expenditure, the MCF lies between 1.1 and 1.2 according to the results of this paper. For the central government, the question may arise whether to increase transfers to senior citizens, children ..., or to supply some public good. In these cases, the 1.1 to 1.2 figure is relevant if one ignores equity.

According to the results of this paper, wage income taxation should not be used to finance public projects. If one does use it, the MCF is very large. The crucial assumption behind this argument is the labour supply of women. But as long as the labour supply response to higher wages is positive at all, wage tax financing is more expensive than poll tax financing. This is because of the preference relation assumed. VAT-financed public expenditure lies in the middle of the other two. Given the labour supply elasticity of women, the MCF of VAT financing is measured at around 1.35-1.40.

The assumption of large price elasticities of trade is important for the results. If these were smaller, the MCF could easily have been below unity.

If one accepts the premises of the results (including the indifference to equity), the evidence on the MCF put forward here may serve as one of many inputs to cost-benefit analyses. Other aspects, like the direct effects of the project on private supply and demand schedules and thus on tax revenues, need also be accounted for, of course.

The large differences in MCF between ways of financing indicate that welfare in Norway will increase if we decrease the level of transfer spending and use the revenue saved to decrease the wage income tax. This conclusion is also conditioned on the indifference to equity. A modified conclusion is that a wage income tax that does not contribute to redistribution should be reduced.

The difference between the MCF's as calculated in this paper, and the values found for the US is insignificant. There is no tendency for the MCF of Norway to be higher than that of the US. If there is a tendency, it leans in the other direction. When Ballard, Shoven and Whalley (1985) change all taxes (proportionally), they get an MCF of 1.39 given a labour supply elasticity of 0.3 (and a savings elasticity of zero like in this paper). This compares to my figures for the VAT, although I use a much larger labour supply elasticity (for women). The Wildasin (1984) figures of 0.95 - 1.23 are lower than mine, but his 0.95 figure is a result of assuming backwards bending labour supply. The 1.23 figure applies to an uncompensated elasticity of zero, in which case labour income financing in the present paper reduces to poll tax financing. 1.23 is quite high compared to my results for poll tax financing. Given that tax rates on average are higher in Norway than in the US, it is a little puzzling why the results on MCF are so close. Other differences between models include the use of the the econometric approach to CGE modelling in this paper, differences in functional forms, and level of detail in the modelling of taxes.

The work of this paper can be extended in several ways. Within the framework of the present model, one extension is to allow for functional forms to describe consumer preferences that are flexible enough to let the data decide, for instance, which is the less costly alternative of a wage income tax or a lump sum tax. To employ flexible functional forms for consumer preferences is however data demanding, and may require the analyst to work with very aggregate consumer groups. Further, in CGE models it is important for the functions to have reasonably large domains.

Another extension of the present paper is to account for the intertemporal aspect of taxation.

This would allow studying taxation of capital and savings. The intertemporal aspect is however present no matter which tax finances expenditure.

Accounting for external effects is a further extension. Some of the motivation for excise taxes on tobacco, alcohol and possibly some other goods like gasoline is that consumption of these goods yields negative external effects. As these effects are not modelled at present, the model treats the taxes as distorting, and the MCF is pushed up.

Appendix: Derivation of the utility function

The utility function of this model is one which generates the following behavioural relations:

Labour supply equation

$$l^h = d\left(1 - a^h\left(\frac{q}{\Pi_i p_i^{\beta_i}}\right)^{b^h}\right). \tag{9}$$

where h is household and

$$\begin{aligned}
l^h &= \text{labour supply, household h} \\
d &= \text{total time} \\
q &= \text{nominal wage, net of tax, household h} \\
\beta_i &= \text{positive constant} \\
a^h &= \text{positive constant} \\
b^h &= \text{negative constant} \\
\Pi_i p_i^{\beta_i} &= \text{a homogenous of degree one price index of commodity prices}
\end{aligned}$$

Commodity demands equation

$$p_i c_i^h = p_i \gamma_i^h + \beta_i(r^h + ql^h - \Sigma_i \gamma_i^h p_i) \tag{10}$$

where

$$\begin{aligned}
p_i &= \text{price, good } i \\
c_i^h &= \text{consumption of good } i\text{, household h} \\
r^h &= \text{lump sum income, household h} \\
ql^h &= \text{wage income, household h} \\
\gamma_i^h &= \text{constant, positive, good } i\text{, household h}
\end{aligned}$$

The commodity demand system is LES.

Proposition 1 *The system of equation (9) to (10) is generated by the (indirect) utility function*

$$V^h(p_1 \ldots p_n, q, r^h) = \frac{r^h}{\Pi_i p_i^{\beta_i}} + \frac{q}{\Pi_i p_i^{\beta_i}} d\left(1 - \frac{a^h}{b^h + 1}\left(\frac{q}{\Pi_i p_i^{\beta_i}}\right)^{b^h}\right) - \frac{\Sigma_i \gamma_i^h p_i}{\Pi_i p_i^{\beta_i}} \tag{11}$$

Proof: Apply Roy's lemma.

Remark: The utility function (11) is equivalently written

$$U^h = \frac{r^h + ql^h - \Sigma_i \gamma_i^h p_i}{\Pi_i p_i^{\beta_i}} + \frac{b^h}{b^h+1} \frac{q}{\Pi_i p_i^{\beta_i}} (d - l^h) \tag{12}$$

$$\left(where \quad l^h = d \left(1 - a^h \left(\frac{q}{\Pi_i p_i^{\beta_i}} \right)^{b^h} \right) \right) \tag{13}$$

The expenditure function corresponding to (12) is

$$e^h(p_1 \ldots p_n, q, U^h) = \Sigma_i \gamma_i^h p_i - ql^h - \frac{b^h}{b^h + l^h} q(d - l^h) + \Pi_i p_i^{\beta_i} U^h \tag{14}$$

Measuring expenditure in base year prices p_i^0, q^0, yields the base year money metric utility function

$$e^h(p_1^0 \ldots p_n^0, q^0, U^h) = \Sigma_i \gamma_i^h p_i^0 - q^0 l^{h\,0} - \frac{b^h}{b^h+1} q^0 (d - l^{h\,0}) + \Pi_i p_i^{0\,\beta_i} U^h \tag{15}$$

Equation (15) is the utility function actually implemented in the model.

Note that

$$e^h(p_1^0 \ldots p_n^0, q^0, U^{h\,0}) = r^{h\,0} \tag{16}$$

Using (15) and (12), the marginal utility of lump sum income, λ^h, is given as

$$\lambda^h = \lambda = \frac{\Pi_i p^{0\,\beta_i}}{\Pi_i p_i^{\beta_i}} \tag{17}$$

λ is treated as a constant in simulations.

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