# Discussion Paper

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# Optimal Taxation in Applied General Equilibrium Models Adopting the Armington Assumption

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## Abstract

The paper derives formulas for optimal taxation in a stylized Applied General Equilibrium (AGE) model, and shows their potential empirical significance. Because of the Armington-assumption for exports and imports, the formulas are quite different from those of the 'textbook' model of optimal taxation. As a result, tax reforms that increase welfare in the 'textbook' model, may not do so in the AGE model. An empirical example for Norway is given.

# 1 Introduction

This paper concerns optimal taxation in a stylized Applied General Equilibrium (AGE) model[1]. The aim of the paper is to provide an explicit analytical background for tax policy experiments analyzed by Applied General Equilibrium models[2]. I derive rules for optimal taxation in a stylized AGE model, and show how these rules may guide the interpretation of an actual policy experiment.

The paper shows that the rules for optimal taxation in the AGE model are quite different from those of the 'textbook' models, which typically are partial models or models of closed economies[3]. To use the 'textbook' model to interpret what happens in the AGE-model, therefore may lead to quite misleading results. For instance, it is a rule of thumb from the 'textbook' model that intermediates should not be taxed in second best. In the AGE-model, taxing intermediates creates no problems, and may even be a necessary part of first best.

More generally, tax policy suggestions that raises welfare when assessed by the 'textbook' model, may not do so when evaluated by the AGE-model. This has considerable practical relevance. For instance, one of the experiments of the empirical part of this paper demonstrates that a policy shift to pure lump sum taxation actually lowers welfare in a model of the Norwegian economy.

[1]The first AGE model was constructed by Johansen (1960). Comprehensive surveys of AGE-models and their applications are offered by Shoven and Whalley (1984) and Borges (1986). Bergman (1985) surveys the nordic tradition. Whalley (1988) surveys some tax policy models. Robinson (1989) surveys models of developing countries.

[2]Tax policy applications of AGE models were pioneered by Harberger (1962), (1966), focusing on incidence effects and welfare costs of capital taxation. Capital taxation remains one of the most popular areas of study, see eg. Shoven and Whalley (1972), Jorgenson and Yun (1986, 1989), Fullerton and Henderson (1989), Goulder and Summers (1989). Other themes include the marginal cost of taxation (eg. Hansson and Stuart (1985), Ballard, Shoven and Whalley (1985)), welfare effects of reform in indirect taxation (eg. Clarete and Whalley (1987), Ballard, Scholz and Shoven (1988)), and consumption taxation (eg. Fullerton, Shoven and Whalley (1983), Makin (1989)).

[3]The standard reference is Diamond and Mirrles (1971). Mirrles (1986) gives an up to date exposition. Dixit (1985) surveys models of open economies. Drèze and Stern (1987) state results on optimal taxation relative to arbitrary restrictions on available instruments.

The question arises as to why such peculiarities occur. The paper indicates that the modelling of foreign trade is essential.

According to the authoritative survey of Shoven and Whalley (1984), the research program of AGE economics is "to convert the Walrasian general equilibrium structure from an abstract representation of the economy into realistic models of actual economies" (p.1007). One concession to reality that modelers usually make, at least when assuming mobile factors of production, is to drop the "small open economy" assumption common of analytical models of open economies. The reason is that this assumption leads to excessive specialization of production, and does not allow for imports and exports of the "same" good.

As an alternative, the so-called Armington assumption is often employed, see Armington (1969). This assumption says that domestically consumed goods are composites of domestic and foreign varieties. Exported goods are varieties of composite goods on the world market. Relative prices determine the relative size of each variety in a composite. If the price of, say, the foreign variety rises, the foreign share falls. Countries therefore face downward sloping demands for their exports, a fact which curbs the tendency to specialize in a few industries.

The Armington assumption therefore contributes to a diversified structure of production in equilibrium. Over the years, this has been important practical argument for employing the assumption. Moreover, it has been justified as an approximate model of competition in diversified products[4].

The downward-sloping export demand curves create differences between optimal taxation in the AGE-model, and the 'textbook' model. The reasoning is familiar from the theory of optimal tariffs as formulated by Johnson (1951-52) (who credit Bickerdike (1906) for the first suggestion of the idea): Whenever possible, a country should use tariffs to improve its terms of trade. It is perhaps not very surprising that this argument can be applied in the case of Armington export demand functions, and one can certainly find verbal comments to that effect in the literature (Whalley and Yeung (1984), Hirte

[4]See Norman (1990) for a critical evaluation of this claim.

and Wiegard (1988) de Milo and Robinson (1989)). However, it seems fair to say that the AGE- literature as a whole has not *emphasized* the effect of the Armington assumption upon results. The price elasticities of trade are for instance not typically included in the sensitivity analyses so common in the literature. Neither are they discussed in most studies of actual of hypothetical tax reforms, except, possibly, for studies of tariff reductions (Whalley (1980)). By the results of the present paper, the AGE literature should direct more attention to the effect of the Armington assumption on all kinds of tax policy experiments, and even on other normative experiments in the field of environmental policy, deregulation, development etc.

The structure of the paper is as follows. Section 2 sets out the analytical model employed. Section 3 derives 'first' and 'second' best optimal taxation in the model, and points to different ways of implementing the optima. Section 4 gives some indication of the practical importance for the results derived. Using a large scale, empirically estimated AGE model, the sensitivity of optimal tax calculations to the elasticities of foreign trade is demonstrated. Section 5 concludes, and gives some opinions on the fruitfulness of the Armington assumption as opposed to other alternatives.

# 2 The model

This section sets out the model. Consumers' and producers' behaviour are examined, and the equations describing their behaviour is combined to describe the full model.

## 2.1 Consumer behaviour

There are $k$ consumers, indexed $1 \ldots h \ldots k$. Market behaviour of consumer no. $h$ in our economy can be described as the outcome of a two stage utility maximization process.

At the top level, she solves the problem[5]

$$\max\ U^h(c_1^h,\ldots,c_n^h,l^h) \text{ s.t. } \sum_{j=1}^{n} p_j^c c_j^h - q_l^c l^h = r^h \tag{1}$$

where

$$p_j^c = b_j^c(1+t_j^c) \tag{2}$$

$$q_l^c = q_l(1-t_l^c) \tag{3}$$

$$r^h = q_k^c K^h + T^h \tag{4}$$

$$q_k^c = q_k(1-t_k^c) \tag{5}$$

$c_j^h$ is consumption of composite good $j$ by consumer $h$, $l^h$ is labour supply of consumer $h$, $p_j^c$ is the consumer price of composite $j$, $b_j^c$ is the price net of tax of the composite. $t_j^c$ is the consumer tax on the composite, $q_l^c$ is the net of tax wage rate of consumers (all consumers earn the same hourly wage), $q_l$ is the gross wage rate received by consumers, $t_l^c$ is the consumer tax rate on wage income.

$r^h$ is total lump sum income of consumer $h$, consisting of capital income and government grants. $K^h$ is an exogenous amount of capital owned by consumer h. $q_k^c$ is the consumer net of tax rate of return to capital. $q_k$ is the gross rate of return, and $t_k^c$ is the consumer tax on capital income. $T^h$ is the lump sum grant directed at consumer h.

Utility maximization yields demand functions for consumer composites and a labour supply function

$$c_j^h = c_j^h(p_1^c \ldots p_n^c, q_l^c, r^h) \tag{6}$$

$$l^h = l^h(p_1^c \ldots p_n^c, q_l^c, r^h) \tag{7}$$

[5]In this paper, superscript $h$ is used for consumers, superscript $j$ for producers, subscripts $j$ or sometimes $i$ are used for goods/varieties (goods from the point of view of producers, varieties from the point of view of consumers). Superscript $c$ denotes prices/taxes which are common to all consumers. Subscript $m$ denotes the good/variety m. The subscript $d$ is used to denote a domestic variety, $f$ for foreign variety and $a$ for an exported good. The variable $p$ is used for purchasers' price of goods/varieties, and $q$ for inputs. $b$ is used for sellers' price, and for Armington price indices. All taxes, $t$, are 'ad valorem', as is standard in AGE models.

and an indirect utility function $V^h(p_1^c \dots p_n^c, q_l^c, r^h)$.

In accordance with the Armington approach to foreign trade, we assume each consumer composite to consist of a foreign and a domestic variety, that is $c_j^h = c_j(c_{dj}^h, c_{fj}^h)$ where $c_{dj}^h$ is the domestic variety and $c_{fj}^h$ is the foreign variety. For simplicity, we assume the same aggregator function $c_j(\cdot)$ for all consumers. At the bottom level, consumer $h$ solves the problem

$$\max\ c_j(c_{dj}^h, c_{fj}^h) \text{ s.t. } p_{fj}^c c_{fj}^h + p_{dj}^c c_{dj}^h = a_j^h \tag{8}$$

where total outlays on composite no.$j$, $a_j^h$, is given from the solution to the top level problem[6]. The consumer price of the imported variety, $p_{fj}^c$, is defined as $p_{fj}^c = b_{fj}(1+t_{fj})$ ie. the world market price plus a tariff. The consumer price of the domestically produced variety, $p_{dj}^c$, is defined as $p_{dj}^c = b_{dj}(1+t_{dj})$, the producer output price adjusted for an ad valorem tax.

$c_j(c_{dj}^h, c_{fj}^h)$ is homogeneous of degree 1. Associated with the problem (8) is the expenditure function $e(p_{fj}^c, p_{dj}^c, c_j^h)$, which takes the separable form $b_j^c(p_{fj}^c, p_{dj}^c)c_j^h$ because of homogeneity. $b_j^c(\cdot)$ is a homogeneous of degree one price index of $c_j^h$, usually a CES-function in applied work. Using Shepards lemma, the sub demand functions corresponding to problem (8) for foreign and domestic varieties of a consumer composite are

$$c_{fj}^h = \frac{\partial b_j^c}{\partial p_{fj}^c} c_j^h \tag{10}$$

$$c_{dj}^h = \frac{\partial b_j^c}{\partial p_{dj}^c} c_j^h \tag{11}$$

$\frac{\partial b_j^c}{\partial p_{fj}^c} c_j^h$ is the import demand equation for consumer variety j. Imports depend on the *relative* price of the domestic versus foreign variety (since $b_j^c$ is homogeneous of degree one), and on the domestic activity level (represented by $c_j^h$). It may be that $\frac{\partial b_j^c}{\partial p_{fj}^c} \equiv 0$ for

[6]The problem (8) is equivalent to

$$\max\ c_j(c_{dj}^h, c_{fj}^h) \text{ s.t. } p_{fj}^c(1+t_j^c)c_{fj}^h + p_{dj}^c(1+t_j^c)c_{dj}^h = \tilde{a}_j^h \tag{9}$$

where $\tilde{a}_j^h$ is equal to $a_j^h(1+t_j^c)$. It may be more plausible to assume that the consumer allocates $\tilde{a}_j^h$ at the 'top' level, than $a_j^h$. Because of homogeneity, equation (9) yields the same demands as equation (8).

some variety j. In that case, the composite j is domestically produced only. If $\frac{\partial b_j^c}{\partial p_{fj}^c} \equiv 1$, the good is imported only.

## 2.2 Producer behaviour

The economy consists of n single output producers indexed $1 \ldots j \ldots n$. Outputs are produced according to constant returns to scale technologies. Production behaviour in an industry can be described as if a representative producer maximizes unit profits:

$$\max\ b_{dj} f^j(zl_j, zk_j, zm_j) - q_l^j zl_j - q_k^j zk_j - q_m^j zm_j \tag{12}$$

where $f^j(\cdot)$ is the unit production function, and $zl_j$, $zk_j$ and $zm_j$ are unit scale inputs of labour, capital and material input. The latter is a produced, composite good, ie. it consists of an imported and a domestically produced variety. The former two are non-produced: Labour is supplied by consumers according to their labour supply schedules, while real capital is assumed to be exogenously supplied. $q_l^j = q_l(1+t_l^j)$ is the producer price of labour input, with $t_l^j$ the producer tax and $q_l$ the price paid to consumers. Similarly, $q_k^j = q_k(1+t_k^j)$ with $t_k^j$ the tax and $q_k$ the price paid to consumers. Finally, $q_m^j = b_m^j(1+t_m^j)$ with $t_m^j$ the tax and $b_m^j$ the net of tax price index.

The outcome of the profit maximization process is (homogeneous of degree zero) unit scale input demand functions

$$zl_j = zl_j(q_l^j, q_k^j, q_m^j) \tag{13}$$

$$zk_j = zk_j(q_l^j, q_k^j, q_m^j) \tag{14}$$

$$zm_j = zm_j(q_l^j, q_k^j, q_m^j) \tag{15}$$

and a unit profit function

$$\pi = b_{dj} - e^j(q_l^j, q_k^j, q_m^j) \tag{16}$$

In most AGE-models, material input $zm_j$ is a price independent constant (Leontief technology). Our model is more flexible on this point, it contains Leontief technology as a special case only.

Regarding the composition of material input, we may use the same method as in the case of consumer behaviour to derive

$$zm_{fj} = \frac{\partial b^j_m}{\partial p^j_{fm}} zm_j \tag{17}$$

$$zm_{dj} = \frac{\partial b^j_m}{\partial p^j_{dm}} zm_j \tag{18}$$

where $b^j_m = b^j_m(p^j_{dm}, p^j_{fm})$ is the price of composite material input, net of tax on the composite (but including taxes on the varieties). Note that the domestic variety of material output is produced in industry $m$.

## 2.3 The complete model

The formal model is

$$b_{dj} = e^j(q^j_l, q^j_k, q^j_m) \qquad j = 1, \ldots, n \tag{19}$$

$$\sum_{h=1}^{k} l^h(p^c_1 \ldots p^c_n, q^c_l, r^h) = \sum_{j=1}^{n} zl_j(q^j_l, q^j_k, q^j_m) y_j \tag{20}$$

$$K = \sum_{j=1}^{n} zk_j(q^j_l, q^j_k, q^j_m) y_j \tag{21}$$

$$\sum_{j=m+1}^{n} p_{aj}\, a_j(p_{aj}) = \sum_{j=1}^{n} b_{fj} \frac{\partial b^c_j}{\partial p^c_{fj}} \sum_{h=1}^{k} c^h_j(p^c_1 \ldots p^c_n, q^c_l, r^h) + b_{fm} \sum_{j=1}^{n} \frac{\partial b^j_m}{\partial p^j_{fm}} zm_j(q^j_l, q^j_k, q^j_m) y_j \tag{22}$$

$$y_j = \frac{\partial b^c_j}{\partial p^c_{dj}} \sum_{h=1}^{k} c^h_j(p^c_1 \ldots p^c_n, q^c_l, r^h) + g_j \qquad j = 1, \ldots, m-1 \tag{23}$$

$$y_m = \frac{\partial b^c_m}{\partial p^c_{dm}} \sum_{h=1}^{k} c^h_m(p^c_1 \ldots p^c_n, q^c_l, r^h) + g_m + \sum_{j=1}^{n} \frac{\partial b^j_m}{\partial p^j_{dm}} zm_j(q^j_l, q^j_k, q^j_m) y_j \tag{24}$$

$$y_j = \frac{\partial b^c_j}{\partial p^c_{dj}} \sum_{h=1}^{k} c^h_j(p^c_1 \ldots p^c_n, q^c_l, r^h) + a_j(p_{aj}) + g_j \qquad j = m+1, \ldots, n \tag{25}$$

where

$$p_j^c = b_j^c(p_{fj}^c, p_{dj}^c)(1 + t_j^c) \tag{26}$$

$$p_{fj}^c = b_{fj}(1 + t_{fj}) \tag{27}$$

$$p_{dj}^c = b_{dj}(1 + t_{dj}) \tag{28}$$

$$q_l^c = q_l(1 - t_l^c) \tag{29}$$

$$q_l^j = q_l(1 + t_l^j) \tag{30}$$

$$q_k^j = q_k(1 + t_k^j) \tag{31}$$

$$q_m^j = b_m^j(p_{fm}^j, p_{dm}^j)(1 + t_m^j) \tag{32}$$

$$p_{fm}^j = b_{fm}(1 + t_{fm}^j) \tag{33}$$

$$p_{dm}^j = p_m(1 + t_{dm}^j) \tag{34}$$

$$p_{aj} = b_{dj}(1 + t_{aj}) \tag{35}$$

and the relation between lump sum income and lump sum taxes are given as

$$r^h = q_k^c K^h + T^h \tag{36}$$

$$q_k^c = q_k(1 - t_k^c) \tag{37}$$

Equations (19) are the no-profit conditions associated with pure competition and constant returns to scale. Equation (20) says that demand for labour equals the supply of labour. $y_j$ is total domestic output of good/variety j. Equation (21) says that total demand for real capital is equal to total supply of real capital, $K$, where $K = \sum_{h=1}^{k} K^h$.

Equation (22) gives the current account. Balance in trade of goods and services is assumed. The foreign variety of the material input, and the foreign varieties of the consumption goods are imported at given world prices. To pay for imports, the goods labeled $m+1$ to n are exported. (Thus there are $n-m$ export-competing industries). $a_j$ denotes exported quantities, $p_{aj}$ export prices. Though exporters of an industry assume they can sell all of their exports at a given price, export quantities are declining functions of export prices on the macro level. A rationalization of this constellation is that each exporter is small relative to the market he is in. The export demand function is therefore flat from the point of view of each individual exporter, even though it declines for the industry as a whole[7]. The export demand functions are Armington-export demand functions where prices of the 'world' variety of each good and any other explanatory variables (such as market indicators) are constant. Foreign exchange is numeraire in the model.

Equations (23) to (25) are the commodity balance equations. The $m-1$ first goods may either be used for consumption (the domestic variety of the consumption composite) or for exogenous public expenditure, $g_j$. Good m may in addition be used as a factor of production (the domestic variety of composite material input). The $n-m$ last goods may be used for consumption, public expenditure or for exports. Equations (26) to (37) define the relation between prices and taxes.

There are $11n-m+4$ equations in the model[8] and $11n-m+4$ endogenous variables ($b_{dj}$, $q_l^j$, $q_k^j$, $q_m^j$, $p_{aj}$, $y_j$, $p_j^c$, $p_{fj}^c$, $p_{dj}^c$, $p_{fm}^j$, $p_{dm}^j$, $q_l^c$, $q_l$, $q_k$ and one of the tax, transfer and government expenditure variables $g_j$, $t_{fj}$, $t_{dj}$, $t_j^c$, $t_l^j$, $t_k^j$, $t_{fm}^j$, $t_{dm}^j$, $t_m^j$, $t_{aj}$, $t_l^c$, $r^h$). Imbedded in the model is the public budget constraint

$$\sum_{j=1}^{n} t_j^c b_j^c c_j + \sum_{j=1}^{n} t_{fj} b_{fj} c_{fj} + \sum_{j=1}^{n} t_{dj} b_{dj} c_{dj} + t_l^c q_l L + q_l \sum_{j=1}^{n} t_l^j z l_j y_j + q_k \sum_{j=1}^{n} t_k^j z k_j y_j +$$
$$\sum_{j=1}^{n} t_m^j b_m^j z m_j y_j + b_{fm} \sum_{j=1}^{n} t_{fm}^j z m_{fj} y_j + b_{dm} \sum_{j=1}^{n} t_{dm}^j z m_{dj} y_j +$$
$$\sum_{j=m+1}^{n} t_{aj} b_{dj} a_j + t_k^c q_k K - \sum_{h=1}^{k} T^h - \sum_{j=1}^{n} b_{dj} g_j = 0 \quad (38)$$

where

$$c_j = \sum_{h=1}^{k} c_j^h \quad (39)$$

$$c_{fj} = \sum_{h=1}^{k} c_{fj}^h \quad (40)$$

$$c_{dj} = \sum_{h=1}^{k} c_{dj}^h \quad (41)$$

[7]Note that industries producing for the domestic market are treated symmetrically. They, too, are assumed to take the price as given, but face downward sloping market demand curves.

[8]Excluding eq. (36) and (37). It proves convenient to collect all lump sum income in one variable, $r^h$.

$$L = \sum_{h=1}^{k} l^h \tag{42}$$

Equation (38) makes it clear that the public sector runs a balanced budget. This is an implication of Walras law, which in this model can be stated as follows: As consumers keep within their budget constraint, profits are zero and the current account is balanced, the public budget must be balanced as well.

The model is a stylized AGE model. This means that AGE models typically incorporate some of the crucial features of the present model. AGE models generally contain some exporting industries, and some importing[9]. At least one produced input is generally assumed, while one factor, labour, generally is supplied by consumers.

The Armington model of foreign trade is employed by eg. Holtsmark et.al.(1991), Boadway and Treddenick (1978), Whalley (1980), Dixon et.al. (1982), Jorgenson and Wilcoxen (1989), Goulder and Summers (1989), Bovenberg (1989). Others, eg. Dervis, De Milo and Robinson (1982) and Ballard et. al. (1985) use the assumption of downward sloping demand for exports without referring to the Armington theory[10]. The assumption of an exogenous supply of capital is employed eg. by Stuart (1984), Hansson and Stuart (1985) and Holtsmark et. al. (1991).

Moreover, the capital stock of a given period is predetermined in the models of eg. Jorgenson and Wilcoxen (1989), Goulder and Summers (1989) and Bovenberg (1989). These, and similar dynamic models consist of equations like those of the present model,

[9] The model and the closed economy: Assume $m = n$, that is no exporting industries, and $\frac{\partial b_j^c}{\partial p_{fj}^c} \equiv 0$, $\frac{\partial b_m^j}{\partial p_{fm}^j} \equiv 0$ for all $j$, that is no imports either. The model is then one of a closed economy.

[10] The model and the CET: The constant elasticity of transformation (CET) formulation of exports was first suggested by Powell and Gruen (1968), and has recently become popular in the AGE-literature (eg. de Milo and Robinson (1989), Robinson (1989)). It says that production is a composite of two varieties, one for domestic use, and one for exports. By a suitable change of interpretation, the present model captures this assumption. Consider the consumption aggregate $c_j = c_j(c_{dj}, c_{fj})$. Inverting this equation for a non-exporting industry with sero government demands yields $y_j = c_{dj}(c_j, c_{fj})$, where $y_j = c_{dj}$. When $c_{fj}$ is negative, it is an export variety, and the function is a CET export transformation function. Rules for taxation of imports derived in the present model will for this reason apply to CET exports as well. Government demand and Armington exports can be included in the new commodity balance equations (which must be formulated in what we in this paper call aggregates) without problems.

plus equations for savings and capital accumulation. The model of this paper is therefore similar to the intratemporal subset of these models, and the conditions we derive, will be a subset of the conditions to be derived in the case of intertemporal models.

# 3 First and second best optimal taxation

This section characterizes necessary conditions for 'first' and 'second' best optimal taxation. 'First best' denotes the case where all kinds of tax instruments, including individual specific lump sum taxes, are available. 'Second best' denotes the case where a general poll tax and a full set of distorting taxes are available, but individual lump sum taxes are ruled out.

In both cases, one finds the structure of optimal taxation through maximizing the social welfare function $W(V^1(p_1^c \ldots p_n^c, q_l^c, r^1) \ldots V^k(p_1^c \ldots p_n^c, q_l^c, r^k))$ s.t. the model (19) to (25)[11]. The price structures facing the agents of the economy are under government control, thanks to the assumption of a full set of distorting taxes. Equations (26) to (35), representing the relations between prices, are therefore not included as constraints. We make use of the following Lagrangian:

$$\begin{aligned}
\mathcal{L} &= W(V^1(p_1^c \ldots p_n^c, q_l^c, r^1) \ldots V^k(p_1^c \ldots p_n^c, q_l^c, r^k)) \\
&- \mu \left[ \sum_{j=1}^{n} \theta_j (b_{dj} - e^j(q_l^j, q_k^j, q_m^j)) \right. \\
&+ \omega \left( \sum_{j=1}^{n} y_j z l_j(q_l^j, q_k^j, q_m^j) - \sum_{h=1}^{k} l^h(p_1^c \ldots p_n^c, q_l^c, r^h) \right) \\
&+ \rho \left( \sum_{j=1}^{n} y_j z k_j(q_l^j, q_k^j, q_m^j) - K \right) \\
&+ \sum_{j=1}^{n} b_{fj} \frac{\partial b_j^c}{\partial p_{fj}^c} \sum_{h=1}^{k} c_j^h(p_1^c \ldots p_n^c, q_l^c, r^h) + b_{fm} \sum_{j=1}^{n} y_j \frac{\partial b_m^j}{\partial p_{fm}^j} z m_j(q_l^j, q_k^j, q_m^j)
\end{aligned}$$

[11]Note that the public budget constraint is implied by the model equations, as asserted above. If the public budget constraint were to be an explicit constraint in the maximization problem, it could substitute for one of the model equations.

$$
\begin{aligned}
& - \sum_{j=m+1}^{n} p_{aj}\, a_j(p_{aj}) \\
& + \sum_{j=1}^{n} \nu_j \Big( \frac{\partial b_j^c}{\partial p_{dj}^c} \sum_{h=1}^{k} c_j^h(p_1^c \ldots p_n^c, q_l^c, r^h) + g_j - y_j \Big) + \sum_{j=m+1}^{n} \nu_j a_j(p_{aj}) \\
& + \nu_m \sum_{j=1}^{n} y_j \frac{\partial b_m^j}{\partial p_{dm}^j} zm_j(q_l^j, q_k^j, q_m^j) \Big]
\end{aligned} \tag{43}
$$

The Lagrangian multipliers of the problem are of course the social values of marginally relaxing the corresponding constraints. They can be interpreted as shadow prices. In particular, $\mu$ is the social value of marginal unit of foreign exchange, or the shadow exchange rate. We are interested in welfare rankings. Obviously, different scalings of the welfare function yield different valuations of a unit of foreign exchange, or different values of $\mu$, without changing the ranking. We can fix the scale by making $\mu$ (or some other variable) the numeraire of the shadow price system. Choosing $\mu$ as the numeraire has the benefit of making market prices and shadow prices be measured in the same unit, i.e. foreign exchange.

$\omega$ is the shadow wage rate, $\rho$ is the shadow rate of return, and $\nu_j$ is the shadow output price of good/variety j ($\nu_m$ to $\nu_n$ appear twice).

The controls of the maximization problem are the composite consumer prices $p_j^c$ and wage rate $q_l^c$, the good/variety prices $b_{dj}$, the input prices $q_l^j$, $q_k^j$ and $q_m^j$, output quantities $y_j$, prices of domestic and imported consumption varieties, $p_{dj}^c$ and $p_{fj}^c$, prices of the domestic and imported varieties of material input, $p_{dm}^j$ and $p_{fm}^j$, and export prices $p_{aj}$. In second best taxation, a uniform poll tax completes the set of available tax instruments for the government. In the first best case, we allow, as mentioned, for individual lump sum taxes to be set optimally.

Notice that the consumer net of tax rate of return to capital $q_k^c$ is not included in the set of controls, neither in 'second best' nor 'first best'. The reason is that in the model, taxing capital will just amount to a special kind of a poll tax, namely one where consumers are taxed according to their share of the total capital stock. The effect on savings and capital formation is disregarded. Discussing the 'optimal' rate of capital taxation of consumers based on this assumption is rather fruitless[12].

## 3.1 First order conditions and a simple implementation scheme

This section derives the first order conditions and presents a simple structure of taxes to implement the optimum.

### 3.1.1 Import tariffs

We start by considering the issue of whether there should be tariffs on imports. The point of departure is the following four first order conditions:

$$\frac{\partial \mathcal{L}}{\partial p^c_{fj}} = -\mu \sum_{h=1}^{k} c^h_j (b_{fj} \frac{\partial^2 b^c_j}{\partial p^{c\,2}_{fj}} + \nu_j \frac{\partial^2 b^c_j}{\partial p^c_{dj} \partial p^c_{fj}}) = 0 \quad (44)$$

$$\frac{\partial \mathcal{L}}{\partial p^c_{dj}} = -\mu \sum_{h=1}^{k} c^h_j (b_{fj} \frac{\partial^2 b^c_j}{\partial p^c_{fj} \partial p^c_{dj}} + \nu_j \frac{\partial^2 b^c_j}{\partial p^{c\,2}_{dj}}) = 0 \quad (45)$$

$$\frac{\partial \mathcal{L}}{\partial p^j_{fm}} = -\mu y_j z m_j (b_{fm} \frac{\partial^2 b^j_m}{\partial p^{j\,2}_{fm}} + \nu_m \frac{\partial^2 b^j_m}{\partial p^j_{dm} \partial p^j_{fm}}) = 0 \quad (46)$$

$$\frac{\partial \mathcal{L}}{\partial p^j_{dm}} = -\mu y_j z m_j (b_{fm} \frac{\partial^2 b^j_m}{\partial p^j_{fm} \partial p^j_{dm}} + \nu_m \frac{\partial^2 b^j_m}{\partial p^{j\,2}_{dm}}) = 0 \quad (47)$$

$$j = 1 \ldots n$$

These conditions say that, for a given level of composite demand, the net social cost of increasing the price of a variety is zero. That is, the marginal benefit of switching to a variety is equal to the marginal cost of switching away from the other.

To interpret the equations, it is helpful to apply Euler's theorem and Young's theorem on the partial derivatives of the $b^c_j$ and $b^j_m$ functions, recalling that $b^c_j$ and $b^j_m$ are

[12]Generally, an exogenous consumer rate of return to capital is not compatible with an exogenous consumer tax-rate of capital. Recall the relation $q^c_k = (1 - t^c_k)q_k$. The pre tax rate of return $q_k$ is endogenous in the model. Any changes in this rate will carry over to changes in consumer income, unless the consumer capital tax rate is endogenous. It must function as a 'buffer'. An exception is the case of an exogenous 100 per cent tax, which leaves the consumer rate of return constant at zero no matter the pre tax rate. This special case relates the model to the literature on optimal taxation in decreasing returns (or fixed factor) economies, which assumes 100 per cent profits taxation for the optimal taxation results to hold (Stiglitz and Dasgupta (1971)).

homogenous of degree one functions with derivatives homogenous of degree zero:

$$p^c_{fj}\frac{\partial^2 b^c_j}{\partial p^{c\,2}_{fj}} + p^c_{dj}\frac{\partial^2 b^c_j}{\partial p^c_{dj}\partial p^c_{fj}} = 0 \tag{48}$$

$$p^c_{fj}\frac{\partial^2 b^c_j}{\partial p^c_{fj}\partial p^c_{dj}} + p^c_{dj}\frac{\partial^2 b^c_j}{\partial p^{c\,2}_{dj}} = 0 \tag{49}$$

$$p^j_{fm}\frac{\partial^2 b^j_m}{\partial p^{j\,2}_{fm}} + p^j_{dm}\frac{\partial^2 b^j_m}{\partial p^j_{dm}\partial p^j_{fm}} = 0 \tag{50}$$

$$p^j_{fm}\frac{\partial^2 b^j_m}{\partial p^j_{fm}\partial p^j_{dm}} + p^j_{dm}\frac{\partial^2 b^j_m}{\partial p^{j\,2}_{dm}} = 0 \tag{51}$$

$$j = 1 \ldots n$$

Comparing equations (44) – (47) to (48) – (51), it is clear that $p^c_{fj} = k_{1j}b_{fj}$, $p^c_{dj} = k_{1j}\nu_j$, $p^j_{fm} = k^j_1 b_{fm}$, $p^j_{dm} = k^j_1 \nu_m$ at the optimum, with $k_{1j}$ and $k^j_1$ commodity and activity specific constants which are there because only relative prices matter in import demand (and domestic demand) functions. In particular, $k_{1j}$ and $k^j_1$ may be equal to unity for all j (a price normalization). If this is the case, $p^c_{fj} = b_{fj}$, $p^c_{dj} = \nu_j$, $p^j_{fm} = b_{fm}$, $p^j_{dm} = \nu_m$, and we can write

$$b_{fj}\frac{\partial b^c_j}{\partial p^c_{fj}} + \nu_j\frac{\partial b^c_j}{\partial p^c_{dj}} = b^c_j(p^c_{fj}, p^c_{dj}) \tag{52}$$

$$b_{fm}\frac{\partial b^j_m}{\partial p^j_{fm}} + \nu_m\frac{\partial b^j_m}{\partial p^j_{dm}} = b^j_m(p^j_{fm}, p^j_{dm}) \tag{53}$$

Equations (52) and (53) are useful for future reference.

Let us pursue an implication of the result that the purchaser's prices of imports are equal to the world market prices of imports, $p^c_{fj} = b_{fj}$, $p^j_{fm} = b_{fm}$. From equations (27) and (33) we find

$$t_{fj} = p^c_{fj}/b_{fj} - 1 = 0 \tag{54}$$

$$t^j_{fm} = p^j_{fm}/b_{fm} - 1 = 0 \tag{55}$$

In other words: Tariffs on imports should be zero. This of course is a familiar rule for a price-taking economy, but it may be little more surprising to find it show up in a model that adopts the Armington approach to foreign trade. The notion that domestic competitors do not lose all sales to importers when their price rises, seems to indicate that there is some market power somewhere to be exploited by tariffs. This is not correct however, and the reason is that imported goods are imported at a *given* world price. The price elasticity of demand for domestic vis a vis foreign varieties is a reflection of preferences (consumers) and technology (producers), and is irrelevant for tariff policy. We will notice below that it is a different story for exports.

### 3.1.2 Producer taxation

To derive the conditions for producer taxation, we follow a detailed procedure, starting by establishing the equality between market factor prices and shadow market prices. We then establish that industry output prices are equal to shadow prices on output. That granted, we can wind up the rates of producer taxation, first on domestically produced material input, then on composite material input, and finally on labour and capital input.

Start by considering the following four equations,

$$\frac{\partial \mathcal{L}}{\partial b_{dj}} = -\mu\theta_j = 0 \tag{56}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial q_l^j} &= -\mu(-\theta_j zl_j + \omega y_j \frac{\partial zl_j}{\partial q_l^j} + \rho y_j \frac{\partial zk_j}{\partial q_l^j} + (p_{fm}\frac{\partial b_m^j}{\partial p_{fm}^j} + \nu_m \frac{\partial b_m^j}{\partial p_{dm}^j}) y_j \frac{\partial zm_j}{\partial q_l^j}) \\ &= 0\end{aligned} \tag{57}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial q_k^j} &= -\mu(-\theta_j zk_j + \omega y_j \frac{\partial zl_j}{\partial q_k^j} + \rho y_j \frac{\partial zk_j}{\partial q_k^j} + (p_{fm}\frac{\partial b_m^j}{\partial p_{fm}^j} + \nu_m \frac{\partial b_m^j}{\partial p_{dm}^j}) y_j \frac{\partial zm_j}{\partial q_k^j}) \\ &= 0\end{aligned} \tag{58}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial q_m^j} &= -\mu(-\theta_j zm_j + \omega y_j \frac{\partial zl_j}{\partial q_m^j} + \rho y_j \frac{\partial zk_j}{\partial q_m^j} + (p_{fm}\frac{\partial b_m^j}{\partial p_{fm}^j} + \nu_m \frac{\partial b_m^j}{\partial p_{dm}^j}) y_j \frac{\partial zm_j}{\partial q_m^j}) \\ &= 0 \qquad\qquad j = 1 \ldots n\end{aligned} \tag{59}$$

where equation (56) shows that the zero profit constraint is not binding. That is, the marginal value of letting prices diverge from costs, is zero. The reason is that purchasers' prices are optimal. If not, there would be a premium on the zero profit condition, welfare could be improved if prices were allowed to diverge away from costs in the direction of optimal prices. As it is, the government possesses the power, through taxation, to present optimal price structures to each agent, and is not limited by the no profit condition in any way.

Using equation (56) and (53) and making some obvious eliminations, we reach the more convenient form

$$\frac{\partial \mathcal{L}}{\partial q_l^j} \; : \; \omega \frac{\partial zl_j}{\partial q_l^j} + \rho \frac{\partial zk_j}{\partial q_l^j} + b_m^j \frac{\partial zm_j}{\partial q_l^j} = 0 \tag{60}$$

$$\frac{\partial \mathcal{L}}{\partial q_k^j} \; : \; \omega \frac{\partial zl_j}{\partial q_k^j} + \rho \frac{\partial zk_j}{\partial q_k^j} + b_m^j \frac{\partial zm_j}{\partial q_k^j} = 0 \tag{61}$$

$$\frac{\partial \mathcal{L}}{\partial q_m^j} \; : \; \omega \frac{\partial zl_j}{\partial q_m^j} + \rho \frac{\partial zk_j}{\partial q_m^j} + b_m^j \frac{\partial zm_j}{\partial q_m^j} = 0 \tag{62}$$

where $b_m^j = b_m^j(p_{fm}^j, p_{dm}^j)$.

Equations (60), (61) and (62) say that, for a given level of output, the net social cost of increasing an input price is zero at the optimum. That is, the cost of the inputs one switches into when a price increases, is exactly matched by the reductions in costs of the inputs one switches away from.

Note that $\frac{\partial zl_j}{\partial q_l^j} = \frac{\partial^2 e^j}{\partial q_l^{j\,2}}$ etc. Now use the same technique as above, apply Eulers theorem and Youngs theorem on the derivatives of the $e^j(q_l^j, q_k^j, q_m^j)$, that is on the $zl_j$, $zk_j$ and $zm_j$ functions. Thus, it is clear that $q_l^j = k_{2j}\omega$, $q_k^j = k_{2j}\rho$, $q_m^j = k_{2j}b_m^j$, where $k_{2j}$ is some (industry specific) constant which is there because only relative prices matter in the input demand functions. In particular, $k_{2j}$ may be unity (a price normalization), which is the alternative pursued here.

Consider the following first order condition,

$$\frac{\partial \mathcal{L}}{\partial y_j} = \mu(\nu_j - \omega zl_j - \rho zk_j - (p_{fm} \frac{\partial b_m^j}{\partial p_{fm}^j} + \nu_m \frac{\partial b_m^j}{\partial p_{dm}^j}) zm_j) = 0 \tag{63}$$
$$j = 1 \ldots n$$

This equation says that production should break even at shadow prices, a very general property of constant returns models (Diamond and Mirrles (1976)).

Using equation (53) and the result that $q_l^j = \omega, q_k^j = \rho, q_m^j = b_m^j$, the condition simplifies to

$$\frac{\partial \mathcal{L}}{\partial y_j} : \nu_j - q_l^j zl_j - q_k^j zk_j - b_m^j zm_j = 0 \tag{64}$$

from which is obvious that $\nu_j = b_{dj}$, the output prices of industries are equal to the shadow prices of production at the optimum.

We are now in a position to determine the optimal taxes to face producers. Begin by considering the optimal tax on the domestic variety of intermediate inputs. From equation (34) and the fact that $p^j_{dm} = \nu_m$:

$$t^j_{dm} = p^j_{dm}/p_m - 1 = 0 \tag{65}$$

ie. there should be no taxation of the domestic variety of material input. Now consider the tax on composite material input. From equation (32) and the fact that $q^j_m = b^j_m$,

$$t^j_m = q^j_m/b^j_m - 1 = 0 \tag{66}$$

In other words, there should be no taxation of composite material input. Combined with the zero tax rate on the domestic variety of material inputs, and the rule of no tariffs on the imported variety, we can conclude that material inputs should not be taxed.

This is of course a very familiar rule from the theory of optimal taxation in models with production. The intuition is that *if* demand and supply did not face the same prices, aggregate production could be increased by making them equal.

Next, consider the taxes on labour and capital input. It follows from the result that $q^j_k = \rho, q^j_k = \omega$, that $q_k = k_k\rho$, $q_l = k_l\omega$ where $k_k$ and $k_l$ are two constants which concern the overall division of taxation of labour and capital between producers and consumers. In particular, they may be unity (equality between shadow price and net of tax input price). If that is the case, the tax rates on labour and capital input will be

$$t^j_k = \rho/q_k - 1 = 0 \tag{67}$$

$$t^j_l = \omega/q_l - 1 = 0 \tag{68}$$

ie. neither capital nor labour are taxed. Since material inputs is not taxed either, we can conclude that production should be left untaxed altogether. An implication is that the marginal rates of transformation between inputs will be equal for all producers. This is the Diamond and Mirrles rule of efficiency in production in an optimally taxed general equilibrium model (Diamond and Mirrles (1971)). The intuition is that the government should bring production to the production possibility frontier, and use its ability to distort all relative consumer prices to bring in the required revenue and fulfill equity concerns.

When production is untaxed, the market values of the marginal products of inputs will of course be equal. This observation is useful for the discussion of section 3.2.

### 3.1.3 Taxes on exports

Next, we consider taxation of exports. The relevant rule is derived from the following first order condition, which applies to exporting industries only:

$$\frac{\partial \mathcal{L}}{\partial p_{aj}} = -\mu(\nu_j \frac{da_j}{dp_{aj}} - (a_j + p_{aj}\frac{da_j}{dp_{aj}})) = 0 \qquad j = m+1 \ldots n \tag{69}$$

The following way of writing this equation is helpful:

$$\frac{\partial \mathcal{L}}{\partial p_{aj}} : \nu_j - p_{aj}(1 - \frac{1}{e_j}) = 0 \qquad j = m+1 \ldots n \tag{70}$$

where $e_j = -\frac{p_{aj}}{a_j}\frac{da_j}{dp_{aj}}$, the (absolute value of the) price elasticity of demand for good j.

Equation (70) says that at the optimum, the relationship between shadow product price and world market price is described by means of a simple elasticity rule equal to that of monopoly pricing. The country should behave like a monopolist vis a vis the foreign market.

The tension between this optimum condition and the price taking behaviour of competitive exporters in an unregulated market, motivates a tariff. The size of the tariff is found by using equations (35) and (70) and the fact that $b_{dj} = \nu_j$,

$$t_{aj} = \frac{1}{1 - 1/e_j} - 1 = \frac{1}{e_j - 1} \tag{71}$$

The tariff, when formulated as a fraction of the net of tariff export price, should equal one over the price elasticity of export demand minus one. This is the 'optimal tariff' of Johnson (1951-52).

The economic rationale for the tariff is to exploit the monopoly power inherent in the assumption of downward sloping demand curves. Because of the microeconomic structure assumed within the exporting industries, no single agent is able to make use of the industry's position vis a vis the world market. In other words, the market power is not 'internalized' by the agents of the industry, and there is an external effect type incentive for the government to intervene.

However, this incentive depends on the value of the elasticities. Imagine the demand elasticities being parametrically increased to infinity. This will make the model approach 'the small open case' on the export side. The optimal tariffs are zero in the limiting case of a completely price taking economy. The empirical relevance of the optimal tariffs will therefore be greater when the terms of trade elasticities are 'small'.

Notice the important difference in the model between imports and exports. Exports should be subject to a tariff, while imports should not. The reason for the difference is simply that exports are traded at prices that depend on export volume (or rather the other way around), while we recall that imports are traded at given world prices. In other words, the Armington approach assumes the market power to belong to exporters, not to importers. In some AGE models, like the BFSW model of Ballard et. al. (1985) for instance, import prices depend on imported quantities. In that case, non-zero import tariffs will be optimal.

### 3.1.4 Consumer taxation

Consider next the conditions for taxation of consumers, and start by identifying the tax on the domestic composite of the consumer composites. From equation (28) and the results that $p^c_{dj} = \nu_j, \nu_j = b_{dj}$ we have:

$$t_{dj} = p^c_{dj}/b_{dj} - 1 = 0 \qquad (72)$$

ie. the tax on the domestic variety should be zero. Since there is to be no taxation of imports either, the conclusion is that consumption within the composites should not be taxed. The conditions for taxation of the composites are given as

$$\frac{\partial \mathcal{L}}{\partial p^c_j} = \sum_{h=1}^{k} W_h V^h_j - \mu \sum_{i=1}^{n} \left( b_{fi} \frac{\partial b^c_i}{\partial p^c_{fi}} + \nu_i \frac{\partial b^c_i}{\partial p^c_{di}} \right) \sum_{h=1}^{k} \frac{\partial c^h_i}{\partial p^c_j} + \mu\omega \sum_{h=1}^{k} \frac{\partial l^h}{\partial p^c_j} = 0 \qquad (73)$$

$$\frac{\partial \mathcal{L}}{\partial q_l^c} = \sum_{h=1}^{k} W_h V_l^h - \mu \sum_{i=1}^{n} \left( b_{fi} \frac{\partial b_i^c}{\partial p_{fi}^c} + \nu_i \frac{\partial b_i^c}{\partial p_{di}^c} \right) \sum_{h=1}^{k} \frac{\partial c_i^h}{\partial q_l^c} + \mu\omega \sum_{h=1}^{k} \frac{\partial l^h}{\partial q_l^c} = 0 \quad (74)$$

$$\frac{\partial \mathcal{L}}{\partial r} = \sum_{h=1}^{k} W_h V_r^h - \mu \sum_{i=1}^{n} \left( b_{fi} \frac{\partial b_i^c}{\partial p_{fi}^c} + \nu_i \frac{\partial b_i^c}{\partial p_{di}^c} \right) \sum_{h=1}^{k} \frac{\partial c_i^h}{\partial r^h} + \mu\omega \sum_{h=1}^{k} \frac{\partial l^h}{\partial r^h} = 0 \quad (75)$$

where the symbol $r$ is used to denote a general element of lump sum income, a (negative) poll tax. Equation (73) says that the welfare weighted gain to consumers of lowering a price (through lowering a tax) is to be equal to the social cost, valued at shadow prices, of the extra demands and labour supply induced. Equation (74) says the same for the case of an increase in the consumer wage rate. Equation (75) says that the welfare weighted gains from an increased uniform grant should be equal to the social costs of the extra demands and labour supply induced.

After a few manipulation involving equation (52), Roys identity, the definition $W_h V_r^h = \beta^h$, the result that $\omega = q_l$, equations (26) and (29) to define consumer prices, and the consumer budget constraint, equations (73) to (75) may be written

$$\frac{\partial \mathcal{L}}{\partial p_j^c} : \sum_{i=1}^{n} b_i^c t_i^c s_{ij} + q_l t_l^c s_{lj} + \frac{\mu - \tilde{\alpha}_j}{\mu} c_j = 0 \quad (76)$$

$$\frac{\partial \mathcal{L}}{\partial q_l^c} : \sum_{i=1}^{n} b_i^c t_i^c s_{il} + q_l t_l^c s_{ll} - \frac{\mu - \tilde{\alpha}_l}{\mu} L = 0 \quad (77)$$

$$\frac{\partial \mathcal{L}}{\partial r} : \mu = \frac{1}{k} \sum_{h=1}^{k} \alpha^h \quad (78)$$

where

$$s_{ij} = \sum_{h=1}^{k} s_{ij}^h \quad (79)$$

$$\tilde{\alpha}_j = \sum_{h=1}^{k} \frac{c_j^h}{c_j} \alpha^h \quad (80)$$

$$\alpha^h = \beta^h + \mu \left( \sum_{i=1}^{n} b_i^c t_i^c \frac{\partial c_i^h}{\partial r^h} + q_l t_l^c \frac{\partial l^h}{\partial r^h} \right) \quad (81)$$

Equation (76) to (78) are the 'standard' formulas for consumer taxation. The number of consumer tax rates is one more than is actually needed to implement the optimum. For instance, if it is possible to tax all consumer demands and labour supply, it is unnecessary to use uniform poll taxes as well. Any common factor in the taxation of consumer demands and labour supply will have the same effect. Formally, one of the first order conditions (76) to (78) can be derived from the others. This means that the government has one degree of freedom in setting the actual values of the consumer tax rates. It can for instance decide not to prescribe any poll taxes, while taxing goods proportionally. Or it can decide not to tax some good. A uniform poll tax plus taxes on the remaining goods and supply of labour suffices to implement an optimum. Or it can decide on some intermediate solution.

It is not straightforward to judge from equations (76) and (78) what the actual optimal tax rates on consumer composites will be. However, almost all AGE-models that have been built to date, assume weak separability between labour and other goods, and linear Engel curves for goods with intercepts being either identical, or depending on observable characteristics only[13]. In these cases, one can apply the results of Deaton (1979) or Deaton and Stern (1986), which say that uniform indirect taxes on goods combined with poll taxes depending only on observable characteristics (identical when characteristics are identical), implements the optimum.

In the first best economy, condition (75) is substituted by the $k$ consumer specific conditions

$$\frac{\partial \mathcal{L}}{\partial r^h} \;:\; \mu - \alpha^h = 0 \tag{82}$$

When equation (82) applies, $t_j^c = t_l^c = 0$ in equations (76) and (77). The government brings in the required revenue and reach its equity objectives purely with the aid of individual specific lump sum taxes.

### 3.1.5 Main conclusions

This result finalizes the derivation of the simplest structure of optimal taxation in the AGE model. We summarize that the structure is characterized by the following three principles:

[13]To my knowledge, the only model *not* making this assumption is Wilcoxen (1988), Jorgenson and Wilcoxen (1989). They maintain the separability assumption and have intercepts depending only on observable characteristics, but the Engel curves for their five 'main' goods are translog, ie. non-linear.

- leave the production sector untaxed.
- tax consumer composites according to the standard principles (implying uniform taxation, given the preference structure of most models)
- levy a (sector specific) tariff on exports

Some implications of these principles have already been drawn. Let us notice at this point that tariffs are part of first best taxation. The reason is, obviously, that tariffs in this model will not distort the domestic economy, just hurt foreigners.

The tariff part of the optimal taxation structures is likely to bias the results of policy experiments using AGE models. For instance, the 'marginal cost of public funds' when funds are raised by tariffs, is likely to be small (Vennemo(1990)). But even the projected benefits from removing distortions in the production sector are likely to be affected by the terms of trade effects.

## 3.2 Alternative optimal structures

Because of its simplicity, the optimal taxation structure involving zero producer taxation, standard optimal consumer taxation and a tariff, as concluded in the last section, is a 'natural' structure by which to implement the optimum conditions in the AGE model. But it is by no means the only one. We have already mentioned that a multitude of combinations of consumer indirect taxes, consumer wage tax and a general consumer poll tax may implement an optimum. The present section addresses the same kind of phenomenon with respect to producer taxation, tariffs etc. and shows alternative tax structures that may implement the optimum. We start by giving an example of an optimal taxation structure in which export tariffs are implicit only. Afterwards, we summarize the information of the first order conditions by giving a general implementation scheme, showing the relation between taxes that must hold at the optimum. An example of an "artificial" tax structure that nevertheless is optimal, is given.

### 3.2.1 Optimal taxation without tariffs

If, for some reason, export tariffs are zero in the model, the question arises of whether factor taxation will act as implicit tariffs. We will look closer at this possibility.

In the comments to equations (60) to (62), we derived the conditions that $q_l^j = k_{2j}\omega$, $q_k^j = k_{2j}\rho$, $q_m^j = k_{2j}b_m^j$, where $k_{2j}$ is some industry specific constant. Consider now the case of $k_{2j} = \frac{1}{1-1/e_j}$ in the exporting industries, while $k_{2j} = 1$ in the non-exporting industries. In the exporting industries, we will now have the following set of equations (using equation(70)):

$$p_{aj} = \nu_j \frac{1}{1-1/e_j} = b_{dj}/k_{2j}\frac{1}{1-1/e_j} = b_{dj}\frac{1-1/e_j}{1-1/e_j} \tag{83}$$

from which it follows that $p_{aj} = b_{dj}$, ie. tariffs *should* be zero.

On the other hand, factor inputs should be taxed at a tariff equivalent rate:

$$t_m^j = q_m^j/b_m^j - 1 = k_{2j} - 1 = \frac{1}{e_j - 1} \tag{84}$$

$$t_l^j = q_l^j/q_l - 1 = k_{2j} - 1 = \frac{1}{e_j - 1} \tag{85}$$

$$t_k^j = q_k^j/q_k - 1 = k_{2j} - 1 = \frac{1}{e_j - 1} \tag{86}$$

Apparently, parallel ad valorem industry specific input taxes in exporting industries are able to substitute perfectly for actual tariffs. The economic intuition is as follows: By taxing factors, costs rise in the taxed industries, and output prices must follow suit (because of the no profit condition). Prices rise by the same rate as if they were subject to a tariff. The result is unchanged export quantities as compared with directly taxed exports. In the input markets, the input taxes cancel when calculating marginal rates of transformation. These stay unchanged, therefore, and the production efficiency theorem continues to hold. In fact, nothing changes in the quantity solution, which is not strange, provided the optimum is unique.

The effect on the formal character of the optimal tax structures is great, however. For one thing, it is no longer true that intermediate inputs should be untaxed. We have driven a tax wedge between the producer of material inputs and consumers of material inputs in the exporting industries. Second, exporting industries should face different prices on their inputs, which is to say that the market values of the marginal products of different industries should differ. Both of these results violate familiar 'rules of thumb' from optimal tax theory.

The question of differential taxation of capital has received some attention in the public finance literature recently (see eg. Feldstein (1990), Hagen and Kanniainen (1990)). This literature emphasizes either the heterogeneity of capital, or the difficulties of taxing all kinds of capital income as reasons for differential taxation of capital. The results of this section show that in the absence of tariffs, terms of trade effects in exports yields another reason for differential taxation of capital. (See Vandendorpe (1972) for an earlier analysis along such lines.) The next section shows that the result is easily extended to the case of exogenous (not necessarily optimal) tariffs.

For completeness it should be added that domestic consumption of varieties produced by exporting industries should be ***subsidized***. If not, domestic consumers would be subject to an implicit tariff as well, and that was never the objective. The size of the subsidy (which we model as a negative tax) is:

$$t_{dj} = p^c_{dj}/b_{dj} - 1 = \nu_j/b_{dj} - 1 = 1/k_{2j} - 1 = -1/e_j \tag{87}$$

ie. it is just equal to one over the price elasticity of export demand.

### 3.2.2 A general structure of taxation

The last section shows that the tax rates of the AGE model cannot be uniquely determined, there are at least two alternatives. More generally, the optimal tax rates are functions of all the k-variables — for each non-zero value of these variables, a new set of optimal taxes results. The best we can do is to determine the optimal tax rates as functions of the k variables. These are

$$t_{fj} = k_{1j} - 1 \tag{88}$$

$$t^j_{fm} = k^j_1 - 1 \tag{89}$$

$$t^j_{dm} = k^j_1/k_{2j} - 1 \tag{90}$$

$$
\begin{aligned}
t_m^j &= k_{2j}/k_1^j - 1 && (91)\\
t_k^j &= k_{2j}/k_k - 1 && (92)\\
t_l^j &= k_{2j}/k_l - 1 && (93)\\
t_{aj} &= \frac{1}{(1-\frac{1}{e_j})k_{2j}} - 1 && (94)\\
t_{dj} &= k_{1j}/k_{2j} - 1 && (95)
\end{aligned}
$$

while the tax rates on consumer composites and labour supply implicitly are determined from the equations

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial p_j^c} &: \sum_{i=1}^{n} b_i^c \hat{t}_i^c s_{ij} + q_l \hat{t}_l^c s_{lj} + \frac{\mu - \tilde{\alpha}_j}{\mu} c_j = 0 && (96)\\
\frac{\partial \mathcal{L}}{\partial q_l^c} &: \sum_{i=1}^{n} b_i^c \hat{t}_i^c s_{il} + q_l \hat{t}_l^c s_{ll} - \frac{\mu - \tilde{\alpha}_l}{\mu} L = 0 && (97)\\
\frac{\partial \mathcal{L}}{\partial r} &: \mu = \frac{1}{k} \sum_{h=1}^{k} \alpha^h && (98)
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{t}_j^c &= t_j^c + 1 - \frac{1}{k_{1j}} && (99)\\
\hat{t}_l^c &= t_l^c - 1 + \frac{1}{k_l} && (100)
\end{aligned}
$$

All together there are $3n+2$ independent k-variables. Thus one can achieve $3n+2$ independent goals regarding tax-rates. Choosing all k-variables to be unity yields the particularly simple tax-structure of section 3.1.

To demonstrate the possibilities of the general formulation, consider the following set of assumptions, which has no particular economic justification: $k_1^j = k_{2j} = k_k = k_l = 1$, and $k_{1j} = 2$. It immediately follows from equation (88) that there should be a 100 per cent tariff on all imports[14]. On the other hand, the domestic variety should also be subject to a 100 per cent tax. This way, the relative price of the domestic versus imported varieties stays the same.

[14]Special rules apply for good m. To simplify, it is assumed that the government has the power to levy a tariff exclusively on consumer purchases of this good.

In consumer taxation, the formulas will now read

$$\frac{\partial \mathcal{L}}{\partial p_j^c} \quad : \quad \sum_{i=1}^{n} b_i^c \hat{t}_i^c s_{ij} + q_l t_l^c s_{lj} + \frac{\mu - \tilde{\alpha}_j}{\mu} c_j = 0 \tag{101}$$

$$\frac{\partial \mathcal{L}}{\partial q_l^c} \quad : \quad \sum_{i=1}^{n} b_i^c \hat{t}_i^c s_{il} + q_l t_l^c s_{ll} - \frac{\mu - \tilde{\alpha}_l}{\mu} L = 0 \tag{102}$$

$$\frac{\partial \mathcal{L}}{\partial r} \quad : \quad \mu = \frac{1}{k} \sum_{h=1}^{k} \alpha^h \tag{103}$$

where

$$t_i^c = \hat{t}_i^c - 1 \tag{104}$$

ie. tax-rates on consumer composites should be 100 percentage points lower than before. This is to compensate for the increase in price that is transmitted from the taxes on the varieties.

While it may seem inconvenient to actually implement a system which adds and subtracts the same tax wedge at different points of the system, it is perfectly legitimate from a formal point of view. Perhaps it greatest practical importance is to remind the analyst that one cannot judge from a single tax rate, or a small subset of tax-rates how far from 'optimal taxation' an economy — or a model of an economy — is. In particular, one cannot criticize a tax system for being inefficient just by looking at its tariff rates.

The points demonstrated in this section is known in the public economics literature as 'indeterminacy of taxes', or the 'independent normalization property', see eg. Mirrles (1986) pp. 1226. The point is that as long as the price structures of each agent are fully controlled by the government (and only relative prices matter), each can be normalized independently, with the relation between the numeraires forming exogenous taxes.

Notice, however, that an export tax and an import subsidy will not be the same thing in (a two industry version of) the present model. Thus the symmetry theorem of Lerner (1936), which is possibly the most famous case of indeterminacy of taxes, cannot be derived in the present context. The reason is that the quantity implications of an export tax are different from that of an import subsidy.

# 4 Empirical illustrations

It sometimes happens that problems that are of theoretical interest, turn out to be of limited practical importance. The sign of an effect may be ambiguous in theory, but in practice, it (nearly) always goes one way.

In this light, it is of interest to investigate whether the differences in optimal taxation between the standard 'text-book' model and the AGE-model that adopts the Armington assumption, have significant practical effects. As an example, we will in this section examine the effects of a hypothetical tax reform that makes the pre, and post tax user cost of capital equal. We will in other words eliminate the distortions in the capital market. This kind of tax reform is sometimes called to 'level the playing field'. The industry values of the marginal product of capital, evaluated at market prices, will be equal after the reform has taken place, while differences prevail before the reform. The difference in welfare between the two equilibria may be interpreted as the cost of distortion in the capital market.

In the AGE model that adopts the Armington assumption, market values of the marginal products of capital should only be equal provided exporters at the same time are subject to a tariff that reflects the ability of the country to improve its terms of trade (compare the comments following equations (68) or (86)). The effect of 'levelling the playing field' is therefore ambiguous in theory. This makes it a nice testing ground of what the Armington assumption means in practice.

The analysis of this section is not without empirical content, and should carry some merit of its own. We measure the combined effects of interasset, intersector, and interindustry distortions in the capital market, using a large scale, multi-industry, estimated AGE model of the Norwegian economy. The analysis, which is based on Holmøy and Vennemo (1990), is comparable to that of Fullerton and Henderson (1989), who in a similar fashion measure the combined effects of interasset, intersector, and interindusty distortions in the capital market. They, too, use a large scale multi industry AGE model of an open economy. Intertemporal issues are not emphasized. Their work is richer than the present in that they focus on the effect of varying the elasticity of substitution between non corporate and corporate capital, and substitution between assets. The present analysis by contrast assumes an elasticity of substitution of zero in both cases. On the other hand, Fullerton and Henderson do not discuss to what extent the price elasticities of export demand influence their results.

## 4.1 The model

The model we use for the simulations is, with minor modifications, an applied version of the the stylized model eq. (19) to eq. (37). The most important difference is that the empirical model is a single consumer model, and that labour supply is exogenous. The actual model features capital accumulation, but we focus here on the static 'year one' solution with exogenous capital supply. Below is a brief description of the model[15].

Start with the user cost of capital formulas. They are of the traditional Jorgenson type, ie. the user cost per NOK invested for an asset $i$ in an institutional sector $c$ of industry $j$ is written

$$\rho_{cij} = \frac{1 - u_c z_i}{1 - u_c}(r_{cj} + \delta_{ij}) \tag{105}$$

where $\rho_{cij}$ is the user cost, $u_c$ is a sector specific tax-rate, $z_i$ is the asset specific present value of depreciation allowances, $r_{cj}$ is a sector and industry specific alternative rate of return, and $\delta_{ij}$ is actual depreciation. $r_{cj}$ is a weighted average of the after tax rates of return to equity and debt. Like Fullerton and Henderson, we construct a composite user cost of capital for an industry by weighting together separate user costs for each asset and institutional sector. We identify four assets; structures, fishing boats, transport equipment and machinery. Simultaneously we identify three institutional sectors; the corporate sector, the non-corporate sector and owner occupied housing. It turns out that the tax system 'subsidizes' the user costs of capital in Norway (the effective tax rates on real capital are lower than on financial capital).

There are 31 industries in the model, 21 of which have endogenous behaviour. (The other 10 are 7 public industries, drilling for oil and gas, production of oil and gas, and ocean transport). Production is a function of labour, real capital, material and energy

[15]A detailed account of a slightly enlarged version of the model is found in Holtsmark et. al. (1991).

input. Substitution between factors are allowed at the level of gross production. The cost functions are of the Generalized Leontief flexible form type, and are estimated on Norwegian National Accounts data by Bye and Frenger (1985).

Material input, energy input and consumption goods are Armington composites, ie. they consist of an imported and a domestic variety in a CES-aggregate. Import share equations are estimated by Svendsen (1990) on a 1970-87 data set.

Export demand equations are introduced for most exporting industries, relating exports to the relative price of Norwegian versus world market prices of the exports. A loglinear functional form is chosen. This implies that the price elasticity of demand is a constant, and the optimal tariff is a constant fraction of the cost of production, see equation (71). The exports of oil and gas related activities, and of ocean transport are exogenous, as are the export prices of these industries.

The export price elasticities are estimated on a 1968 to 1987 data set by Lindquist (1990). The estimates are presented in table 1 along with the optimal tariffs calculated from equation (71). The estimated elasticities, which range between 1.23 and 3.41, with an average of around 1.8, are slightly higher than other estimates of price elasticities of export demand[16]. Nevertheless, the optimal tariffs are quite high, and generally above 100 per cent.

(table 1 about here)

The model consists of N identical consumers, each equipped with a LES consumption system. The parameters of the consumption system are taken from the Norwegian macroeconometric model MODAG (see Cappelen and Longva (1987) for a description). They are estimated indirectly by using estimates of income elasticities from various sources (mainly cross-section studies) and budget shares and a formula for translating these estimates to the parameters of the LES-system.

[16] An approximate central case in the compendium of trade elasticities provided by Stern, Francis and Schumacher (1976) is 1.4. The elasticities used by Wilcoxen (1988), who cite Cline et.al. (1978), range between 0.63 and 2.14.

Welfare is measured by means of the money metric utility function of the LES system. The function is evaluated in base year (1986) prices. The presentation focuses on percentage changes in welfare between two equilibria, defined as

$$\Delta U = \frac{e(r, U(1)) - e(r, U(0))}{e(r, U(0))} \tag{106}$$

where $e(\cdot)$ is the expenditure function, r is the base year price vector (which actually is a vector of one's), and superscript 0 and 1 refer to the pre and post reform equilibria respectively. $e(r, U(0))$ is equal to pre reform consumption outlays.

## 4.2 The cost of distortion with and without tariffs

(table 3 somewhere in this section)

We perform four experiments. Endogenous lump sum taxation will be a feature of all of them. Thus, we are in a world that is capable of first best optimality.

As our first experiment, we calculate the cost of distortion in the capital market in a 'first best' economy, *including tariffs.* The capital market distortion is the only distortion in the economy. The experiment illustrates the welfare effect of removing the distortion. We know from theory that this effect must be positive.

The optimum is implemented by setting all taxes on factors and goods except for the capital market equal to zero, and assume the optimal tariffs of table 1. Thus we make use of the simple implementation scheme of section 3.1.

Given these assumptions, the distortion in the Norwegian capital market turns out to be 0.94 per cent of the welfare level in the reference scenario. That is, the utility level calculated as a money metric in base year prices is 0.94 per cent higher in the alternative scenario than in the reference scenario. Since welfare in the reference scenario is equal to consumption, another interpretation is that "real consumption" would rise 0.94 per cent if the distortions were removed.

This is not a great number perhaps, given that the circumstances are the best possible (no negative second best effects, positive sign of welfare change guaranteed).

However, it is of a comparable size to that found by Fullerton and Henderson (1989). They conclude that the "results indicate that distortions between sectors or among industries are much smaller than previously thought. Distortions among assets are larger, but the total of all these welfare costs is still below one percent of income." (p.391). Their highest published welfare gain is 0.74 per cent of expanded (to include the value of leisure time) national income. When the elasticities of asset substitution and incorporation are both unity, welfare change is only .007 per cent. (No results are given for the case of zero elasticities, which would compare best to our experiment). This is lower than found in experiment one, but note that their denominator is higher. Another reason for their lower figure may be that they do not correct for the price elasticity of exports.

As such, their simulations compare better to our second experiment, where we construct a reference scenario with capital market distortion, no taxes on goods and factors, *but no tariffs.*

In the alternative scenario we remove the capital market distortion. This makes the alternative scenario equal to what would have been optimal in a text-book model of first best taxation. We know however, that this is not the case in the AGE model, because of the ability of the country to improve its terms of trade. In fact, what should be done (in the absence of tariffs) is to tax all factors at tariff equivalent rates, compare equations (84) to (86). Ie. capital should be taxed at different rates in different industries.

When comparing welfare in the two scenarios, we find, interestingly, that utility *falls* by 1.08 percent. The existing system of capital taxation, despite being the result of the fairly uncoordinated actions of overlapping fiscal authorities over many years[17], is closer to the optimum than a system characterized by zero tax rates and an "equal playing field".

This is the case despite the fact that the experiment represents a movement towards the optimum (represented by equations (84) to (86)), as subsidies on capital are reduced

[17]The need for a through reform of the Norwegian system of capital taxation has recently been forcefully argued by Aarbakke et. al. (1989).

to zero without changing any other tax-rates. But welfare decreases. This demonstrates, in an empirical context, the highly non linear relation between tax optima and tax reforms. It is well known that a movement towards the optimum not necessarily increases welfare. Here is a demonstration of this possibility.

What explains the result is that the price of capital in exporting industries falls, despite the removal of subsidies. One reason why it falls is that the most subsidized industry, housing, is a sheltered industry. When subsidies on housing are eliminated, demand for housing capital falls. In order to restore some of the housing demand, which is necessary since aggregate capital is exogenous, the price of capital falls quite a lot. For the average exporting industry, the net effect on the price of capital is comprised of two opposite factors; the rise in price because the industry specific subsidy is removed, and the fall in the economy wide price of capital. The latter is larger.

When the price of capital falls in exporting industries, terms of trade losses are incurred. These lead to an overall welfare loss, despite the partially beneficial effects of production efficiency.

Experiment one and two demonstrate that the optimal capital tax structure *is* vulnerable to the terms of trade implications of taxation when the price elasticities of export demand take on values that are derived from econometric studies. But what if the elasticities are higher?

Experiments three and four rework experiments one and two, with one important difference: The export price elasticities are multiplied by a factor of 7.5. Table 2 shows the optimal tariffs that apply now. At around 5 to 10 percent, they are certainly considerable, although not at all as large as before.

(table 2 about here)

In experiment three, the tariffs of table 2 are introduced to the model, which again is undistorted apart from capital market distortions. We are again moving from a non-optimal to an optimal situation, and the question is not if there is a welfare gain, just how large it is. Notice that the only difference between experiment one and experiment three is that tariffs are smaller in experiment three, corresponding to smaller price elasticities of exports.

The welfare gain turns out to be 1.26 per cent of reference case welfare/consumption. This is around 0.3 percentage points higher than in experiment one. An explanation for the somewhat higher figure may be that larger price elasticities of trade induce larger shifts in the sectoral allocation. The model is closer to the small open economy. Thereby the economy responds more to changes in relative prices. It is known from public finance theory that high elasticities of demands increase the costs of a given distortion. This makes it possible to reap more gains when conditions are bettered, and a larger figure should be expected in experiment three. On the other hand, a welfare gain of 1.26 per cent clearly is in the same range as that suggested by experiment one.

The interesting question now is whether a capital tax reform constitutes a welfare gain without tariffs, given these larger price elasticities. This is the theme of experiment four. A priori, there are two effects at work. The production efficiency effect is the reason the reform is beneficial in experiment three, and a monopoly in trade effect works in the direction of maintaining the present system of taxation.

It turns out a welfare gain of 1.13 per cent is realized. Increasing the openness of the economy therefore transforms the welfare loss to a welfare gain, despite the considerable terms of trade effects that remain. Note that the gain is lower than in experiment three, indicating that the terms of trade argument is still valid, but its empirical impact has been reduced to 0.15 per cent (the difference between experiments three and four). In other words, tariff rates of ten per cent (or less), contribute insignificantly to welfare, according to this model exercise.

Taken together, the experiments underline the message that the price elasticities of exports are crucial for explaining welfare effects of tax reform as measured by AGE models. Estimated, 'low' price elasticities make the set of optimal capital tax rates highly non-uniform, and 'levelling the playing field' decreases welfare. Optimal tariffs add 2 percentage points to welfare in this case (the difference between experiments one and two). When elasticities are 'high' however, capital tax reform yields the correct sign of welfare change, while the contribution of the price elasticities is down to 0.15. The welfare impact of reform is in all cases in the same range as that found by Fullerton and Henderson (1989). The results from our five experiments are summarized in table 3.

# 5 Concluding comments

The paper has argued that optimal taxation of AGE models that apply the Armington assumption, in an important way is different from optimal taxation in the 'text-book' model. It is optimal to tax exporters in order to bring home the monopoly profit of the country on the foreign market. This tax is a feature of first (as well as second) best taxation, as it is non-distorting from the domestic point of view. It just hurts foreigners.

The other main feature of the Armington approach to foreign trade, imports as varieties of composites in consumption and material inputs, creates no problems compared to the 'textbook' model. The reason for the difference is that market power falls on exporters, not importers.

A simple implementation scheme for optimal taxation is one where production is left untaxed, exporters are subject to a tariff, and consumers are taxed according to the standard formula for optimal taxation — pure lump sum taxation in the first best case, some combination of taxes on consumption composites and labour supply, and a general poll tax in the second best case. Given the typical AGE modelling of consumer preferences, indirect taxation should be uniform.

There is however, an infinite number of optimal taxation structures. Factor taxation can substitute for tariffs on exports. Even a large tariff levied on imports may consistent with optimality, provided it is neutralized by other taxes in the system. This demonstrates the indeterminacy of taxation within the model. An implication is that assessments of the 'goodness' of a tax system should not be made on the basis of a particular subset of tax rates, for instance whether or not there are tariffs on imports.

The empirical importance of the elasticity of export demand on taxation can be very large. We have demonstrated that, using time series estimates of the price elasticities of export demand, the structure of taxation is totally dominated by the need for taxes to act as implicit tariffs. Thus 'levelling the playing field' with respect to capital taxation will, in the empirically estimated model of the Norwegian economy, lead to a lower level of welfare. This clearly suggests that AGE-modelers should direct more attention to the way the modelling of foreign trade in general, and in particular the price elasticities of foreign demand, affect their results.

When the elasticities of export demand are raised to levels that imply optimal tariffs of about 5 to 10 per cent, their empirical significance almost disappear. This demonstrates that policy experiments on taxation are vulnerable to the elasticities of trade. In addition, the finding is interesting in its own right, since tariffs of 5 to 10 per cent often is considered to be large. In terms of their impact on the domestic economy, this paper does not support such a view.

Besides warning AGE modelers to be aware of the effect of elasticities of foreign demand, this paper naturally adds to the skepticism against the Armington assumption. Few people are convinced that tariff rates on exports of well above 100 per cent actually are optimal, *despite* the overwhelming econometric evidence of "small" responses to price changes in the Armington-relation[18]. Or to put it differently, on this matter the empirical prediction of the model seems to be just plain wrong.

Yet it is because of its ability to rationalize the facts that the Armington assumption came into fashion in the first place. Recall that it explains trade patterns better than the 'small open economy' assumption, and that it prevents excessive specialization from taking place. These are facts of economic life that we want our models to incorporate.

[18]Some will add that if it was true that industries had considerable market power, the present model would not fit the facts anyway, as producers then would collude, and reap the tariff revenue for themselves. The government should respond to this by imposing a profits tax, not tariffs. Ie. when the price elasticities are low, tariffs are close to zero, and when they are high, tariffs are zero because of collusion. Given the estimated market power, collusion would however give exporters much larger profits than is actually registered. The real problem therefore seems to lie with the price elasticities of export demand.

The alternatives to the Armington model are not altogether satisfactory either. The 'small open economy' assumption with mobile factors is unsatisfactory because of the specialization and net trades aspects. The alternative assumption of industry specific fixed factors, which basically stops specialization by inducing decreasing returns to scale, will in an optimal taxation framework naturally favour (direct or implicit) taxation of the fixed factor. This is hardly more desirable than the optimal tariffs of the Armington.

Yet another alternative is the CET formulation, which we have claimed is a special case of our model where consumption of foreign varieties is negative (compare footnote 10). The terms of trade problem will not occur in this model, but we can have export tariffs in a second best optimum (just as we can have import tariffs when consumption of foreign varieties is positive). It has been claimed that the CET formulation "may severely understate the cost of protection" (de Milo and Robinson (1989), p.50), since it is only an approximation to a 'full' imperfect competition model. The CET model may in fact be taken to assume that the home country produces for the domestic market of others (and even is a price taker on that market), while it is alone in producing for its own domestic market.

Eventually, an imperfect competition/increasing returns framework seems a most promising alternative. There are many ways to model imperfect competition however, and it may still take some time before it is ready for serious empirical work. Besides, the structure of optimal taxation in imperfect competition models is by no means simple and intuitive (see eg. Myles (1989) or Konishi (1990) for two recent contributions).

The practical model builder has to choose between imperfect alternatives. For those of us who for the time being are continuing to work with the Armington-formulation, the main lesson of this paper is to be aware of the terms of trade element, to perform sensitivity analyses to check its importance, and eventually to neutralize it, either by artificial tariffs, or by larger than estimated price elasticities of demand.

Table 1: Estimated price elasticities and optimal tariffs

| | *Price elasticities absolute values* | *Optimal tariffs per cent* |
|---|---|---|
| Purchased commodities from agriculture and fishery | 1.230 | 435 |
| Beverages and tobacco | 1.700 | 143 |
| Textiles and wearing apparels | 1.783 | 127 |
| Timber, wood and wooden products | 1.700 | 143 |
| Printing and publishing | 2.416 | 71 |
| Chemical and mineral articles | 2.522 | 66 |
| Pulp and paper | 3.412 | 41 |
| Industrial chemicals | 1.390 | 256 |
| Metals | 1.844 | 118 |
| Machinery and equipment | 1.906 | 110 |
| Domestic transport services | 1.700 | 143 |
| Wholesale and retail trade | 1.700 | 143 |

Table 2: Increased price elasticities and optimal tariffs

| | *Price elasticities absolute values* | *Optimal tariffs per cent* |
|---|---|---|
| Processed commodities from agriculture and fishery | 9.22 | 12.2 |
| Beverages and tobacco | 12.75 | 8.5 |
| Textiles and wearing apparels | 13.37 | 8.1 |
| Timber, wood and wooden products | 12.75 | 8.5 |
| Printing and publishing | 18.12 | 5.8 |
| Chemical and mineral articles | 18.92 | 5.5 |
| Pulp and paper | 25.59 | 4.1 |
| Industrial chemicals | 10.43 | 10.6 |
| Metals | 13.83 | 7.8 |
| Machinery and equipment | 14.30 | 7.5 |
| Domestic transport services | 12.75 | 8.5 |
| Wholesale and retail trade | 12.75 | 8.5 |

Table 3: Welfare cost of distortions in the capital market

| | *Percentage money metric utility cost of capital market distortions at 1986 prices* |
|---|---|
| Estimated terms of trade elasticities, optimal tariffs | 0.94 |
| Estimated terms of trade elasticities, no tariffs | -1.08 |
| 7.5 times larger terms of trade elasticities, optimal tariffs | 1.26 |
| 7.5 times larger terms of trade elasticities, no tariffs | 1.13 |

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