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A SHORT RUN DYNAMIC EQUILIBRIUM MODEL OF THE NORWEGIAN PRODUCTION SECTORS

BY

PETTER FRENGER

CONTENTS

ABSTRACT

The paper develops a short run neoclassical model of the production sectors of the Norwegian economy using the short run G.L. cost function. Emphasis is put on the relationship between the numerical model and modern duality theory which allows us to draw useful conclusions about the model as a whole. The model is then made dynamic by the introduction of the flexible accelerator theory of investment and its convergence towards long run equilibrium is analyzed.

1 INTRODUCTION

The paper develops a short run neoclassical model of the production sectors of the Norwegian economy, the short run being characterized by the fact that the capital stock is given and is specific to each sector. The technology of each sector is represented by a three level production function, the upper level being described by a short run Generalized Leontief (GLS) cost function.

Much emphasis is put in the paper on the relationship between the numerical model and modern duality theory which allows us to draw important and useful conclusions about the properties of the model as a whole. It is shown that the resulting model may be regarded as being derived from a short run restricted profit function for the ensemble of production sectors. This type of functional representation has been utilized in economic dynamics by Lau (1976), Cass (1976), and Cass and Shell (1976), and in international trade where the gross national product function was introduced by Samuelson (1953) and is used extensively in the recent textbooks by Dixit and Norman (1980) and Woodland (1982). The use of sector specific capital relates the model to the specific factor theory of international trade as developed by Mussa (1974) and Mayer (1974), and summarized in Jones and Neary (1984). The assumption of sector specific capital makes it likely that every sector will be producing in the short run. In the long run, with capital adjusting optimally, the model degenerates into a convex programming problem as presented in Diewert and Woodland (1977). We have restricted the attention to modelling the behavior of the production sectors in the belief that a detailed analysis and a compact representation of this central block of a national economic model is a useful exercise prior to its integration into a full model.

In section 2 we present the formal model, while section 3 introduces the explicit functional forms which we utilize in its parametrization. Section 4 describes the determination of the parameters of the model. We then use the reduced form elasticities to summarize the properties of the short run model. And the last section introduces a simple investment theory intended to connect the short run and the long run, and analyzes the convergence of the model to a long run equilibrium.

2 THEORETICAL FRAMEWORK

This section will present the formal structure of the short run model, while the explicit functional forms will be introduced in the next section. There are m production sectors each producing a single output y using intermediate inputs x_A, imported inputs x_B, raw materials v, labor L, and capital stock K. The technology of an arbitrary production sector is described by the linear homogeneous production function

$$y_k = f^k(x_{Ak}, x_{Bk}, v_k, L_k, K_k) \quad , \qquad k = 1,\ldots,m, \tag{2.1}$$

where x_{Ak}, x_{Bk} and v_k are n_A, n_B and n_v dimensional vectors, respectively, and L_k and K_k are scalars. In the short run the capital stock is considered fixed, and using duality theory, we will represent the technology by the short run cost function

$$V^k(y_k, p_A, p_B, p_v, p_L, K_k) = \tag{2.2}$$

$$= \min \{ p_A x_{Ak} + p_B x_{Bk} + p_{vk} v_k + p_{Lk} L_k \mid y_k = f^k(x_{Ak}, x_{Bk}, v_k, L_k, K_k) \},$$

which expresses the minimum cost of producing the output y_k given the prices p_A, p_B, p_v and p_L of the factors which are variable in the short run and the fixed capital stock K_k. Each sector regards itself as a price taker on the market for domestically produced inputs and resources. This may be justified by assuming that each sector consists of a number of firms each of which is too small to exercise any monopoly power. The short run cost function is concave and linearly homogeneous in the input prices and convex in K. We will further assume that it is jointly convex and linearly homogeneous in y_k and K_k reflecting the assumption that the production function (2.1) is concave and linearly homogeneous in the inputs.

The cost minimizing demand for the variable factors is given by the price derivatives of (2.2)

$$x_{Ak} = \frac{\partial}{\partial p_A} V^k(y_k, p_A, p_B, p_v, p_L, K_k) \quad ,$$

$$x_{Bk} = \frac{\partial}{\partial p_B} V^k(y_k, p_A, p_B, p_V, p_L, K_k) \,,$$

$$v_k = \frac{\partial}{\partial p_V} V^k(y_k, p_A, p_B, p_V, p_L, K_k) \,, \tag{2.3}$$

$$x_{Lk} = \frac{\partial}{\partial p_L} V^k(y_k, p_A, p_B, p_V, p_L, K_k) \,.$$

We will additionally assume that each sector sets its price equal to marginal cost

$$p_{Ak} = \frac{\partial}{\partial y_k} V^k(y_k, p_A, p_B, p_V, p_L, K_k) \,. \tag{2.4}$$

Such a price setting rule implies that each sector maximizes its short run profit, and insures an inter-sectoral efficient allocation of the variable inputs.

The set of equations (2.3) and (2.4) completely describes the short run behavior of each production sector. Using duality theory and the theory of conjugate convex functions we can obtain an equally compact representation of the ensemble of production sectors of the economy. This representation will treat the ensemble as a single multiple output producing unit, which may be represented by a cost function.

Equation (2.4) implies that the sector maximizes short run profit

$$\Pi^k(p_A, p_B, p_V, p_L, K_k) = \sup_{p_{Ak}} \{p_{Ak} y_k - V^k(y_k, p_A, p_B, p_V, p_L, K_k)\} \,, \tag{2.5}$$

and this function is the support function of the short run production possibility set of sector k. Using the fact that the production possibility set of the whole economy is the sum of the production possibility sets of the individual sectors, and the support function of a sum of sets is the sum of the support functions of each set [1], we obtain the short run social profit function

$$\Pi(p_A, p_B, p_V, p_L, K) = \sum_k \Pi^k(p_A, p_B, p_V, p_L, K_k) \quad , \tag{2.6}$$

where $K = (K_1, \ldots, K_m)$ is the vector of sector specific capital. The function summarizes the net behavior of the economy: its price derivatives are the net supply of the various commodities while it is not possible to obtain information about the gross output of sectors and interindustry deliveries from Π alone.

The profit function represents an economy which has a given vector of capital stock and considers itself a price taker on the output and the other input markets. In other situations other formulations may be more appropriate. From Π one can derive, using the conjugacy correspondence, the social short run cost function

$$V(x_F, p_B, p_V, p_L, K) = \sup_{p_A} \{p_A x_F - \Pi(p_A, p_B, p_V, p_L, K)\} \ , \tag{2.7}$$

giving the minimum cost of producing the final demand vector x_F, or the gross national product function

$$G(p_A, x_B, v, L, K) = \sup_{p_B, p_V, p_L} \{\Pi(p_A, p_B, p_V, p_L, K) + p_B x_B + p_V v + p_L L\} \tag{2.8}$$

giving maximum national income derivable from a given vector of imported inputs x_B, given resources v and given labor supply L, in addition to the fixed vector K of capital stock. Which function one chooses will depend on the problem at hand, and particularly on the specification of which variables are exogenous and which are endogenous.

In the model below we will partition the output vector $x_F = (x_F^1, x_F^2)$ and the associated price vector $p_A = (p_A^1, p_A^2)$ and assume that the economy is a price taker for the x_F^1 goods, while the quantities of the x_F^2 goods are given exogenously [2)]. We further assume that the supply of imports is infinitely elastic, while the economy has a fixed supply v of the resources and a fixed quantity L of labor. In this case the short run profit function Π is

$$\Pi(p_A^1, p_A^2, p_B, p_V, p_L, K) \tag{2.9}$$

and the problem is best represented by the short run restricted profit function

$$H(p_A^1, x_F^2, p_B, v, L, K) =$$

$$\sup_{p_A^2, p_V, p_L} \{\Pi(p_A^1, p_A^2, p_B, p_V, p_L, K) - p_A^2 x_F^2 + p_V V + p_L L\} , \tag{2.10}$$

which will be convex in the price variable and concave in the quantity variables. The derivatives with respect to prices give the net demand and the derivatives with respect to quantities give the shadow prices. Using this derivative properties and the homogeneity of H of degree one in the price and the quantity variables, allows us to write down the basic national income identity

$$p_A^1 x_F^1 - p_B x_B = - p_A^2 x_F^2 + p_V v + p_L L + p_S K , \tag{2.11}$$

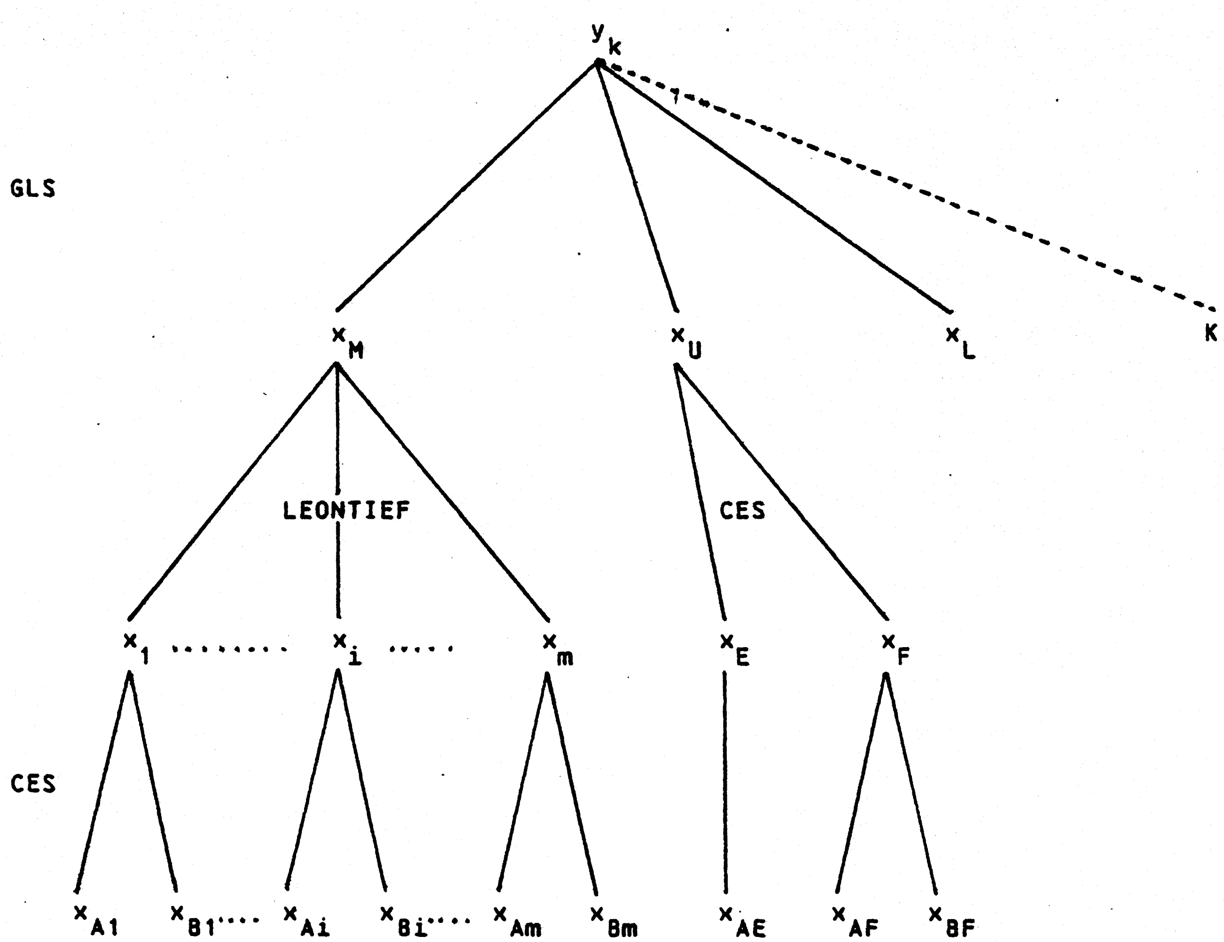
where the left hand side is obtained by differentiating H with respect to prices and the right hand side with respect to quantities, p_S being the shadow price of the capital stock. The identity expresses the equality between the net revenue from the sale of the variable factors and the net payment to the fixed factors, the latter being valued in terms of their shadow prices.

3 THE SHORT RUN MODEL

This section will present the actual functional form chosen for the short run cost function $V^k(y_k, p_A, p_B, p_V, p_L, K_k)$ [see (2.2)] and this will implicitly define the functional form of the restricted profit function H.

The technology of an arbitrary production sector will be represented by a three-level cost function. At the upper (or third) level gross output is produced by means of material inputs x_M, energy inputs x_U, labor inputs x_L, and by the fixed capital stock K. The upper level technology is described by a short run Generalized Leontief (GLS) cost function. At the middle level material inputs are produced by inputs of intermediate goods (except electricity and fuel) using fixed coefficients, while energy is a CES composite of the electricity and fuel inputs. At the bottom (or first level) each input is, at least in principle, a CES composite of the domestic and the imported "source" of that good. This three level production structure is schematically represented in figure 3.1, which also shows that

Figure 3.1: Schematic representation of the technology of the model.



there is no import of electricity. A more detailed presentation of the data, and the commodity and sector classification is given in section 4 below and in appendix A.

The GLS cost function is derived from the Generalized Leontief (GL)[3] cost function using the Legendre transformation. In the general case, with n inputs and capital as the fixed n'th input, the GLS cost function is

$$V(y,p,K) \quad = \quad \sup_{p_k} \{\, y \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}(p_i p_j)^{1/2} - p_k K \,\}$$

$$= \quad y \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} (b_{ij} + \frac{b_{in}\, b_{jn}}{\frac{K}{y} - b_{nn}})(p_i p_j)^{1/2} \quad . \qquad (3.1)$$

Defining the coefficients

$$d_{ij} \quad = \quad b_{ij} + \frac{b_{in}\, b_{jn}}{\frac{K}{y} - b_{nn}} \quad , \qquad i,j = 1,\ldots,n-1, \qquad (3.2)$$

shows that the GLS function has the same form as an n-1 input GL function, but the d_{ij} coefficients are functions of the capital-output ratio. And the function is no longer linear in the unknown parameters. The domain of the function is restricted to the set of K and y such that $K/y > b_{nn}$[4]. If $K < y\, b_{nn}$, then the output y cannot be produced with the given capital stock and any quantity of the variable inputs. The function is concave in prices and convex in y and K.

The demand for the variable factors are given by the price derivatives of (3.1)

$$x_i(y,p,K) \quad = \quad \frac{\partial V(y,p,K)}{\partial p_i} \quad = \qquad i = i,\ldots,n-1,$$

$$= \quad y \sum_{j=1}^{n-1} (b_{ij} + \frac{b_{in}\, b_{jn}}{\frac{K}{y} - b_{nn}}) \; (\frac{p_j}{p_i})^{1/2} \quad , \qquad (3.3)$$

while the short run input coefficients are

$$a_i(y,p,K) \quad = \quad \frac{x_i}{y} \quad = \quad \sum_{j=1}^{n-1} (b_{ij} + \frac{b_{in}\, b_{jn}}{\frac{K}{y} - b_{nn}})(\frac{p_j}{p_i})^{1/2} \quad . \qquad (3.4)$$

The shadow price of the capital stock is given by

$$p_S = -\frac{\partial}{\partial K} V(y,p,K) = \frac{\left[\sum_{i=1}^{n-1} b_{ik}\, p_i^{1/2}\right]^2}{\left[\frac{K}{y} - b_{nn})\right]^2} . \qquad (3.5)$$

Combining these last two equations we see that the marginal cost can be written

$$\frac{\partial v}{\partial y} = \sum_{i=1}^{n-1} p_i a_i + p_S a_K , \qquad (3.6)$$

where a_K is the capital output coefficient K/y.

Expressed explicitly in terms of the chosen functional forms, the input coefficients of the three stages for sector k are

3rd stage (GLS) [see (3.4)]

$$a_{ik} = \frac{x_{ik}}{y} = \sum_j (b^k_{ij} + \frac{b^k_{iK}\, b^k_{jK}}{\frac{K_k}{y_k} - b^k_{KK}}) (\frac{p_{jk}}{p_{ik}})^{1/2}, \qquad i,j = M,U,L. \qquad (3.7)$$

2nd stage (Leontief and CES)

$$a_{ik} = \frac{x_{ik}}{x_{Mk}} = \text{constant} , \qquad i\epsilon M \qquad (3.8)$$

$$a_{Ek} = \left[\delta^E_K (P_E)^{1-\sigma_k} + (1-\delta^E_k)(P_{Fk})^{1-\sigma_k} \right]^{\frac{\sigma_k}{1-\sigma_k}} \delta^E_k (P_E)^{-\sigma_k} , \qquad (3.9)$$

$$a_{Fk} = \left[\delta^E_k (P_E)^{1-\sigma_k} + (1-\delta^E_k)(P_{Fk})^{1-\sigma_k} \right]^{\frac{\sigma_k}{1-\sigma_k}} (1-\delta^E_k)(P_{Fk})^{-\sigma_k}, \qquad (3.10)$$

1st stage (CES)

$$m^A_{ik} = \frac{x^A_{ik}}{x_{ik}} = \left[(1-\delta^B_{ik})(p^A_i)^{1-\sigma_i} + \delta^B_{ik}(p^B_i)^{1-\sigma_i} \right]^{\frac{\sigma_i}{1-\sigma_i}}$$

$$(1-\delta^B_{ik})(p^A_i)^{-\sigma_i} \quad , \qquad i\varepsilon M,\ i=F. \tag{3.11}$$

$$m^B_{ik} = \frac{x^B_{ik}}{x_{ik}} = \left[(1-\delta^B_{ik})(p^A_i)^{1-\sigma_i} + \delta^B_{ik}(p^B_i)^{1-\sigma_i} \right]^{\frac{\sigma_i}{1-\sigma_i}}$$

$$\delta^B_{ik}\ (p^B_i)^{-\sigma_i} \quad , \tag{3.12}$$

where M represents the set of material inputs. Using the above expressions we can determine the prices for material, fuel, and energy inputs

$$p_{Mk} = \sum_{i\neq E,F} a_{ik}\ (m^A_{ik}p^A_i + m^B_{ik}p^B_i)\ , \tag{3.13}$$

$$p_{Fk} = m^A_{Fk}\ p^A_F + m^B_{Fk}\ p^B_F \quad , \tag{3.14}$$

$$p_{Uk} = p_E\ a_{Ek} + p_{Fk}a_{Fk} \quad . \tag{3.15}$$

The price of labor is determined endogenously so as to insure full employment and is assumed to be the same for all sectors. The above may be supplemented by an expression for the shadow price of capital [see (3.5)]

$$p_{Sk} = \frac{\left[\sum\limits_{i=M,U,L} b^k_{iK}\ p^{1/2}_{ik}\right]^2}{\left[\frac{K_k}{y_k} - b^k_{KK}\right]^2} \quad . \tag{3.16}$$

This price, or more particularly the ratio of p_{Sk} to the price of capital services p_{Kk}, is a useful indicator of the capacity utilization of the sectors and of its discrepancy from long run equilibrium. It also makes it easier to write down the pricing equation [see (3.6)]

$$p_{Ak} = \sum_{i=M,U,L} p_{ik}a_{ik} + p_{Sk}a_{Kk}, \tag{3.17}$$

giving the equality between output price and marginal cost.

The complete system (3.7)-(3.17) is equivalent to the more compact, but less revealing, formulation (2.3) and (2.4), and the former will exhibit all the duality properties described in the previous section. In particular there exists a short run restricted profit function H [see (2.10)] which can represent the model, an issue which we will return to in section 5.

4 DATA AND ESTIMATION

The model contains 12 commodities and 9 production sectors, which are listed in appendix 1. This appendix also shows the relationship of the commodity and sector classifications utilized in this paper to those of the principal models operated by the Central Bureau of Statistics. The gross output of each production sector and the net output of each commodity is presented below in table 5.1.

The domestic commodities are divided into three groups as outlined in section 2:

i) Commodities for which the economy is a price taker:
   35 - Raw materials,
   40 - Gasoline and heating oil (F),
   60 - Shipping services,

ii) Commodities for which the economy is a quantity taker:
   10 - Products of agriculture, forestry, and fishing,
   20 - Consumer goods,
   47 - Investment goods,
   55 - Buildings,
   75 - Services,
   90 - Government goods and services.

iii) Raw materials:
   65 - Crude oil,
   71 - Electricity (E).

Gasoline and heating oil are also called fuel and are designated by F when used as inputs. Similarly, electricity is often represented by an E. There are no imports of commodities 55, 60 and 90 and the raw material 71. The other commodities can also be imported, and the substitutability of the domestically produced and the imported commodity is described by the lower level CES functions (3.11) and (3.12). Only crude oil is regarded as being a homogeneous good which is traded at an internationally given price. The 12th good of the model is

03 Non-competing imports,

of which there is no domestic production [5]. We have settled on this division because it seems to be an appropriate one in which to analyze the foreign trade sector of the Norwegian economy, but the partition will, as emphasized in section 2, depend upon the problem at hand.

The base year of the model is 1975 and most fixed coefficients, such as import shares and input-output coefficients, are taken directly from the national accounts of that year. The behavioral relations are estimated on national acounting data for the period 1962 to 1981. The choice of 1975 as a base year is somewhat unfortunate since it underestimates the magnitude of the oil sector compared to its current level.

Each sector of the model produces a single output, and we have identified the quantity of the output of that commodity with the gross output of the sector. The commodity flows may be regarded as having been evaluated in producer's prices, though greater attention should have been given to the question of valuation, the preparation of base year data and the treatment of multiple outputs.

There are three set of relationships which have been (or will be) estimated empirically. These are

i) the import functions,
ii) the energy functions,
iii) the GLS functions.

The other coefficients of the model have been determined directly from the national accounts for the base year.

The import functions have not yet been estimated. We have assumed that the import shares m_{ik}^{B} and domestic shares m_{ik}^{A} are constant for most commodities and sectors, i.e. that $\sigma_{ik}=0$ in (3.11) and (3.12). But for the 26 largest import flows we have assumed that the elasticity of substitution is 2 and the import shares δ_{ik}^{B} set equal to their actual value in 1975. [6]

The energy substitution has been estimated using the relationship

$$\ln \frac{x_{Ek}}{x_{Fk}} = \ln \frac{\delta_k^E}{1-\delta_k^E} - \sigma_k \ln \frac{p_E}{p_F} , \qquad (4.1)$$

which gives the logarithm of the ratio of the cost minimizing inputs of electricity and oil as a linear function of relative prices. The results [7] are presented in table 4.1.

Table 4.1: Regression results for the estimation of energy substitution.

Sector	$\ln \frac{\delta^E}{1-\delta^E}$	σ	SER	SSR	RSQ	DW
10	-1.244 (0.041)	0.617 (0.137)	0.172	0.534	0.530	0.480
20	-0.241 (0.023)	0.553 (0.096)	0.101	0.185	0.647	1.670
35	0.975 (0.077)	0.096 (0.206)	0.284	1.451	0.012	0.686
47	0.110 (0.042)	0.918 (0.218)	0.183	0.601	0.497	0.313
55	-1.588 (0.105)	1.876 (0.484)	0.416	3.111	0.455	0.492
75	-0.998 (0.057)	0.677 (0.154)	0.151	0.412	0.516	0.932
90	-0.689 (0.048)	0.735 (0.215)	0.214	0.828	0.393	0.323

Regression period: 1962 to 1981.

The GLS functions have been estimated using the coefficient form (3.4). We have estimated the set of three equations a_{Mk}, a_{Uk}, and a_{Lk} simultaneously using full information maximum likelihood. Estimates for four of the 9 sectors are presented in table 4.2. Two of these estimates where obtained by including a Hicks' neutral exponential technical change in the factor demand equations. The functions in table 4.2 are, with the exception of the early years for sector 55, concave for all observations in the sample period. For the remaining 5 sectors we were only able to get the "right" results for the GLS function after imposing suitable restrictions or by using "extraneous" information.

Table 4.2 Parameter estimates for the GLS function for select sectors: 1962 - 1981*

		20 - Manufacture of consumer goods	47 - Manufacture of investment goods	55 - Construction	60 - Shipping
b_{MM}		.2382 (.0492)	-.0970 (.0664)	-.0991 (.0524)	.3175 (.0507)
b_{MU}		.0060 (.0034)	.0161 (.0065)	-.0044 (.0023)	.0041 (.0083)
b_{ML}		.3166 (.0314)	.6549 (.0640)	.6517 (.0465)	.2963 (.0543)
b_{UU}		.0044 (.0035)	-.0059 (.0051)	.0014 (.0017)	-.0005 (.0071)
b_{UL}		.0046 (.0052)	.0013 (.0052)	.0047 (.0033)	.0067 (.0059)
b_{LL}		-.1241 (.0303)	-.3354 (.0646)	-.3489 (.0484)	.0180 (.0718)
b_{MK}		.0726 (.0324)	.0820 (.0363)	.0316 (.0112)	.0607 (.0143)
b_{UK}		.0009 (.0010)	-.0064 (.0049)	.0008 (.0003)	.0018 (.0008)
b_{LK}		.0141 (.0111)	-.0325 (.0341)	-.0027 (.0082)	.0659 (.0175)
b_{KK}		.3205 (.0271)	.3239 (.0441)	.0472 (.0034)	2.9637 (.0207)
trend		.0089 (.0033)		.0137 (.0033)	
FCN		32.5989	31.6021	28.3365	34.9684
R^2	M	.3566	-3.5271	.7847	-.3276
	U	.5059	.8184	.5993	.1183
	L	.9836	.9197	.9449	.5671
DW	M	.440	.126	.719	1.084
	U	1.376	1.296	1.848	1.018
	L	1.931	.343	1.607	.797
SER	M	.0369	.0778	.0248	.1010
	U	.0013	.0010	.0004	.0044
	L	.0079	.0375	.0182	.1306
LHS MEAN	M	.5871	.5495	.5297	.5776
	U	.0161	.0101	.0020	.0097
	L	.2398	.3736	.3594	.3725

) "Full information maximum likelihood (FIML) estimates using the program package TROLL. The numbers in parenthesis are asymptotic standard errors. FCN is the scaled form of the negative of the (concentrated) log-likelihood function. R^2, DW, SER, and LHS MEAN are single equation statistics.

5 ELASTICITIES OF THE MODEL

A key feature of the short run model is the fact that the short run equilibrium may not be a long run equilibrium since the existing capital stock need not be optimal. The departure from equilibrium in sector k may be measured by the ratio p_{Sk}/p_{Kk} of the shadow price of capital to the user cost of capital. This ratio is given in column 4 of table 5.1: it should be 1 at a long run equilibrium. Sectors 35, 75, and 90 are producing at close to long run equilibrium, while 60 seems to have a substantial amount of excess capacity and 40 produces well above full capacity.

How fast p_{Sk} will change as output changes will depend on the steepness of the marginal cost function, i.e. on the second derivative of the sectoral profit function Π^k [see (2.5)] with respect to the output price. A measure of this responsiveness is provided by the price elasticity of supply, which is presented in the fifth column of table 5.1.[8]. The table shows substantial differences in the estimated elasticities of supply: it is rather low (.48) for agricultural products, while it is very high for construction (22.66) and shipping (6.45).

Table 5.1: Summary description of production sectors: computed values for the base year of the model.

sector	gross output* y	final demand* x_F	capacity pressure p_S/p_K	elasticity of supply
10	14 091	2 678	.67	.48
20	54 227	32 859	2.38	.99
35	24 444	16 337	1.18	1.24
40	7 143	3 203	11.20	.77
47	28 524	15 270	.67	3.14
55	25 961	17 700	.37	22.66
60	12 042	11 859	.04	6.45
75	85 343	50 618	.92	1.70
90	30 562	29 657	1.13	.77

* Measured in million kroner (base year prices).

In section 2 we emphasized that the model could conviniently be summarized by the restricted profit function $H(p_A^1, x_F^2, p_B, v, L, K)$ [see (2.10)] and that the endogenously determined supplies and prices are given by the first derivatives of H

$$
\begin{aligned}
x_F^1 &= \frac{\partial H}{\partial p_A^1}, & p_A^2 &= -\frac{\partial H}{\partial x_F^2}, \\
x_B &= -\frac{\partial H}{\partial x_F^2}, & p_v &= \frac{\partial H}{\partial v}, \qquad (5.1) \\
p_L &= \frac{\partial H}{\partial L}, & p_S &= \frac{\partial H}{\partial K}.
\end{aligned}
$$

We have assumed that the output price p_{v65} of the raw material "crude oil" is determined on the international market.

The resulting model is best summarized by the elasticities of the net suppply and price equations. The partial derivatives of the supply and price equations are, except for the sign, given by the elements of the Hessian matrix H as illustrated by the following submatrix of H

$$
H_{AA} = \begin{bmatrix} \dfrac{\partial^2 H}{(\partial p_A^1)^2} & \dfrac{\partial^2 H}{\partial p_A^1 \partial x_F^2} \\[2ex] \dfrac{\partial^2 H}{\partial x_F^2 \partial p_A^1} & \dfrac{\partial^2 H}{(\partial x_F^2)^2} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial x_F^1}{\partial p_A^1} & \dfrac{\partial x_F^1}{\partial x_F^2} \\[2ex] -\dfrac{\partial p_A^2}{\partial p_A^1} & -\dfrac{\partial p_A^2}{\partial x_F^2} \end{bmatrix} . \qquad (5.2)
$$

The full Hessian of the restricted profit function is, except for the sign of some of the elements, nothing but the Jacobian of the reduced form of the model. The fact that the computed first derivatives of the net supply and price equations (5.1) form, again allowing for the sign convention, a symmetric matrix is a confirmation of the existence of the H function, and a restatement of Samuelson's reciprocity condition.[9)]

Normalizing the elements of H_{AA} gives the matrix of supply elasticities

$$E_{AA} = \begin{bmatrix} \dfrac{\partial x_F^1}{\partial p_A^1} \dfrac{p_A^1}{x_F^1} & \dfrac{\partial x_F^1}{\partial x_F^2} \dfrac{x_F^2}{x_F^1} \\ \\ \dfrac{\delta p_A^2}{\delta p_A^1} \dfrac{p_A^1}{p_A^2} & \dfrac{\delta p_A^2}{\delta x_F^2} \dfrac{x_F^2}{p_A^2} \end{bmatrix} \tag{5.3}$$

of the endogenous net supplies x_F^1 and output prices p_A^2 with respect to the exogenous output prices p_A^1 and the exogenous final demand x_F^2, and these elasticities are given in table 5.2 below. The full matrix of elasticities

Table 5.2: Elasticities of supply.

Endog. var.	Exogenous variables								
	PA35	PA40	PA60	XF10	XF20	XF47	XF55	XF75	XF90
XF35	1.50	-0.05	-0.61	-0.05	-0.48	-0.18	-0.26	1.00	-0.62
XF40	-0.27	3.10	-0.68	-0.04	-0.41	-0.13	-0.19	-1.47	-0.60
XF60	-0.84	-0.18	4.62	-0.13	-1.14	-0.49	-0.61	-2.74	-1.91
PA10	0.26	0.04	0.50	0.33	0.95	0.11	0.24	0.69	0.52
PA20	0.24	0.04	0.41	0.09	1.00	0.10	0.21	0.63	0.43
PA47	0.20	0.03	0.39	0.02	0.22	0.28	0.13	0.51	0.34
PA55	0.27	0.04	0.46	0.05	0.45	0.13	0.20	0.69	0.46
PA75	0.28	0.08	0.55	0.04	0.35	0.13	0.18	1.17	0.55
PA90	0.27	0.05	0.61	0.04	0.38	0.14	0.20	0.88	1.86

of (5.1) is given in appendix 8. Increasing the exogenously given output prices increases the supply of the "own" good and reduces the supply of the other goods, at the same time as it increases all the endogenously determined output prices. Increasing the exogenous output demand decreases the net supply of the endogenously determined outputs. The first three elements on the diagonal represent the price elasticities of supplies of the three goods whose price is exogenous and may be compared with the corresponding elements in the fifth column of table 5.1, though they are not strictly comparable since the elasticities in table 5.1 are measured with respect to the net output of the sector, while the elasticities in table 5.2 are normalized with respect to the net output of all the production sectors, i.e. net of the intermediate inputs. The difference is also due to the simultaneity of the whole model and that the prices of most of the outputs, electricity, and labor, which each producer regards as fixed, now become determined by the model. The price elasticity increases for sectors 35 and 40, and decreases for sector 60.

6 DYNAMIC MODEL

From one year to the next it becomes possible to alter the capital stock, and we assume that in the long run, and when determining his investment, the producer tries to to minimize long run total cost

$$C^k(y_k,p_A,p_B,p_V,p_L,p_{Kk}) = y_k\, c^k(p_A,p_B,p_V,p_L,p_{Kk})$$

$$= \min_{p_{Kk}} \{p_{Kk}K_k + V^k(y_k,p_A,p_B,p_V,p_L,K_k)\}, \qquad k=1,\dots,m, \qquad (6.1)$$

where p_{Kk} is the user cost of capital in sector k and c^k is the long run unit cost function. The first equality follows from the assumed linear homogeneity of the production function. Assuming myopic expectations we obtain from (6.1) the sector's desired capital input coefficient

$$a^d_{Kk} = \frac{\partial}{\partial p_{Kk}} c^k(p_A,p_B,p_V,p_L,p_{Kk}) \quad , \qquad (6.2)$$

and the sector's desired capital stock $K^d_k = y_k a^d_{Kk}$. The rate of net investment is determined by a flexible accelerator so that gross investment in sector k is

$$z_k = \mu_k \left[y_k\, a^d_{Kk}(p_A,p_B,p_V,p_L,p_{Kk}) - K_k \right] + \delta_k K_k \ , \qquad (6.3)$$

where μ_k is the rate of adjustment and δ_k is the depreciation rate. In the example below we have rather arbitrarily assumed that μ=0.5 in all sectors. K_k is, contrary to national accounting practice, the capital stock at the beginning of the period. Whether net investment is positive or negative will depend upon whether the desired capital input coefficient a^d_{Kk} exceeds or not the actual input coefficient a_{Kk}, or equivalently whether the shadow price of capital p_{Sk} is greater or smaller than its service price p_{Kk}.

Investment in sector k is a composite of the goods produced by the economy, the composition being described by the vector $(b_{1k},\dots,b_{nk})$ which is normalized so as to sum to unity. Investment goods may be either imported or produced domestically. We assume that this proportion is constant and described by the import share matrix m_{BJ} and the domestic share matrix $m_{AJ} = E - m_{BJ}$, E being a matrix of ones. A gross investment level z_J implies a demand for domestic commodities equal to $(m_{AJ} \bullet B)z_J$ and a demand for imported goods equal to $(m_{BJ} \bullet B)z_J$.

The cost of a new unit of capital equipment is

$$p_J = (m_{AJ} \bullet B)' \, p_A + (m_{BJ} \bullet B)' \, p_B \, , \tag{6.4}$$

while the user cost of capital is

$$p_K = (rI + \hat{\delta})p_J = (rI + \hat{\delta}) \left[(m_{AJ} \bullet B)' p_A + (m_{BJ} \bullet B)' p_B \right] , \tag{6.5}$$

r being the rate of interest. The description is completed by the commodity balance equations for the $n_A + n_V$ domestic commodities

$$y_i = \sum_k \frac{\partial}{\partial p_{Ai}} V^k(y_k, p_A, p_B, p_V, p_L, K_k) + \sum_k (m_{AJ})_{ik} b_{ik} z_{Jk} + x_{Ci} \, , \tag{6.6}$$

and by the import demand equation for the n_B imported commodities

$$x_{Bi} = \sum_k \frac{\partial}{\partial p_{Bi}} V^k(y_k, p_A, p_B, p_V, p_L, K_k) + \sum_k (m_{BJ})_{ik} b_{ik} z_{Jk} \, . \tag{6.7}$$

The above equations describe the static model, i.e. the single period model describing behavior in an arbitrary period t with fixed capital stock. The dynamic behavior is then induced by the relation between the capital stock and gross investment

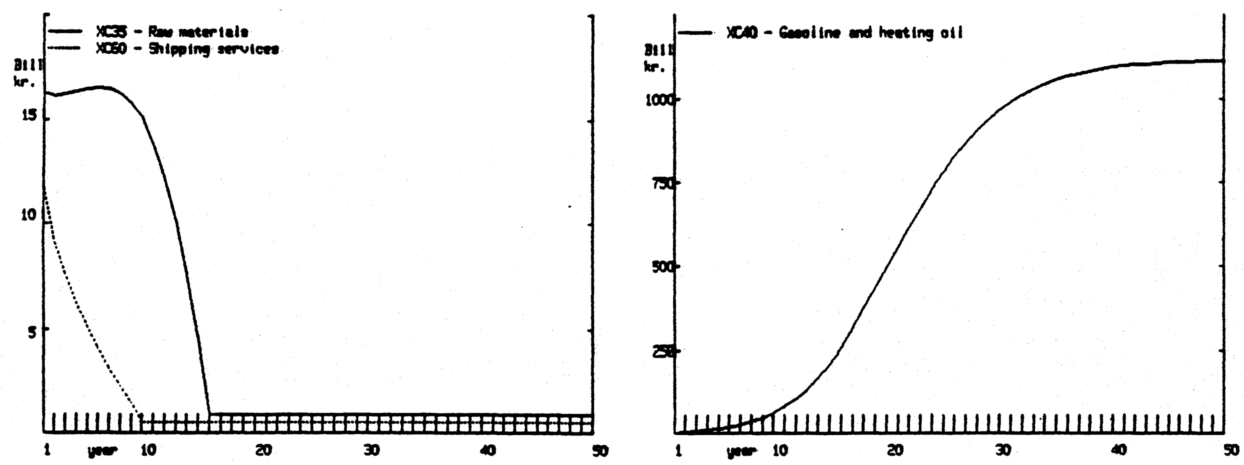
$$K_{t+1} = z_t + (1 - \delta)K_t \, . \tag{6.8}$$

The base year (year 1) of the model is not in long run equilibrium as is evident from table 5.1: p_S/p_K deviates from 1 in all sectors.

To see what happens as the economy evolves over time, we have simulated the model for 100 years holding the level of all the exogenous variables p_A^1, x_F^2, p_B and p_V constant. The model reached a level close to long run equilibrium after about 50 years so we will concentrate on the shorter time span.

The development is dominated by the very productive (and profitable) sector 40 "oil refining". This sector uses very little of the two domestic inputs electricity and labor, while over 70 per cent of the value of its inputs consists of the raw material crude oil, for which the economy is a price taker. Thus an expanding oil refining sector will only to a modest degree be faced with increasing marginal cost. As shown in fig. 6.1 the economy's net supply of oil and gasoline increases from 3.2 billion kroner in period 1 to 1116 billion kroner in year 50, measured in base year prices. The parameters for sector 40 have not been estimated for lack of data.[10] This obviously greately reduces the empirical applicability of the following results, but it illustrates well the large changes which the model is capable of simulating.

Fig. 6.1: Output level of price taking industries.



This expanding oil refining sector draws resources away from the other two price taking sectors. First from the shipping sector which was rather unprofitable even in the base year, and then from the metals sector. Since the computer program does not allow the introduction of nonnegativity constraints into the model, net output did eventually become negative. We have then fixed the net output at the level of the last year for which it was positive.[11] This also insures that sufficient production still goes on to produce intermediate inputs and meet the demand for investment.[12]

The adjustments in the other sectors are less dramatic. Table 6.1 shows the changes in some of the main endogenous variables from period 1 to period 50 (the table gives the ratio of the value of the variables in these

Table 6.1: Change in select endogenous variables from period 1 to period 50.

sector	gross output y	employ-ment L	capital stock K	output price p_A
10	1.07	0.89	1.06	2.89
20	1.23	0.76	1.48	2.11
35	0.26	0.24	0.29	2.54*
40	160.01	170.42	191.21	1.00
47	1.99	0.66	1.61	2.05
55	1.72	0.79	1.25	2.34
60	0.09	0.06	0.07	2.58*
65				1.00
71				11.98
75	1.61	1.30	1.69	2.70
90	1.04	0.98	1.13	2.79

* These factors are not 1.00 since output was endogenized during the simulation period to avoid negative output levels (see text).

two years). Employment declines in all the price setting sectors, with the output price in all these sectors increasing significantly. The wage rate increased from 1.00 to 3.47. It is interesting to note that we end up with complete specilization among the three price taking seectors, with a single "exportable" commodity being produced, even though we have two non-traded "resources": electricity and labor.

7 CONCLUDING REMARKS

The primary purpose of this paper has been to describe the implementation of a short run model using the short run GLS function, and to develop a consistent representation of the short run and the long run technology. This also makes it possible to analyze the dynamic path from a short run to a long run equilibrium.

Emphasis has also been put on the relationship between the numerically implemented model and modern duality so that the latter may be used to analyze and draw conclusions about the former.

Interesting directions for future work would be to implement the model of the production sectors in a full Walrasian model with an explicit consumption sector, and to pay greater attention to the dynamic formulation of the model. The latter could perhaps be done either by studying its relationship to Hamiltonian dynamics, or by utilizing optimal control methods. The concavity-convexity of the restricted profit function would probably guartantee a unique optimal solution, but the model may still be too large for numerical optimization.

APPENDIX 1. COMMODITY AND SECTOR CLASSIFICATION

The following two tables present the commodity and sector classification utilized in the model. The three last columns show how our classification corresponds to that of the quarterly model KVARTS, the medium term model MODAG and the long run model MSG, and to the classification of the national accounts data base AARNR.

Table A.1: Commodity classification.

Code	Name of commodity	Other classifications		
		KVARTS	MODAG/MSG	AARNR
10	Products of agriculture, forestry, and fishing	10	11,12,13	12,13,21,22
20	Consumer goods	15,25	16,17,18,26, 27,28	16,17,18,26, 27,28
35	Raw materials	Δ30	32,33,34,37, 43	32,33,34,37, 43
40	Gasoline and heating oil	Δ30	41,42	41,42
47	Investment goods	45,50	45,50	46,47,48,49
55	Buildings	55	55	55
60	Shipping services	60	60	60
65*	Crude oil	66,67	66,67,68 2)	66,67,68,69
71*	Electricity	Δ70	72,73	71
75	Services	Δ70,80	74,79,81,82, 83,84	
90 1)	Government goods and services	90	91,92,93,94, 95	91,92,93,94, 95
03	Non-competing imports	00,01,02, 05,06,07	00,01,02,05, 06,07 3)	

* This commodity is treated as a raw material in the model.

Δ The symbol Δxx means that the commodity contains parts of Kvarts commodity xx.

1) GLSMOD commodity 90 includes all production of goods and services by central and local government. These are treated as sector-sector flows by the other classifications.

2) Commodity 68 divided into commodities 68 and 69 from 1982.

3) Changes in classification in 1982.

Table A.2: Classification of production sectors.

Code	Name of sector	Other classifications		
		KVARTS	MODAG/MSG	AARNR
10	Agriculture, forestry, and fishing	10	11,12,13	12,13,21,22
20	Manufacture of consumer goods	15,25	16,17,18,26, 27,28	16,17,18,26, 27,28
35	Mining and manufacturing of raw materials	Δ30	31,34,37,43	31,34,37,43
40	Oil refining	Δ30	40	40
47	Manufacture of investment goods	45,50	45,50	45,48,49
55	Construction	55	55	55
60	Shipping	60	60	60
65*	Crude oil extraction	65	65 1)	66,68,69
71*	Electricity generation	Δ70	72,73	71
75	Production of services	Δ70,80	74,79,81,82, 83,84	61,63,75,76, 77,78,79,83, 86,87,88,89, 44,51,52,53, 54,56,57 2)
90	Government	90	91,92,93,94, 95	91S,92S,93S, 94S,95S,96S, 97S,98S,91K, 93K,94K,95K, 97K

* Sector producing one of the raw materials of the model.
Δ The symbol Δxx means that the sector comprises parts of Kvarts sector xx.
1) MODAG/MSG sector 65 has recently been split into sectors 64,68.
2) The constant price adjustment account 58 has not been included.

APPENDIX 2: MODEL ELASTICITIES

Endogenous variables	Exogenous variables: PA35	PA40	PA60	XF10	XF20	XF47	XF55	XF75	XF90	PB10	PB20	PB35	PB40	PB47
XF35	1.503	-0.053	-0.612	-0.048	-0.484	-0.184	-0.259	-1.005	-0.617	-0.056	-0.194	-0.621	-0.029	-0.115
XF40	-0.272	3.099	-0.677	-0.040	-0.406	-0.135	-0.193	-1.466	-0.598	-0.023	-0.133	0.032	-0.542	-0.093
XF60	-0.843	-0.183	4.620	-0.128	-1.144	-0.487	-0.614	-2.742	-1.909	-0.081	-0.362	0.028	-0.122	-0.681
PA10	0.260	0.042	0.501	0.327	0.947	0.112	0.235	0.694	0.525	0.207	0.213	-0.062	0.027	0.044
PA20	0.238	0.039	0.408	0.087	0.996	0.098	0.213	0.626	0.432	0.071	0.319	-0.011	0.024	0.048
PA47	0.202	0.029	0.389	0.023	0.221	0.277	0.134	0.515	0.344	0.014	0.084	0.093	0.019	0.314
PA55	0.268	0.039	0.460	0.045	0.448	0.125	0.198	0.686	0.462	0.032	0.174	0.019	0.026	0.142
PA75	0.277	0.079	0.548	0.036	0.351	0.129	0.183	1.172	0.550	0.021	0.112	-0.027	0.053	0.086
PA90	0.270	0.051	0.607	0.043	0.385	0.137	0.196	0.875	1.856	0.028	0.140	-0.016	0.035	0.091
XB10	0.671	0.053	0.697	0.459	1.720	0.152	0.370	0.928	0.749	-1.250	0.341	-0.191	0.037	0.020
XB20	0.468	0.063	0.635	0.096	1.568	0.184	0.406	0.986	0.771	0.069	-0.946	-0.092	0.038	0.034
XB35	1.285	-0.013	-0.042	-0.024	-0.045	0.175	0.037	-0.199	-0.074	-0.033	-0.079	-1.059	-0.007	-0.008
XB40	0.189	0.702	0.587	0.034	0.328	0.113	0.167	1.280	0.525	0.020	0.103	-0.021	-1.435	0.064
XB47	0.242	0.039	1.043	0.017	0.205	0.600	0.289	0.656	0.439	0.004	0.030	-0.008	0.021	-0.859
XB65	-0.007	0.967	0.004	-0.000	-0.007	-0.005	-0.001	-0.017	0.003	-0.000	-0.008	-0.002	-0.023	-0.015
XB75	0.522	0.186	1.035	0.044	0.542	0.242	0.339	2.715	1.004	0.018	0.122	-0.097	0.123	0.083
XB03	-0.374	-0.090	2.729	-0.079	-0.547	-0.254	-0.310	-1.330	-0.948	-0.056	-0.201	-0.020	-0.060	-0.411
PA71	0.914	-0.151	0.508	0.027	0.382	0.132	0.183	1.441	0.583	-0.001	0.083	-0.160	-0.109	0.070
PL	0.287	0.042	0.690	0.045	0.388	0.150	0.214	0.917	0.674	0.028	0.114	-0.021	0.030	0.086
PS10	0.222	0.004	0.441	0.867	1.907	0.088	0.334	0.601	0.508	0.539	0.339	-0.167	0.006	-0.071
PS20	0.134	0.023	0.289	-0.000	3.661	0.080	0.472	0.536	0.407	-0.019	0.937	-0.107	0.014	-0.073
PS35	4.477	-0.161	-1.751	-0.161	-1.317	-0.427	-0.597	-2.939	-1.770	-0.198	-0.614	-0.737	-0.085	-0.369
PS40	-0.131	6.391	-0.222	-0.017	-0.173	-0.077	-0.078	-0.404	-0.221	-0.011	-0.082	-0.004	-0.106	-0.108
PS47	-0.325	-0.038	0.247	-0.077	-0.495	3.788	0.229	-0.888	-0.577	-0.065	-0.268	-0.212	-0.054	2.064
PS55	0.337	0.073	0.394	0.042	0.465	0.129	2.242	0.916	0.727	0.026	0.165	-0.003	0.046	0.137
PS60	-4.436	-0.964	25.683	-0.678	-6.024	-2.563	-3.230	-14.441	-10.023	-0.427	-1.911	0.139	-0.645	-3.631
PS75	0.319	0.105	0.608	0.023	0.330	0.148	0.211	2.433	0.596	0.008	0.077	-0.066	0.068	0.053
PS90	0.277	0.056	0.636	0.042	0.384	0.140	0.201	0.939	5.761	0.026	0.134	-0.022	0.037	0.084

Endogenous variables	Exogenous variables: PB65	PB75	PB03	V71	L	K10	K20	K35	K40	K47	K55	K60	K75	K90
XF35	0.002	-0.048	0.223	0.244	1.747	0.065	0.067	1.050	-0.009	-0.022	0.007	-0.091	0.380	0.159
XF40	-1.576	-0.088	0.273	-0.206	1.302	0.005	0.057	-0.192	2.176	-0.013	0.008	-0.101	0.638	0.163
XF60	-0.002	-0.132	-2.242	0.187	5.785	0.177	0.198	-0.566	-0.020	0.023	0.011	0.727	0.997	0.504
PA10	-0.000	0.022	-0.253	-0.038	-1.479	-1.359	0.001	0.204	0.006	0.028	-0.005	0.075	-0.145	-0.129
PA20	-0.001	0.025	-0.161	-0.050	-1.162	-0.273	-0.895	0.152	0.006	0.016	-0.005	0.061	-0.193	-0.109
PA47	-0.002	0.025	-0.167	-0.039	-1.008	-0.028	-0.044	0.110	0.006	-0.282	-0.003	0.058	-0.194	-0.089
PA55	-0.000	0.032	-0.191	-0.051	-1.347	-0.101	-0.242	0.145	0.005	-0.016	-0.047	0.069	-0.259	-0.120
PA75	-0.001	0.069	-0.219	-0.106	-1.536	-0.048	-0.073	0.190	0.007	0.017	-0.005	0.082	-0.798	-0.149
PA90	0.000	0.041	-0.248	-0.068	-1.796	-0.065	-0.089	0.182	0.006	0.017	-0.007	0.090	-0.311	-1.452
XB10	-0.001	0.020	-0.397	0.004	-2.047	-1.870	0.110	0.553	0.009	0.052	-0.006	0.104	-0.110	-0.178
XB20	-0.006	0.027	-0.290	-0.053	-1.675	-0.239	-1.127	0.348	0.013	0.044	-0.008	0.095	-0.223	-0.186
XB35	-0.001	-0.019	-0.025	0.089	0.260	0.101	0.110	0.357	0.001	0.030	0.000	-0.006	0.163	0.026
XB40	-0.049	0.075	-0.237	0.193	-1.189	-0.011	-0.046	0.131	0.047	0.024	-0.006	0.088	-0.535	-0.142
XB47	-0.010	0.016	-0.517	-0.040	-1.108	0.043	0.076	0.183	0.015	-0.294	-0.006	0.157	-0.132	-0.102
XB65	-0.910	-0.002	-0.004	0.010	-0.024	0.002	0.009	-0.003	1.000	0.008	0.000	0.001	0.024	0.000
XB75	-0.007	-1.504	-0.481	-0.242	-2.626	-0.001	0.014	0.392	0.018	0.054	-0.005	0.155	-1.397	-0.248
XB03	-0.002	-0.075	-1.440	0.101	3.042	0.152	0.159	-0.231	-0.009	0.027	0.007	0.417	0.548	0.258
PA71	-0.012	0.084	-0.225	-1.532	-1.305	0.014	-0.020	0.683	0.016	0.054	-0.005	0.076	-0.572	-0.157
PL	0.001	0.040	-0.298	-0.057	-2.111	-0.060	-0.049	0.197	0.007	0.026	-0.003	0.103	-0.270	-0.170
PS10	-0.002	0.000	-0.310	0.013	-1.247	-3.793	0.393	0.278	0.007	0.061	-0.004	0.066	0.029	-0.107
PS20	-0.006	-0.003	-0.190	-0.011	-0.595	0.230	-4.945	0.165	0.009	0.051	-0.011	0.044	-0.007	-0.085
PS35	0.004	-0.155	0.587	0.779	5.095	0.345	0.349	-0.749	-0.024	-0.032	0.017	-0.261	1.224	0.467
PS40	-4.787	-0.025	0.084	0.064	0.611	0.030	0.067	-0.083	-0.003	0.032	0.003	-0.033	0.221	0.062
PS47	-0.036	-0.074	-0.239	0.212	2.307	0.264	0.375	-0.112	0.031	-5.780	-0.007	0.040	0.519	0.171
PS55	-0.004	0.024	-0.195	-0.072	-0.981	-0.063	-0.259	0.196	0.008	-0.023	-2.932	0.059	-0.278	-0.176
PS60	-0.009	-0.698	-12.103	0.988	30.456	0.941	1.057	-2.974	-0.107	0.132	0.059	-1.516	5.274	2.650
PS75	-0.007	0.109	-0.275	-0.129	-1.382	0.007	-0.003	0.241	0.012	0.029	-0.005	0.091	-2.451	-0.153
PS90	-0.000	0.040	-0.268	-0.073	-1.800	-0.054	-0.074	0.191	0.007	0.020	-0.006	0.095	-0.316	-5.456

FOOTNOTES

1) See f.ex. Arrow and Hahn (1971), and Rockafellar (1970) corollary 16.4.1.

2) Less extreme assumptions are possible, but they would require the introduction of explicit demand functions.

3) See Diewert (1971).

4) The GLS function can be generalized to any number of fixed factors. It is described in greater detail in two unpublished papers, Frenger (1982,1983), the first analyzing the theoretical properties of the function, while the latter presents empirical estimates of the function and some tentative tests of the GL and the GLS functions.

5) There is, according to the national accounts, a small domestic production of goods which are classified as noncompeting imports. This has been ignored.

6) See Frenger (1980) and Stølen (1983) for analyses and estimates of import share functions at the MODAG/MSG aggregation level.

7) A more detailed analysis of energy substitution at the MODAG/MSG aggregation level is given in Bye (1984).

8) The elasticity allows for the sector's use of its own output as an input.

9) See Samuelson (1953, p. 10) and Woodland (1982, p. 91).

10) It is somewhat unfortunate that sector 40 is the only one for which the technology has not been estimated. Lack of data lead us to assume the same technology (same shadow elasticities of substitution) for sector 35 and 40. This may, among other things, lead to an overestimate of the substitutability among the inputs.

11) To achieve this we have made XC exogenous and PA endogenous at this point. Setting XC equal to zero leads to several difficulties with the investment levels etc., so this alternative was avoided.

12) The model allows, as formulated, for negative gross investment. This is rather plausible for the shipping sector where sale of used ships is common. But in the model negative gross investment will lead to a negative production of the component commodities and this is not very realistic.

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