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A REVIEW OF VARIOUS APPROACHES

ETTERSPØRSEL ETTER NATURGASS

EN OVERSIKT OVER ULIKE MODELLOPPLEGG

ØYSTEIN OLSEN AND KJELL ROLAND

STATISTISK SENTRALBYRÅ
CENTRAL BUREAU OF STATISTICS OF NORWAY

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STATISTISK SENTRALBYRÅ
OSLO - KONGSVINGER 1988

ISBN 82-537-2665-1
ISSN 0801-3845

EMNEGRUPPE
12 Energi

ANDRE EMNEORD
Energibærere
Gass
Naturgass
Modeller
Statisk/dynamiske modeller
Økonometri

Forsidebilde: Per Anders Rosenkvist/Sam foto

Preface

This study surveys different models and approaches for modeling the demand for natural gas. When analysing energy use it is important to take into account substitution possibilities between different energy carriers and between energy and other goods or factors. In particular, one should stress the interrelationship between energy use and capital equipment. This calls for a dynamic model, describing how the consumers adjust their capital stocks to changes in e.g. prices and incomes. In the residential sector, energy use is related to the stock of dwellings. A preferable procedure may then be to analyse the energy use decision as a discrete-continuous choice.

A major part of this work was carried out when the authors visited Lawrence Berkeley Laboratory in February 1986, and enjoyed the hospitality of International Energy Studies. A special thanks to the group leader, Lee Schipper.

Central Bureau of Statistics, Oslo, 8 July 1988

Gisle Skancke

Forord

Denne studien gir en oversikt over ulike metodeopplegg for å modellere etterspørselen etter naturgass. I slike analyser er det av avgjørende betydning å ta hensyn til mulighetene for substitusjon mellom ulike energibærere og mellom energi og andre varer eller innsatsfaktorer. Spesielt er det viktig å ta hensyn til sammenhengen mellom etterspørselen etter energi og bruk av realkapital. Dette kaller på en dynamisk modell, som beskriver hvordan energibruken justeres over tid som en følge av endringer i for eksempel priser og inntekter. I husholdningssektoren er energibruk spesielt knyttet til boligkapitalen. Husholdningenes energietterspørsel kan beskrives ved teorien for diskret-kontinuerlig valg.

Store deler av dette arbeidet ble utført da forfatterne besøkte Lawrence Berkeley Laboratory (LBL) i februar 1986. En spesiell takk rettes til Lee Schipper, som leder gruppen for International Energy Studies ved LBL.

Statistisk Sentralbyrå, Oslo, 8. juli 1988

Gisle Skancke

Contents

1. INTRODUCTION

Consumption of natural gas in Western Europe has increased considerably during the last 25 years. The first period of rapid expansion was initiated by the discovery of the Groningen field in the Netherlands in the early 1960's. Average yearly growth in consumption of natural gas in Western Europe over the period 1960 to 1979 was 16,5 per cent. The price hike on crude oil in 1979-80 and the following turmoil in all energy markets initiated a more stagnant phase in the development of the natural gas market; total consumption slightly declined over the next four years, but the share of gas in total energy use remained roughly stable. In 1984, the market started to grow again. This year, many observers believe, represents a turning point and the start of a period of mature but stable growth in the demand for natural gas.

Projections of future natural gas demand carried out in the past were in general based on rather simple methodologies and varied considerably depending on the market conditions at the time the projections were made. Projections made during the period of rapid expansion (1960's and 1970's) tended to prolong existing trends of high growth rates and increase in natural gas' share of total energy consumption. After the period of stagnation in gas demand was entered in 1979, projections were gradually scaled down. In this period, both consumption of natural gas and total energy use were projected to grow very moderately (1-2 per cent per year) compared to what was experienced in the previous years.

From a Norwegian point of view, it is of particular interest to shed light on the demand side of the natural gas market. This interest arises from Norway's role as a net exporter of oil and gas. As a relatively young and inexperienced producer of natural gas with no direct access to final consumers, knowledge of the market is limited and information not always easily obtainable. In order to increase its role in the natural gas market, huge front-up investments in projects with lead time of 7-10 years are necessary. Thus, there exists a need for assessing the demand for natural gas in the *long run.*

Against this background, the Division of Petroleum Economic Research of the Central Bureau of Statistics, Oslo decided to embark on a research program with the aim of increasing our knowledge and understanding of the natural gas market in Western Europe. An important part of this program is to develop a formal model framework describing the demand for gas.

On the demand side of the natural gas market, several aspects should be stressed when undertaking such a study. One important feature that should be emphasized is that for many end uses, gas can replace or be replaced by other energy carriers. Thus, it is essential to consider possibilities for interfuel substitution when analysing gas demand. However, interactions between energy and other goods and services should also be taken into account.

In particular, the relationship between energy and various kinds of capital goods should be stressed. A very important factor when analysing natural gas demand is the penetration of the distribution network for gas. In many European countries, considerable uncertainty exists to what degree the network will be extended to include new groups of potential consumers, in particular in the residential sector. Finally, it is still an open question whether changed trends in demand in the late 1970's and again possibly in the mid 1980's are explained by fluctuations in economic growth and energy prices, or if structural relationships of demand have also changed. This question stresses the need for undertaking empirical studies close to the "micro" level, where structural changes may be identified.

The purpose of this paper is twofold; to review the status of natural gas modeling and to serve as a background for the empirical efforts of modeling European gas demand. In the literature, a variety of approaches to modeling natural gas demand is reported. Since there are several common features in the demand structure for different energy goods, the methodologies proposed for studying gas demand are quite often similar to the methodologies recommended when analysing demand for other energy carriers. Thus, in this paper a number of models available for analysing energy demand *in general* are also reported. We refer to natural gas demand when the model reviewed relates directly to this market, and contains features which are believed to characterize the demand for natural gas in particular. On the other hand, when the methodology studied is of the more general kind, the term "energy demand" is used.

It should be stressed in the outset, however, that the paper does not intend to cover the whole spectrum of methodologies available when assessing the structure of natural gas demand. Through chapter 2 and 3 the emphasis is on what are usually classified as *econometric models* of energy demand. Most analysts agree that more detailed engineering studies are useful and necessary as suplements to the models suggested in these chapters. A comprehensive survey of engineering methods is beyond the scope of this study. However, the application of socalled *discrete choice models* (chapter 4) may be regarded as a synthesis between the traditional econometric approach and the engineering approach. In chapter 5, we briefly discuss some methodologies taking into account declining block rate schedules and variations in availability of natural gas in a demand model for natural gas.

2 THE STANDARD ECONOMETRIC APPROACH TO ENERGY DEMAND MODELING

A basic assumption underlying almost all analysis of economic relations is that of *optimizing behaviour*. This means that the economic agents choose values for their control variables so that the highest possible benefit is obtained given technological or other constraints. Households are typically assumed to maximize utility given their budget constraint, while producer behaviour commonly is based on profit maximization restricted by the production technology. In both cases, from the optimizing behaviour, *demand relations* for all input variables are derived. In particular, the demand for *energy* is determined by an equation of the following form:

$$E = f\ (p\ ,y\ ,z\ ) \tag{2.1}$$

where

E is energy consumption,

p is a vector of (relative) prices,

y is the real income - or activity level, and

z is a vector of other independent variables believed to affect energy demand, such as climate, demographic factors etc.

When no lags in the included variables are introduced, a rather simple econometric model is obtained by choosing a specific functional form for (2.1) and adding common stochastic assumptions. Examples of energy demand studies based on this simple one-equation type of model are Anderson (1973), Halvorsen (1973) and Wilson (1971), all focusing, however, on electricity demand. One advantage of choosing a rather simple theoretical structure is that the analysis do concentrate on a limited number of variables assumed to be the most important when studying energy demand. For instance, Halvorsen (1973) discusses in some detail the simultanity problem caused by the varying tariff structure for electricity, and considerable efforts are made in specifying independent price variables in the equation to be estimated. As mentioned above, this problem, which is neglected in most other (more "advanced") studies of energy demand, may also be highly relevant when analysing gas demand. We will return to a brief discussion of the econometric problems caused by various kinds of tariff structures in the gas market in chapter 5 of this paper.

Still, to apply a simple model like (2.1) obviously involves several severe problems. One main weakness is that the model does not distinguish between *long term* and *short term* effects on demand. Since energy consumption is closely related to the operation of capital goods, dynamic elements and differences in substitution possibilities over time are usually judged to be essential to energy demand modeling. The distinction between long run and short run impacts on energy use can be emphasized even within a static framework (cf. below). However, in order to capture the evolution of energy use over time, a *dynamic framework* is obviously needed. A number of dynamic models relevant for studying gas demand will be discussed in chapter 3.

2.1 Interactions with other factors. Dual approaches.

One important theoretical objection to the simple model (2.1) relates to its *partial nature*. Since energy consumption is part of an overall optimization process involving several goods, the demand for all factors "benefiting" consumers or producers should be analysed and estimated *simultaneously*. By neglecting the demand relations for other goods in question, one will generally neglect restrictions between all or some of the parameters in the demand structure. Thus, the calculations do not utilize all available information, and estimators obtained will normally not be the most efficient ones, i.e. their variance will not be minimized. In order to deal with this problem, in recent years several energy studies have specified and estimated *complete systems of demand functions* consistent with the underlying technological and behavioural assumptions. Some early attempts of utilizing this procedure in analysing energy demand were based on a Cobb-Douglas or CES production - or utility function (see e.g. Baxter and Rees (1968) for an application on the manufacturing sector). Despite their widespread use, these functional forms are rather restrictive with respect to substitution properties, especially when the number of factors exceeds two. Thus, the application of simultaneous methods in demand analysis really gained popularity when socalled *flexible functional forms* were proposed in the literature. The specification most frequently applied is the *translog function* introduced by Christensen, Jorgenson and Lau (1973). Jorgenson (1974) described the indirect utility function by a translog function when analysing consumer demand in the US. In their comprehensive general equilibrium model developed to analyse US energy policy, Hudson and Jorgenson (1974) applied the same functional form to explore the behaviour of production sectors. Later on, a number of studies have applied a similar procedure for studying energy demand, using other data and extending the theoretical model at some points. Other flexible forms, such as the *Generalized Leontief function* (Diewert (1971)) have also been utilized. Among econometric studies of this kind carried out for production sectors, we will mention Berndt and Wood (1975,79), Fuss (1977), Griffin and Gregory (1976), Longva and Olsen (1983) and Bye and Frenger (1985). In addition to Jorgensons' study of household behaviour, the works by Roedseth and Stroem (1976) and Pindyck (1980) should be mentioned.

As already indicated, the majority of "complete" behavioural studies focusing on energy demand have applied socalled *dual approaches*. Instead of working with production functions or (direct) utility functions and necessary conditions for optimization, the economic behaviour is described in terms of *cost functions* and *indirect utility functions* respectively. Some of the advantages by utilizing the dual approach may be revealed by presenting a production model in general terms. Let the technology between output (y) and a vector of inputs (x) in a production sector be described by the relation

$$y = F(x) \quad . \tag{2.2}$$

By assuming cost minimizing behaviour, it can be shown (see e.g. Diewert (1971)) that given that relations fulfill some reasonable conditions, the technology can equivalently be represented by the *dual cost function*

$$C = G(p, y) \quad , \tag{2.3}$$

where p is a vector of factor prices. One main advantage with the dual approach is that having chosen a specific form for the cost function, the input demand functions can be easily derived by Shephard's lemma:1)

$$x_i = g_i(p, y) \equiv \frac{\partial G(p, y)}{\partial p_i} \tag{2.4}$$

The optimal input levels, i.e. the demand functions, may thus be derived directly from the cost function simply by differentiation. This property is obviously very convenient when focus is on the demand for factors of production, for instance energy use.

A production structure frequently applied in econometric studies in recent years, is the socalled KLEM - specification of inputs. Substitution possibilities are assumed to exist between capital (K), labour (L), energy (E) and materials (M). When using a flexible functional form (e.g. Translog or Generalized Leontief), it can not be judged a priori whether

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1)A simple way of seeing this property is the following (Woodland(1982)): Let the optimal input vector when prices are $\bar{p}$ be denoted by $\bar{x}$. The definition of $G(p, y)$ as the minimum of px implies that for every given y the function $\phi(p) = G(p, y) - p\bar{x} \leq 0$, gains its maximum value for $p = \bar{p}$. This, however, means that

$$\frac{\partial \phi(p)}{\partial p_i} \equiv \frac{\partial G(p, y)}{\partial p_i} - \bar{x}_i = 0$$

which leads directly to Shephard's lemma.

these inputs are pairwise complements or substitutes.[1] With respect to the relationship between energy and other inputs, almost all studies that have been undertaken indicate that energy and labour are substitutes. On the other hand, no consensus has been reached regarding the substitution between energy and capital. Some analysis report positive estimates on the cross price elasticities between these two inputs, while others indicate that they are complements (among studies discussing this issue are Berndt and Wood (1975), Griffin and Gregory (1977), Berndt and Wood (1979) and Longva and Olsen (1983)). The majority of results probably belongs to the latter category. The main argument used by their opponents is that many of these studies have employed pure time series data in their calculations, which may tend to capture short term effects. However, the result that energy and capital are complements is obtained in some combined cross section-time series analysis as well (see e.g. Berndt and Wood (1979)). Another argument is to claim that basic engineering knowledge about production processes indicates that energy and capital are substitutes. In our opinion this is probably true at the "micro level", i.e. in a certain production process, but this is not necessarily a strong argument for the same being the case in an aggregate production study, where the most important changes probably are *between* processes and firms. The dispute about this matter will probably continue.

Applied on the household sector, the formal framework presented above is usually slightly modified. In this case, (2.2) can be reinterpreted to indicate *a utility function,* and (2.3) in this case is the *expenditure function,* yielding the minimum expenditure for any given level of utility, y. As already mentioned, the approach for analysing consumer behaviour typically starts out from a specification of the *indirect utility function,* defined as the maximum utility obtained for any given combination of prices and income (expenditure, C). Obviously, the indirect utility function follows directly by "inverting" the expenditure function, i.e. solving (2.3) with respect to y.

$$y = V(p, C) \tag{2.5}$$

When the theory is applied to model consumer behaviour, the relations (2.4) are the socalled Hicksian (compensated) demand equations. While these relations are directly related to the expenditure function, the ordinary demand functions (specifying demand as functions of

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[1] Two inputs i and j are defined as complements if the cross elasticities between input i and j are negative, i.e. if

$$\frac{\partial \log g_i}{\partial \log p_j} < 0 \quad ,$$

and substitutes if the signs of the cross elasticities is positive.

prices and income) can be derived from the indirect utility function by *Roy's identity* :[1]

$$x_i = h_i(p,C) \equiv - \frac{\partial V/\partial p_i}{\partial V/\partial C} \tag{2.6}$$

Generally speaking, the dual approach exploits general properties of the optimization problem. The solution is expressed in terms of "parameters" (prices and activity level) and provides a simple and consistent way of deriving the demand for goods. By utilizing the dual approach, one avoids working with first order conditions in the optimization problem, and given the relevant properties on the cost function, no further conditions will have to be imposed on the derived demand structure.

2.2 Fuel shares models.

So far we have used the word "energy" demand in general terms, without being very specific about the meaning of this concept. In particular, not much has really been said about how *natural gas* enter the models, or how consumption of gas interacts with demand for other energy carriers. In relation (2.2), the input commodity vector x often contain several energy goods. In this case, two different approaches can be chosen. One possibility is to treat each energy carrier separately as independent input to the production or the utility structure. This methodology is based on the presumption that interactions between energy carriers in production or consumption is not distinguishable from interactions between other goods or services. The alternative hypothesis, to be presented in this section, is that different energy carriers in some sence service the same basic need; demand is really not demand for natural gas or heavy fuel oil as such, but for the services produced from the use of energy. With this latter assumption, demand for energy should be modeled in a two - step procedure. In the first step, demand for energy as such is derived. In the second step, this demand is

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[1] The validity of Roy's identity may be explained in the following way: The ordinary demand function for commodity i is derived by inserting the level of optimal utility in (2.4), so that

$$h_i(p,C) = g_i(p,V(p,C)) \equiv \partial G(p,V(p,C))/\partial p_i . \tag{i}$$

Here it should be noted that C is the optimal level given y and vice versa. Furthermore, the correspondence between the two optimization problems implies the following identity to hold:

$$y \equiv V(p,C(p,y)) \tag{ii}$$

This relation provides restrictions between the *functions* $G(p,y)$ and $V(p,C)$. Specifically, by implicitly differentiating through (ii) with respect to p_i :

$$0 = \partial V/\partial p_i + \partial V/\partial C \cdot \partial G/\partial p_i \quad . \tag{iii}$$

Roy's identity is easily obtained by combining (i) and (iii).

allocated between different energy goods depending on relative prices and other relevant variables. Thus, a submodel with equations for *fuel shares* is typically included in the model system. Let E_i denote the consumption of energy carrier no. i. A typical formulation of a fuel share model may then be the following:

$$S_i \equiv \frac{E_i}{E} = f_i(p_i, y, z_i) \tag{2.7}$$

where

S_i is the fraction of energy good i in total energy consumption,

p_i is a vector of relative energy prices and

z_i contains other variables affecting energy demand.

In Federal Energy Administration (1976), a model of this kind is estimated (using log-linear equations) for electricity, natural gas, coal, and petroleum products for the residential, commercial, transportation and manufacturing sector1). A slightly different procedure is to choose one of the fuels, say j, as a "numeraire good" and normalize the consumption levels for the other energy goods against this fuel. In that case, relations of the following type are specified:

$$\frac{E_i}{E_j} = h(p_i, y, z_i) \tag{2.8}$$

An example of this kind of model is the analyses of residential energy demand carried out by Chern et al.(1983). This study distinguishes between four fuels, among which one is natural gas. Share relations similar to (2.8) are specified both for total residential energy demand and energy used for space heating. With respect to the formal specification, the multinomial logit model applied by Baughman and Joskow (1974) is chosen (see also section 5 below). A distinguishing feature of the multinomial logit specification is that the share between fuels i and j depend upon the ratio between the prices of these two energy goods, but not on the prices of other fuels. This implies that important rstrictions are imposed on the cross elasticities between different energy carriers.2) The assumption may e.g. be restrictive in a case where fuels A and B compete in one end use and fuels B and C in another. Clearly, in this

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1) A simple dynamic structure is also added to this model structure.

2) As will be discussed in section 4 of this paper, the multinomial logit model might be a reasonable behavioural specification in the case where there is a *limited (discrete)* number of choice alternatives. However, even in this case the specification may be too restrictive, and it is probably more questionable in a traditional econometric framework where the use of the various fuels (implicitly) are assumed to vary continuously.

case a change in the price of C would affect the share of the total market served by A and B. A more general formulation would of course be to allow all energy prices to be included in the share equation (2.8). An alternative solution would be to model the end uses separately.

Another problem related to undertaking estimation directly either on equation (2.7) or equation (2.8), is the *measurement of energy flows*. The frequently applied procedure is to measure all figures for energy use in a common energy unit, e.g. Peta Joul (PJ), and sum over all fuels to obtain data for total energy use and fuel shares. However, from a strict theoretical point of view this procedure should be based on a certain degree of a priori knowledge of the underlying technology, i.e. how efficient different fuels are in "producing energy". To undertake pure summations implicitly assumes that the various fuels are perfect substitutes in this technological process.

In the last two paragraphs, we have touched upon some principal difficulties of specifying directly relations like (2.7) or (2.8). Basicly, these are *reduced form equations,* and even though they represent integrated systems where the demand for various energy carriers are interrelated with one another, they are not derived from and thus not necessarily consistent with an underlying production or utility structure. For a further discussion of these models it is useful to survey briefly the technical assumptions underlying the two-step procedure in the demand structure mentioned above. We then again start out from a general description of technology and behaviour as e.g. represented by (2.2), (2.3) or (2.4), focusing specifically on the substitution between the energy goods in question. A common procedure is to introduce an assumption of *weak separability* between energy goods in the technology or utility structure. If we let x_i denote an arbitrary element in the structure (2.2), and by convention choose x_1 to represent energy, weak separability implies that this relation can be rewritten in the following way:

$$y = f\,(x_1,\ldots,x_n) \equiv \tilde{f}\,(x_1(x_{11},\,.\,.\,.\,,x_{1m}),\ldots,x_n) \quad , \tag{2.9}$$

where $x_{11},\ldots,x_{1m}$ represent all energy goods involved. x_1 is thus the total level of energy "produced" by the different energy carriers that enter the technology structure. It can be shown (see e.g Berndt and Christensen (1973)) that given that this energy aggregation function is *homothetic,* the complete optimization process can be carried out stepwise. In the first step, the producer decides on the total use of energy and the input levels of the other factors of the technology. In the second step, producers minimize energy costs of producing a certain output of "energy", and thus determines the optimal composition of energy inputs. From this sequential optimization procedure, share equations for the different fuels are easily derived as

$$\frac{x_{1j}}{x_1} = u(x_1)\, v_j(p_{11},\ldots,p_{1m}) \qquad ,j = 1,\ldots,m. \tag{2.10}$$

where $p_{11},\ldots,p_{1m}$ are prices of the various energy inputs.

One advantage with the above procedure is that the concept of "total energy" in this model is defined in a distinct and meaningful way. In addition, introduction of the assumption of weak separability simplifies the estimation of a complete behavioural model like (2.3) or (2.4). The procedure sketched above and derivation of equations of the type (2.9) has therefore been frequently applied in several of the studies refered to earlier, cf. Fuss (1977), Pindyck (1979) and Longva and Olsen (1983).

A limitation on this kind of models should, however, also be noted. In the demand equations (2.10), the "scale effect" of the level of total energy is similar for all fuels included . This is a direct implication of the assumption that the energy aggregation function is homothetic. For the same reason, the relative demand for fuels, i.e. the proportions x_{1j}/x_{11} $,j = 1,\ldots,m-1$, are independent of total energy use. From a strict theoretical point of view, a non-homothetic energy aggregate would imply that the stepwise optimization procedure is no longer valid, and that the demand for fuels must be analysed in conjunction with the other input variables.

3 SHORT RUN VERSUS LONG RUN IMPACTS ON ENERGY DEMAND

As already emphasized, since the use of energy both in households and for production purposes is closely related to the operation of capital goods, an important distinction can be drawn between short term effects and long term effects on energy demand. This recognition actually calls for a dynamic model, where the difference between the short run and the long run is tackled explicitly. A presentation of various dynamic approaches will be undertaken below. However, even within a static modeling framework different interpretations regarding the time perspective of the analyses may be made. Firstly, as already have touched upon, *the kind of data* used in estimating various models are of significance to infer whether long term or short term effects are measured. A view widespread in the literature is that in order to fit a long term demand relation for energy, either cross section data or some sort of pooled time series/cross section data should be utilized, since these tend to capture differences between "states" that have remained stable for some time. Application of pure time series figures, on the other hand, are believed to imply that the estimates are strongly influenced by short term fluctuations in the included variables. Reference may here be made to Griffin and Gregory (1977) and Berndt and Wood (1979). However, the distinction in what may be inferred with respect to the time perspective when time series as opposed to cross sectional data are employed, is not at all clear. As pointed out by Taylor (1975):

> "While the view that cross-sectional observations reflect steady-state variations has some limited validity as a general tendency, it is not correct to say that cross-sectional data never reflect short term, dynamic adjustment. For the latter will be represented to the extent that individual observation units .... are not all at the same point of disequilibrium arising from recent changes in income, prices or other relevant factors. Since income and prices in general do not change at the same time across cities, states and regions, differential disequilibria are almost certain to be reflected in the data." (Taylor (1975), s. 103).

Moreover, time series data do not *necessarily* result in estimated short term relations, if the variables fluctuate over time in a random way.

In some studies, however, the distinction between long term and short term effects on demand is introduced explicitly in the formal theoretical model. Fisher and Kaysen (1962) utilize explicitly the assumption that energy use is related to the operation of capital equipment. In their model, energy consumption (E) is determined by the volume of energy using equipment (K), but in the short run, the utilization rate of the capital stock (u) has to be decided upon. The following short term relation is thus specified:

$$E = u(p, y, z)\, K \tag{3.1}$$

In the long run, consumers want to optimize the level of their stocks of energy using equipment. This optimization procedure is the theoretical justification for the following long run capital demand function:

$$K = K(p, y, r, z) \tag{3.2}$$

where r is an interest rate relevant for the adjustment of the capital stock. Long run energy demand can then be calculated by combining (3.1) and (3.2). (3.2) should be regarded as the optimal level of the capital stock. In Fisher and Kaysen (1962), the structure given by (3.1) and (3.2) is also combined with explicit assumptions of the adjustment process to a new equilibrium , i.e. dynamic relations are added to the model (see also Taylor (1975)).

A related, but more elaborate and consistent (static) model is obtained within the general framework (2.2) by assuming that capital in the short run can be regarded as a ***fixed factor*** in the production process (see e.g. Lau (1976) and McFadden (1978)). Choosing arbitrarily factor x_n to denote capital equipment, producer behaviour is represented by a ***restricted variable cost function:***

$$\overline{C} = \overline{G}(p_1, \ldots, p_{n-1}, y, \overline{x}_n) \tag{3.3}$$

The corresponding set of (short term) demand functions is derived by Shephards lemma:

$$x_i = \frac{\partial \overline{G}}{\partial p_i} \equiv \overline{g}(p_1, \ldots, p_{n-1}, y, \overline{x}_n) \quad i = 1, \ldots, n-1. \tag{3.4}$$

Notice that the short term restricted cost function provides a complete characterization of technology and behaviour; i.e. not only does it describe short term adjustments, it also fully characterizes long term elasticities. Having estimated the long term relations, the short term demand equations can be derived directly. The same is true the other way round: long term equations can be derived from estimated short term relations. The link between the short run and the long run follows from the fact that the only feature distinguishing the two optimization problems is that one more restriction is added in the short run . Mathematical programming theory tells us that in long run equilibrium, the price of the fixed input is simply the negative of the derivative of the restricted cost function, i.e.

$$-p_n = \frac{\partial \overline{G}(p_1,\ldots,p_{n-1},y,x_n)}{\partial x_n} \quad . \qquad (3.5)$$

The optimal long run equilibrium level for the fixed input is obtained simply by solving (3.5) with respect to x_n, which can be substituted into the short term relations (3.4) in order to obtain long run demand relations for the variable inputs.

So far, rather few empirical studies have been undertaken using the restricted cost function approach. A practical problem with this procedure is that it may be difficult to obtain a closed analytical form to the demand equation for the fixed input using (3.5). Brown and Christensen (1981) estimated a restricted translog cost function for the US agriculture sector, but had to utilize numerical procedures in their calculations. Frenger (1984) has, on the other hand, shown how to solve explicitly for x_n when the technology is described by a Generalized Leontief (long term) cost function. Based on this functional form, short term demand equations for aggregated inputs (including energy) in the manufacturing sector are estimated in Bye and Frenger (1985).

The major shortcoming with the approach of estimating a restricted cost function when regards the distinction between short run and long run effects, is that the adjustment path from short term to long term equilibrium is not modeled. In order to include this process formally, an explicitly *dynamic model* is required.

3.1 Dynamic models of energy demand

It is a common view that in order to capture the relationship between energy demand and capital goods and to separate long run and short run effects on demand, it is *necessary* to apply a *dynamic model framework*. Firstly, this is true if the time horizon of the analyses includes the short or medium term. The reason is of course that since investments in capital equipment may be required, the adjustment of energy use is time consuming, and consequently there exist differences in short-,medium- and long term responses to price changes. Secondly, even if the focus is solely on the long term interactions between energy use and other variables, the use of *time series data* may still necessitate estimation of a dynamic model because the existence of time-consuming adjustment processes generate fluctuations in the observed figures around their long run values.

After having stressed these - in our view - basic motives for specifying a dynamic model, it is important to emphasize that a static model is not *necessarily* inferior to a dynamic structure in a case where the focus is mainly on long term impacts and data enabling the estimation of long term parameters are available. On the contrary, in such cases a static model may be preferable. One reason for this is that consistency with economic theory is usually more easily obtained within a static framework. As will be emphasized below, in the most frequently applied dynamic energy models, the "costs" of introducing an explicit time dimension and adjustment lags has often been the lack of other interactions and relationships that follow from assumptions of economic behaviour. Furthermore, it will be seen that many of the most common dynamic specifications are very *simple and ad hoc* in nature. It may be questioned whether these formulations really corresponds to a consistent theory or capture the essence of how energy use depends on stocks of capital goods. In particular, for the residential sector, few models take explicitly into account the dependence of energy demand on *demand for housing services and the stock of dwellings* (vintages and different housing types). Finally, as we shall see, the interrelatedness of energy demand with other inputs is often neglected.

In the following, a selection of various dynamic models used for analysing energy demand will be presented and briefly discussed. As a start, we will sketch the main features of the socalled Balestra-Nerlove model (Balestra and Nerlove (1966), Balestra (1967)). Our motive for paying particular attention to this model is that it is one of the rather few models that has been specifically designed for estimating natural gas demand. Its importance is also stressed by its frequent use in econometric studies over the last two decades. Having discussed the Balestra-Nerlove model, we will then pass on to some related, but slightly different dynamic formulations. This chapter ends with a brief overview of some extensions of traditional dynamic models that have been proposed in the literature. For a more "complete" survey of dynamic energy demand models, the comprehensive exposition and discussion of various approaches given in Berndt, Morrison and Watkins (1981) is recommended.

3.2 The Balestra-Nerlove model.

This model was originally constructed to analyse the use of natural gas in the residential and commercial sector. A main feature of the model is the distinction between *captive and new demand* for energy and natural gas. A large part of the total energy consumption is assumed to be "captive", as it is tied to the existing stock of energy consuming equipment. This feature is represented in the model by the following central relation:

$$G_t = u \cdot K_t \quad , \tag{3.6}$$

where

G is the use of gas ,

u is the utilization rate, and

K is the stock of gas consuming capital.

The subscript t, here and in the following, denotes that variables are dated to period t. An important assumption in the Balestra-Nerlove model is that *the utilization rate is constant.* This is the same as to say that u does not depend on economic or other factors. The utilization rate is determined by the technology chosen, and is not subject to changes after the capital equipment has been installed. Otherwise, equation (3.6) would of course have been a purely definitional relation between gas consumption and the stock of capital. Assuming that the capital stock is depreciated at a constant rate, δ, the following relation holds between the capital stock and new (gross) investments, I_t:

$$K_t = I_t + (1-\delta)K_{t-1} \quad . \tag{3.7}$$

Applying (3.6), a corresponding dynamic equation can be specified for the incremental change in consumption of natural gas (G_t^*), i.e.

$$G_t^* = G_t - (1-\delta)G_{t-1} \quad , \tag{3.8}$$

where $G_t^* = u \cdot I_t$. It is worth noting that due to the assumption that the utilization rate is constant, the "capital variable" can be eliminated from (3.8). This is computationally advantageous since data on energy using equipment is often unavailable.

Gross investments in gas consuming equipment, and thus new demand for gas, is specified as a function of the relative price of gas, P_g, and new demand for total energy, denoted by E^*, i.e.

$$G_t^* = f\,(P_{g,t}\ ,E_t^*) \quad . \tag{3.9}$$

By an "analogous argument"(Balestra and Nerlove (1966), pp.588), a relation similar to (3.8) is derived for the increment in total energy use:

$$E_t^* = E_t - (1-\delta_e\ )E_{t-1} \quad , \tag{3.10}$$

where E_t is the total use of all fuels in period t, and δ_e is the "average" rate of depreciation for energy using equipment. The model is "closed" by specifying a relation explaining the total consumption of energy, E , i.e[1].

$$E_t = F(P_{e,t}, Y_t, H_t) \quad , \tag{3.11}$$

where

P_e is a price index for total energy,

Y is real income, and

H is a vector of socio-economic variables.

By combining (3.10) and (3.11), and inserting the expression for E^* in (3.9), an equation for total natural gas demand is obtained. This equation is then fitted to data by Balestra and Nerlove .

The Balestra-Nerlove model has a rather simple structure that makes it convenient for econometric implementation. In particular, it can be estimated even if data on energy using capital goods are not available. The distinction between new and captive demand has been regarded as the main advantage of the model since this feature is believed to capture the close connection between energy use and capital equipment. However, the model includes some obvious weak points and others that at least need to be discussed.

(i) The principal drawback with the Balestra-Nerlove model is usually emphasized to be the assumption of *a constant rate of equipment utilization.* Changes in prices, incomes etc. may motivate consumers to change the utilization or intensity at which the capital equipment is applied. One example of this is the fact that consumers in most countries adjusted their termostates on their heating and air conditioning equipment as responses to the drastical changes in the fuel prices in the 1970's. A more general model should allow for an endogenous determination of capital utilization.

(ii) Even though the model relates gas consumption to total energy use by (3.9), it is basically *a partial model* of gas demand. It does not explain the interfuel competition of natural gas versus other energy carriers, or the possible substitution between energy and other goods or factors. Furthermore, while only new demand for gas is assumed to be "flexible", total energy demand (i.e. the level) is specified as a function of autonomous variables. This specification poses the problem of consistency between different parts of the model, since gas use is part of the energy aggregate. Embodied in (3.10) is the assumption that for all fuels the utilization rates of the respective capital stocks are constant. However, if these constant rates differ between fuels (which seems reasonable to assume), relation (3.10) must be based on the

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[1] In the original work of Balestra and Nerlove, the price variable P_e in (3.11) was omitted ("since price effects are found to be negligible", Balestra and Nerlove (1967), pp.588). However, in more recent studies using basically the same approach, this variable is included, see e.g. Berndt and Watkins (1977).

rather improbable assumption that the composition of the total stock of energy using equipment is constant over time[1]. Consequently, by specifying (3.11) this must imply that all kinds of energy using capital, including gas equipment, are adjusted to their desired levels. The demand for gas then follows directly from the assumption of constant utilization rates. New demand for gas may be derived directly from (3.8). Thus, there is really no "room" in the model for equation (3.9). This possible inconsistency in the original Balestra-Nerlove model may, however, be avoided by specifying new demand for total energy directly as a function of independent variables.

(iii) The "depreciation rates", δ , δ_e , are usually interpreted as *physical rates of retirement*. This is consistent with the basic idea in the model of capital being frozen after installation, hence the distinction between new and captive demand. Implicitly this presupposes an underlying "vintage structure" with no substitution possibilities "ex post", and that energy using equipment is not being replaced before it is worn out. However, in most studies applying a Balestra-Nerlove type of model, the depreciation rates are not fixed a priori on the base of knowledge of actual survival profiles of capital goods, but rather treated as unknown parameters and estimated together with other coefficients of the model. The following problems may be related to this procedure:

a)Even if the "clay"-assumption is valid for each vintage of energy equipment, estimated rates of retirement would be dependent on the age structure of the total capital stock. Thus, for the model to give correct predictions of future energy use requires a specific composition of the capital stock.

b)Estimates of depreciation rates may also be influenced by "endogenous scrapping" of the energy using equipment. Capital equipment may be subject to scrapping before it is physically worn out because it is unprofitable to operate at existing prices. This is in essence a way of suggesting that another, probably more complicated model is actually in effect. This may of course not prevent good "fit" to data for the Balestra-Nerlove model, but disturbs the interpretations of parameters in the model and indicate that the model does not consist of autonomous, structural equations.

It has been stressed by several authors that the calculated depreciation rates can be tested against other, independent estimates if such figures are available, and thus providing a test of the validity of the specified model. However, a) and b) above show that this is not a simple task when the model is used to analyse aggregate energy demand.

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1)This feature is explicit in the version of the Balestra-Nerlove model presented in Bohi (1981).

(iv) In the study of Balestra and Nerlove, like in many other dynamic analyses of energy demand, the capital concept involved in the model is explicitly stressed to be natural gas - or energy using *equipment*. For the residential sector this would e.g. include stoves, furnaces, appliances etc. The demand for these goods and possible rigidities in the adjustment of these stocks to new equilibrium levels are obviously of significance when analysing energy demand. However, for the residential sector the most important capital good to which energy use is related is probably *the dwelling stock itself*. An important reason for this is the fact that a major part of total energy consumption in households is used for heating purposes. This part of the energy demand is to a large extent explained by the development in the housing stock, i.e. the number of dwellings, their size, the distribution of the stock on various qualities, types etc. A model aiming at explaining residential energy demand should therefore focus specifically on these structural features of the dwelling stock and on factors influencing the demand for housing services. Regarding dynamic elements in the energy market, "lag effects" in the adjustment of the dwelling stock may be more important than rigidities due to investments in energy using equipment. It may be costly to change a heating system once installed in a dwelling, not so much because of the costs of purchasing new equipment as the fact that a heating system is to a certain extent "integrated" in the building at the time of construction.

Even though the capital concept does not occur directly in the Balestra-Nerlove model, the problems mentioned here are of relevance even for this model. Firstly, they are related to (iii) above and the interpretation and measurement of the depreciation rate. Secondly, the discussion is highly relevant for the interpretation and evaluation of equation (3.11), i.e. the relation explaining total demand for energy . Since energy demand is specified as a function of the income level and the population, this relation is obviously "correlated" to the demand for housing services and dwellings. On the other hand, with such an interpretation one would probably prefer additional explanatory variables to be added to this equation, and a more detailed treatment of the demand for dwellings to be included. The main point we would like to emphasize is that when analysing energy demand in the residential sector, particular attention should be paid to determinants of the stock of dwellings and the use of housing services.

3.3 Lagged adjustment models.

This section outlines a number of rather simple dynamic models that have been used in energy demand analyses. Furthermore, we will mention some extensions of the traditional single-equation models which have been suggested.

(i) Koyck lag models.

In the literature, Koyck lag models characterize a class of dynamic models where the dependent variable - lagged one period - occurs as an exogenous variable in the equation to be estimated. This model is typically specified to be linear in the parameters, e.g. of the following form:

$$E_t = \alpha + \beta X_t + \gamma E_{t-1} \quad , \tag{3.12}$$

where X is a vector of independent variables, and α , β and γ are coefficients to be estimated. This structure has been a very popular dynamic formulation in the energy demand literature. A Koyck-relation like (3.12) can be arrived at starting out from several different theoretical models. First, it should be noted that the Balestra-Nerlove model discussed in the previous section actually reduces to a relation similar to (3.12) in the case when the gross purchase of new capital equipment is specified *directly* as a function of independent variables, instead of postulating an equation for the level of total energy demand ((3.11)).

Originally, Koyck (1954) assumed that the demand for the good in question is a function of the independent variables in the present and all previous periods, i.e.

$$E_t = a_0 + a_1 \sum_{i=0}^{\infty} \lambda^i X_{t-i} \quad . \tag{3.13}$$

A characterizing feature of this dynamic structure is that the weights attached to the explanatory variables decline geometrically over time. The term $a_1 \sum_{i=0}^{\infty} \lambda^i X_{t-i}$ may be interpreted as the expectations held by consumers for the values of the independent variables, such as e.g. prices and income.[1] By using the formula for a geometric sum, one easily arrives at a relation similar to (3.12).

An even more commonly applied structure in energy analyses, leading to a Koyck-equation of the type (3.12), is the socalled *flow-adjustment model* applied by Houthakker and Taylor (1970). In this model, *a partial adjustment* mechanism is specified for energy consumption, i.e.

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[1] Obviously, this presupposes that the sum of the periodical weights $w_i = a_1 \lambda^i$ equals one. In (3.13) these weights are assumed to be the same for all the elements included in X. This is done only in order to simplify the notation, and can easily be generalized.

$$E_t - E_{t-1} = \lambda(\tilde{E}_t - E_{t-1}) \quad , 0 < \lambda \leq 1 \ . \tag{3.14}$$

Here $\tilde{E}_t$ is the *desired* level of energy consumption in period t, and λ is an adjustment parameter measuring the proportion of the deviation between the desired energy use and the actual input which is being adjusted in each period. Furthermore, Houthakker and Taylor specified the desired level of energy consumption as a function of the real price of energy and real income, symbolized by a vector of independent variables, X, in the following equation:

$$\tilde{E}_t = a + b \cdot X \ . \tag{3.15}$$

(3.12) follows directly by combining (3.14) and (3.15).

One main *computational* advantage with this flow-adjustment model is that no data on energy using equipment are required. Not even the rate of retirement of the capital stock is included as a structural parameter in this framework, as was the case in the Balestra-Nerlove model. But again, from a theoretical point of view this is at the same time the main weakness of the model. The formal relation between energy use and the capital goods is not included explicitly in the model. It may, however, be reasonable to assume that *implicit* in the model specification is a constant rate of capital utilization, since relation (3.14) must be related to a process of adjusting the capital stock.

Another problem with this specification is that the partial adjustment equation (3.14) is specified largely ad hoc. No relations are included describing how the capital equipment are adjusted over time or explaining what factors are causing or determining the time lags. Furthermore, like the Balestra-Nerlove model, interactions with markets for other goods are neglected. In particular, the model does not take into account the fact that disequilibrium in one market (delays in adjustment caused by (3.14)) could influence the equilibrium situation in other factor markets as well. By thinking in terms of a neoclassical production function, it is obvious that it is not possible for only one factor to deviate from its long term equilibrium.

Even if the reduced form of this model is quite similar to the one derived in the Balestra-Nerlove model, it should be pointed out that the basic assumptions about economic behaviour and also the interpretations of parameters are highly different in the two approaches. In the flow-adjustment model, changes in e.g. relative prices cause changes in consumers' *desired level* of energy use. A reasonable interpretation is that this corresponds to a certain stock of energy using equipment. Gross investments/new purchases of energy are determined so that this new level is reached. In the Balestra-Nerlove model, on the other hand, a price change is assumed to influence *directly* gross investments and new demand for energy, and there will be a net change in capital stock/energy use until gross investment equals the depreciation of the capital stock.

In addition to Houthakker and Taylor (1970), the flow-adjustment specification (3.14), (3.15) has been used by Mount, Chapman and Tyrell (1973), Houthakker, Verleger and Sheehan (1974) and Taylor, Blattenberger and Verleger (1976). In their extensive analyses of consumer demand in the US economy, Houthakker and Taylor (1970) also specified another dynamic structure, known as *the state-adjustment model,* which leads to a Koyck-equation similar to (3.12). This model consists of a behaviour relationship which relates energy consumption to "stocks", income and the relative price of energy, and a relation similar to (3.7), i.e. expressing new purchases of energy as changes in "stocks" plus depreciation. In our opinion, the definition of variables and the interpretation of effects in this model are somewhat dubious. By a reasonable interpretation, the structure is quite similar to the Balestra-Nerlove model presented earlier. Thus, we choose not to discuss the state-adjustment model in further detail in this context.

(ii) Polynomial (Almon-) lag models.

The characterizing feature of the Koyck lag models is that they include the dependent variable - lagged one period - as explanatory variable in the equation to be estimated. Another frequently applied dynamic specification is *the polynomial (Almon) lag model.* In a similar way as in the partial adjustment model (3.14), this model starts out by assuming lags in the independent variables, but now a finit historical time horizon is introduced. By assuming a linear structure, the following model may be specified:

$$E_t = a + \sum_{i=1}^{i=T} b_i X_{t-i} \qquad (3.16)$$

where $a, b_1, b_2, \ldots, b_T$ are coefficients to be estimated. In order to restrict the number of parameters, a priori restrictions on their variations over time are imposed by specifying a *polynomial lag distribution.* If this is of order z (where presumably $z < T$),(3.16) can be written:

$$E_t = a + \sum_{i=1}^{i=T} b(i; \alpha_0, \alpha_1, \ldots \alpha_z) X_{t-i} \quad , \qquad (3.17)$$

The estimation problem is now reduced to estimate the parameters $\alpha_0, \alpha_1, \ldots \alpha_z$. The polynomial dynamic model is used among others by Griffin (1974) in his study of industrial electricity demand.

Like in the other dynamic models presented above, the impact of stocks of equipment on energy consumption is taken care of by a simple dynamic structure, without observing the stock variables directly. This is computationally convenient, but reveals a number of problems and weaknesses as pointed out earlier.

Somewhat critically perhaps, the single equation dynamic models mentioned so far in this section may be characterized as "best-fit methodologies" to a given set of data. In choosing between a Koyck lag model and a polynomial lag structure, the assumed stochastic properties of the included error terms are often decisive. In the former model, the occurence of a lagged dependent variable causes well known econometric problems if there are indications of serial correlation between the error terms.

(iii) An extension of the simple dynamic model.

In all the dynamic models considered so far, the adjustment process in the market for the good studied (natural gas) is either independent of disequilibrium in connected markets or proportional to changes in the capital market (Balestra-Nerlove). More elaborate dynamic adjustment models are, however, suggested in the literature. One important extension is the model of *interrelated disequilibria* introduced by Nadiri and Rosen (1969,1973). This may be regarded as a generalization of the simple partial adjustment model presented above, or it can be interpreted as an approximation to a set of differential equations derived from a dynamic optimization problem. If q is a vector of inputs, the central set of equations in the Nadiri-Rosen model are of the following type:

$$q_t - q_{t-1} = \Lambda(\tilde{q}_t - q_{t-1}) \quad . \tag{3.18}$$

In this expression, Λ is a matrix of adjustment coefficients, where a typical element λ_{ij} measures the change in each period in the i'th input caused by a deviation between desired and actual level of factor j. Taking explicitely into account simultaneous effects between disequilibria in several markets, the Nadiri-Rosen model bypasses a major objection to the simple partial adjustment models, namely their partial nature. While the latter is not consistent with complete demand systems derived from consumer or producer behaviour, the generalized scheme (3.18) may consistently be combined with a traditional, static production model, for example the restricted cost function model outlined above (cf. (3.3-5)). If in this model all inputs - including energy - are reinterpreted to express *desired levels,* the derived demand relations can be substituted into (3.18), thus constituting a simultaneous set of dynamic equations. This methodology is applied by Halvorsen (1976), who specifies a Cobb-Douglas production function with capital, labour and energy as inputs, and combines this structure with the Nadiri-Rosen adjustment process[1]. Using a similar approach, Berndt, Fuss and Waverman (1977) estimated a "dynamic translog model" on US manufacturing data with energy as one of four specified inputs.

The strength of the dynamic structure of Nadiri and Rosen is that it takes into account interrelationships between adjustments of various factors or goods. However, when postulated directly, like the simple partial adjustment model, (3.18) may be characterized as being rather "mechanical", because the speed of adjustment of different factors are represented by constant parameters. The model does not reflect an explicit dynamic optimization procedure, and does not "explain" what causes the rigidities and obstacles to factor substitution. Moreover, it may be shown (see Berndt, Fuss and Waverman (1977)) that the structure (3.18) does not necessarily imply that (direct) short run price elasticities are smaller in absolute value than corresponding long run elasticities, i.e. short run overshooting may occur within the model. Thus, unlike the restricted cost function approach, the Nadiri-Rosen model does not "obey" the socalled Le Chatelier principle. This property is only secured if an explicit optimization model is specified, where the behaviour is "restricted" to a larger extent in the short run than in the long run.

(iv) A dynamic cost of adjustment model.

A more advanced dynamic model (which fulfils the Le Chatelier principle) has been suggested in the literature, originally by Lucas (1967). The exposition of the model in this paper follows the lines of the pionering work of Berndt, Fuss and Waverman (1980). This paper develops a dynamic model of energy demand explicitly incorporating interrelated factors of demand and dynamic optimization on behalf of individual firms. A distinction is drawn between variable and quasi-fixed inputs. A variable factor fulfils the condition that the optimal level at time $t>0$ is independent of its level at time $t=0$. For a quasi-fixed factor this is not the case.

Essential to this model is the idea that adjustment of quasi-fixed factors are costly. Changes in short run energy demand as responses to changes in energy prices, depend on prices on all variable inputs, the production level and the quantities of the inputs fixed in the short run. Furthermore, speeds of adjustment of the quasi-fixed inputs are derived from an economic optimization procedure, where costs of adjusting the level of quasi-fixed inputs are balanced against the advantage of a factor composition more adapted to the structure of relative prices.

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[1] Actually, Halvorsen also included utilization rates for labour and capital as separate inputs in the production structure.

This structure enables the definition of short run (SR), intermediate run (IR) and long run (LR) elasticities of input demand. Short run implies that the quasi-fixed inputs are exogenously given, long run is defined as the point of time where quasi-fixed inputs are fully adjusted to their optimal levels, and in the intermediate run a part of the adjustment process is completed. If adjustments of inputs was costless to the firms and no capacity limitations existed, one would expect firms to adjust all inputs instantaneously. Contrary to this, the present dynamic model assumes that when the producers undertake changes in the quasi-fixed inputs, output falls cet. par. because resources will have to be disposed for the adjustment process to take place.

Formally, the dynamic internal cost of adjustment model is based on the following production technology:

$$y = F(v, x, \dot{x}, t) \quad , \tag{3.19}$$

where

y is the output level,
v is a vector of variable inputs,
x is a vector of quasi-fixed inputs,
t is an index for technical change, and
$\dot{x} = dx/dt$

In this dynamic structure, the presence of adjustment costs mentioned above is reflected by the assumption that $\partial F / \partial \dot{x} < 0$, indicating that resources are consumed when stocks of quasi-fixed inputs are changed.[1] The dynamic optimization problem facing the firm is to minimize total discounted future costs, $C(0)$:

$$C(0) = \int_{t=0}^{\infty} e^{-rt}(wv + q(\dot{x} + \delta x))dt \quad . \tag{3.20}$$

subject to (3.19). In this relation, w and q are vectors of prices for variable and quasi-fixed inputs, respectively. r is a discount rate and δ is a vector of retirement rates for the quasi-fixed inputs, assuming a geometric survival pattern. The formal solution to this dynamic optimization problem is discussed in detail in e.g. Berndt, Fuss and Waverman (1980). Here, we limit ourselves to emphasizing some main points characterizing the solution.

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[1] Among energy studies utilizing this type of explicit dynamic model are Berndt, Fuss and Waverman (1977) and Denny, Fuss and Waverman (1979).

In the short run, all quasi-fixed inputs are given, and the optimization problem consists of minimizing variable costs subject to the production function (3.19). This is in essence a static problem, identical to the one already discussed above; in the present model a restricted cost function of the following type is obtained:

$$G = G(w, x, \dot{x}, y, t) \tag{3.21}$$

By inserting the restricted cost function in the global cost minimization problem (3.20), this problem can be solved straightforwardly by using the Euler conditions. In the case with only one quasi-fixed factor, the first order condition in steady state, i.e. when the quasi-fixed input is adjusted to its optimal level, looks as follows:

$$-G_x(\tilde{x}) = r \; G_{\dot{x}}(\tilde{x}) + \mu \quad , \tag{3.22}$$

where $\tilde{x}$ is the long term (steady-state) solution to the optimization problem (defined as the solution derived when $\dot{x} = 0$), and μ is the user cost of the quasi-fixed factor. In steady state, marginal savings in variable costs from a change in the quasi-fixed factor must equal the costs of a marginal change in the level of this factor, i.e. the user cost plus adjustment costs.

This model has several attractive properties. The distinction between short run, intermediate run and long run responses is explicitly derived from the theoretical model and the speed of transformation from short to long run is based on explicit dynamic optimization. An interesting feature of this approach is that it can be shown (see e.g. Treadway (1971)) that $\dot{x}$ can be derived as an approximate solution to the differential equation system

$$\dot{x} = \tilde{\Lambda}(\tilde{x} - x) \quad , \tag{3.23}$$

where $\tilde{\Lambda}$ is a matrix where the elements are related to properties of the technology. It is seen that (3.23) has a striking similarity with the generalized adjustment scheme of Nadiri and Rosen (1969). However, two important differences exist. Firstly, the present model distinguishes between variable and quasi-fixed factors, and the adjustment process comprises only the latter variables. Secondly, a more fundamental difference is that in this model, where costs of adjustment are explicitely taken into account, the "adjustment parameters" are not constant. Thus, the time path from one equilibrium to another is not exogenously given, but determined *within* the model as functions of prices and the discount rate.

The model (3.19)-(3.23) is far more satisfactory from a theoretical point of view than the rather "mechanical" dynamic adjustment models previously discussed. The "relative costs" of utilizing this kind of model is of course the increased complexity involved in the model structure and the fact that a larger number of coefficients usually will have to be estimated. An empirical application of this kind of model on the European natural gas market is shown in Gjeldsvik and Roland (1986).

3.4 Vintage models and irreversibility in investment decisions

When discussing producers' demand for energy above, we started out from the traditional neoclassical theory of production and then added to this framework various schemes of dynamic adjustment of factor inputs. Implicit in the dynamic models are assumptions that there are rigidities or costs involved in adjusting inputs to e.g. changes in prices. However, a common feature in all the models considered, is that changes in input composition are *reversible* and that *symmetry* prevails between responses to "positive" and "negative" changes in independent variables such as prices. It is commonly argued that for many purposes, the neoclassical theory of production and its embedded flexibility may not be proper as a description of technology and producer behaviour. In particular when the focus of the study is on the *micro level,* a model distinguishing between substitution possibilities *ex ante* and *ex post* may be considered as a more realistic framework for analysing producer behaviour. An attractive alternative is the *putty-clay* framework originally suggested by Johansen (1959) and developed further in Johansen (1972). In this model substitution possibilities between different inputs of the technology are assumed for *new vintages,* i.e. at the time the investment is undertaken (ex ante). On the contrary, once an investment is made and a specific input composition is chosen, this structure is embedded in the vintage of the capital stock through its entire period of operation (ex post).

The putty-clay framework can be characterized as a *structural dynamic model,* involving rigidities of adjustment in the production process and yielding differences between demand effects depending on the time horizon. Thus, it provides a theoretical explanation to the dynamic elements which are modeled more "mechanically" within a traditional neoclassical production framework. To our knowledge, only a few studies using this methodology to analyse energy demand empirically has been undertaken. One reason for this is probably the complexity involved in a vintage model, and the ensuing demands to data implied by adopting this structure. As will be seen, the studies mentioned below are either based on rather strong simplifying assumption to facilitate an empirical implementation, or they are only *calibrated,* i.e. they are not based on econometric investigation, rather parameters are based on literature studies or "consensus" views.

For illustrative purposes, we start by presenting a simplified formal representation of the putty-clay model using the following notation:

$Y(t,s)$ is the production capacity at time t of the vintage installed at time s.

$E(t,s)$ is the energy use at time t related to vintage s.

$K(t,s)$ is the volume of capital services at time t which is related to vintage s.

The *ex ante technology* for new investments can be described by a traditional production function of the neoclassical type, i.e.

$$Y(s,s) = F(E(s,s), K(s,s), s) \tag{3.24}$$

The time index in this relation, s, represents technological change *embodied* in the various vintages of the capital stock. On the base of expectations of future prices the producer undertake an investment decision (see below), i.e. they chooses a set of input coefficients

$e(s,s) = E(s,s)/Y(s,s)$ and $k(s,s) = K(s,s)/Y(s,s)$.

After the investment has been made, the "clay" assumption implies that the ex post technology is of the ordinary Leontief type, i.e. described by the following relations:

$$Y(t,s) \leq Y(s,s) \tag{3.25}$$

$$E(t,s) = e(s,s)\, Y(t,s) \qquad (t > s) \tag{3.26}$$

$$K(t,s) = k(s,s)\, Y(t,s) \tag{3.27}$$

$K(t,s)$ denotes the volume of capital services produced by the capital *actually used*. If starting and stopping costs are assumed away, the producer will operate each vintage at full capacity as long as it yields a positive quasi rent, i.e. as long as the following relation holds:[1]

$$\pi(t,s) = p_y(t) - p_e(t)e(s,s) > 0 \tag{3.28}$$

where $\pi(t,s)$ is the (unit) quasi-rent, $p_y(t)$ denotes the output price and $p_e(t)$ the energy price, both as observed by the producer at time t.

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[1] A generalization of the structure presented here may e.g. be found in Biorn (1985), allowing in the outset the producer to choose a utilization rate, $\xi(t,s)$, where in general $0 \leq \xi(t,s) \leq 1$. In the present simplified framework, the capital units will either be operated at full capacity ($\xi = 1$) or taken completely out of operation ($\xi = 0$). Biorn (1981) also introduces a survival profile describing the *physical retirement* of the capital stock. This aspect is neglected in the following.

As a result of the rigidity in the ex post technology and the "operational rule" (3.28), in a putty-clay framework, the concept of the *economic lifetime* of the capital becomes of central importance. Denoting the actual lifetime of vintage s by $N(s)$, this can be defined as the point of time where the capital equipment is scrapped because further operation is unprofitable. If no starting or stopping costs exist, obviously this requires the quasi-rent of any particular vintage to be zero, i.e.

$$\pi(N(s),s) = 0 \tag{3.29}$$

Furthermore, the future development of prices must be such that $\pi(t,s) \leq 0$ for $t > N(s)$. In the special case where prices change *monotonically* so that $\dot{p}_e/p_e > \dot{p}_y/p_y$, $N(s)$ is uniqually determined by (3.29). When this regularity condition on price movements is not met, an alternative simplifying assumption may be that the economic lifetime is determined by the point of time when the quasi-rent for the *first time* reaches zero. From an engineering point of view it may be argued that when a vintage of equipment has been out of operation for some time, for technical or institutional reasons, this specific vintage will not be reactivated even if prices turn around again and makes it potentially profitable.

At a sectorial level, in general a number of vintages are operated simultaneously. When aggregating over vintages with different efficiencies (input coefficients for energy), the fact that micro units are taken in and out of operation according to (3.28) may yield a *short run relation* between the variable inputs and output for the *sector as a whole* (see Johansen (1972)). Given certain regularity conditions, this relation may behave as a smooth, neoclassical production function. This has been used as a "defence" for the many attempts of estimating neoclassical production models at a rather aggregate level. From a theoretical point of view, however, a preferable procedure is to estimate the input coefficients for the "micro" units and thereafter derive the sector production function from this structure, as is done e.g. in the study of Foersund and Hjalmarsson (1983).

In the putty-clay framework, the (ex ante) choice of input composition in *new capital* is of a more complicated nature than determining the actual operation of a given equipment. Since investments decisions, as opposed to the neoclassical model, are irreversible, the producer has to form *expectations* on future development of prices and other independent factors and to plan carefully the utilization of the vintage at each point of time in the future. Let the time path for prices as expected by the producer at time s be denoted by $\bar{p}_y(t,s)$ and $\bar{p}_e(t,s)$. The *expected quasi-rent* at time t is then given by

$$\overline{\Pi}(t,s) = \overline{p}_y(t,s)F(E(s,s),K(s,s),s) - \overline{p}_e(t,s)E(s,s) \tag{3.30}$$

The objective of the firm is to maximize expected net cash flow over the planned economic lifetime of the capital equipment, $\overline{N}(s)$, i.e. maximize

$$\overline{\Pi}(s) = \int_{s}^{\overline{N}(s)} e^{-r(t-s)}\,\overline{\Pi}(t,s)dt - q(s)K(s,s) \tag{3.31}$$

with respect to the choice variables $E(s,s)$, $K(s,s)$ and $\overline{N}(s)$. The nature and the solution of this optimizing problem is discussed in detail in Johansen (1972) and Biorn (1985).

As indicated by the brief description of the putty-clay framework above, expectations, for example with respect to future prices, play an essential role in this model. Obviously, when actual prices turn out to deviate considerably from what was anticipated, this may result in dramatic changes in profitability. In a case where there is a unanticipated "jump" in input prices, several production units may be taken out of operation and significant changes in capacity utilization of the industry may be observed. Biorn (1985) focuses particularly on how large changes in *energy prices* compared to what was expected at the time of investment affect profitability, scrapping behaviour and revaluation of the capital equipment in a production sector. Specifying the ex ante production function to be of the CES type, and experimenting with different values for the elasticity of substitution, the model is used to explain the actual development observed in many industries following the dramatic increases in oil prices in the 1970's. Thus, this provides a good example of the relevance and usefulness of the putty-clay model as a description of actual producer behaviour.

Among other studies using the putty-clay framework to analyse producer behaviour and energy demand, we will mention the ETA-MACRO model developed by Allan Manne, see e.g. Manne (1981). This is a model constructed to analyse energy-economy interactions, integrating long-term supply and demand conditions. In the "MACRO"-part of the model, the formulation of the production side is formulated as a putty-clay model in the sence that it distinguishes between *existing* and *new* capacity. For the latter, substitution possibilities are assumed, while fixed input coefficients for the inputs, labour, energy and (utilized) capital are assumed in the existing stock. For the increment of the capacity at time t, ΔY, the following nested production structure is assumed:

$$\Delta Y = [a\,(\Delta K^{\alpha} \Delta L^{1-\alpha})^{\rho} + b\,(\Delta E^{\beta} \Delta N^{1-\beta})^{\rho}]^{1/\rho} \tag{3.32}$$

where

ΔK is the volume of new capital,

ΔL is labour input,

ΔE is electricity, and

ΔN is non-electric energy needed for the operation of the new unit.

An autonomous assumption is made with respect to the rate at which capital units are replaced. This is not in strict coherence with the "micro-based" putty-clay framework outlined above, where the scrapping behaviour of the firms is "endogenous", depending on actual profitability of the various vintages of capital equipment. In the calculations presented in Manne (1981), the replacement rate is set to 4 percent. Moreover, the values of the parameters in (3.32) used in these simulations (i.e. the value shares α, β and the substitution parameter ρ) are not based on empirical investigation; instead "reasonable" values are inserted. With the chosen functional form, where the coefficients can be given familiar interpretations, this of course "allows" for experimenting with different parameter estimates.

None of the studies focused upon so far have made any attempt of determining *econometrically* the specified putty-clay production structure. In Hawkins (1978), a vintage model for Australian manufacturing industry is econometrically addressed, utilizing pooled data for five subclasses within the manufacturing sector. The ex ante production structure is represented by a dual cost function of the following type:

$$\Delta X_t = g\,(\Delta Y_t, \bar{P}_t, t) \tag{3.33}$$

where ΔY represents the planned increment in capacity, ΔX is the corresponding optimal use of factor inputs and $\bar{P}_t$ is a vector of expected factor prices. Coming to the specification of the ex post technology and behaviour, instead of tracing the development of each vintage based on relations similar to (3.28), a ***short run production function,*** i.e. a relation between the total, actual output and the actual use of variable inputs, is specified directly. Denoting the total capacity at time t by Yc_t and the corresponding planned use of inputs by Xc_t, the short run adjustment behaviour of factors is described by the following set of equations:

$$X_t / Xc_t = (Y_t / Yc_t)^{\rho} \tag{3.34}$$

Thus, the model developed by Hawkins is not strictly a putty-clay model for the micro units of the kind outlined in the outset of this section. However, a reasonable interpretation is that a detailed ex post structure similar to (3.28) is implicit in the model. This may be used as an argument to *motivate* the choice of the aggregate relation (3.34).1)

The putty-clay framework constitutes a rather extreme formulation in the sence that it is assumed a priori that *no* substitution possibilities exist ex post. A natural extension of this model is obviously to allow for *some flexibility* also after an investment decision has been made. This is the guideline for the model of producer behaviour suggested by Fuss (1977c). This model describes a production process with arbitrary degrees of substitutability between pairs of factors, but allowing for differences in flexibility ex ante and ex post. This more general framework is denoted a *putty-semiputty* model. In order to distinguish between substitution possibilities before and after an investment is undertaken, the *restricted cost function approach,* described in brief terms in section 3 is applied. The cost structure is assumed to be of the Generalized Leontief type, enabling the putty-clay hypothesis to be tested econometrically as a special case of this structure. The putty-semiputty framework is used to investigate the production structure in a sample of US electricity generating plants.

In Beltramo (1985), a modified putty-clay framework is used to project the demand for natural gas in US manufacturing at a regional level. For each vintage, ex ante substitution possibilities are assumed between energy and non-energy inputs, and in addition interfuel substitution is specified between natural gas and oil. The ex ante optimization problem then consists of choosing factor requirements per unit of output to minimize the present value of future costs associated with each vintage of capital. A rational expectation hypothesis is used to support the application of actually observed prices in the estimation process. Ex post, fixed coefficients are assumed to prevail for total energy and non-energy inputs. However, due to considerable dual fuel capability between oil (residual and distillate) and natural gas used for heat and power, these inputs are assumed to be substitutable also ex post. The short run oil and gas submodel employs BTU market share formulas for natural gas and oil, which are determined by price comparisons between the fuels. The parameters of the model are calculated econometrically. However, like many other studies of this kind, sufficient

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1) In our opinion, there are other features with the model specified by Hawkins which makes its interpretation as a putty-clay model more problematic. In order to relate the vintage structure to variables at the sector level, assumptions are made regarding the development of the the output capacity and the corresponding input levels (the Yc_t and Xc_t -variables) over time. More precisely, it is assumed that the existing capacity, Yc_t , decays at a *constant, exponential rate,* δ. As was stressed above, an important feature of the putty-clay framework is that production units may be taken out of operation, not necessarily when they are physically worn out, but because they are economically unprofitable to operate. There is therefore little reason to believe that the "survival profile" of the capacity is exponential; strictly the lifetime of the capital should be treated as an endogenous variable. Equally problematic is the assumption made by Hawkins that the levels for total, planned inputs, Xc_t , develop over time according to the the same exponential rate ("..because factor proportions are fixed...". equation (5) pp. 481). In the stylized putty-clay framework, fixed coefficients prevail *for each vintage,* but this will in general not be the case for the total capacity, since efficiences should be assumed to vary between the different capital units.

vintage data are not available. In order to implement the model, output and inputs associated with each vintage have to be related to figures at the sector level. The following additional assumptions are accordingly made:
i) Ex post, the ratio of output to capital services is fixed.
ii) Capital decays (geometrically) at a constant rate.
iii) All surviving capital vintages are utilized at the same rate.
These assumptions are similar to those (implicitly) adopted by Hawkins (1978). From a strict theoretical point of view, they may be said to violate some basic ideas of the putty-clay framework, namely that scrapping of capital units is endogenous and that capacity utilization in general differ between vintages - discretely when vintages are taken completely out of operation.

Another example of a generalization of the strict putty-clay model (with no possibilities of substitution ex post), is the study of Peck and Weyant (1983). This model was constructed by the authors to make long term forecasts of *electricity demand, but* to the extent this procedure is considered useful, it is probably equally well suited for studies of future natural gas demand. The authors stress that their aim is to construct a simple and transparent model easy to use in projecting long run trends in energy consumption. Like most other studies, the methodology is based on the observation that energy is closely related to the capital stock and therefore can not be adjusted to changing conditions in the short run. The *intensity* in the use of the equipment can, however, be varied dependent on the development in the energy prices. The following (ex post) relation is assumed to describe the intensity by which capital equipment installed in year s is used:

$$I_{s,t} = \left(\frac{P_t}{P_s}\right)^{-d} P_s^{-a} Q_s^{b} R_s^{c} \tag{3.35}$$

where

$I_{s,t}$ is the energy intensity in period t of the equipment installed at time s,
P_i and Q_i are prices of electricity and "other energy", respectively, in year i (i=s,t),
R_s is income per household in period s, and
a, b, c, d are parameters (all assumed to be positive).

Even though it is not stated clearly in their paper , both prices and income should be interpreted as real variables, i.e. normalized against some average price index. At the point of time when investment is made, the initial intensity is determined as

$$I_{s,s} = P_s^{-a} \, Q_s^b \, R_s^c \quad . \tag{3.36}$$

If the (relative) price of gas changes over time, the utilization of the capital equipment will be modified due to the relative price term on the right hand side of (3.35).

Estimates of price- and income elasticities of electricity demand in this study are based on a survey of recent empirical studies. Mean estimates are then employed in the benchmark projections for energy demand. This procedure obviously has its weaknesses, because it is not at all clear how traditional estimates of "average" price- and income elasticities estimated on long time series may be applied in a "vintage" framework like (3.35), distinguishing explicitly between the effect of a price change on the initial choice of technology ($-a$) and the impact on the rate of utilization of the equipment installed earlier ($-d$).

By inserting values for price- and income variables for every year in equation (3.35), the average intensity, $\overline{I}_t$, can be calculated by weighting the intensities with the corresponding vintages of capital equipment. If S_t denotes the vintage installed in period t and δ is the (constant) rate of retirement, total energy consumption in period t, E_t can be written as:

$$E_t = \overline{I}_t \sum_{r=0}^{\infty} S_{t-r}(1-\delta)^r \tag{3.37}$$

In order to apply equation (3.37), data on various vintages of energy using equipment are required. In practice the procedure may therefore be difficult to implement directly, and in any case it "would violate the desire to employ a simple and transparent methodology"(Peck and Weyant, pp.25). In order to avoid these difficulties, Peck and Weyant undertake two major simplifications: Firstly they assume that the stock of energy using equipment at each point of time is proportional to some aggregate measure of economic activity. For the household sector the number of households is recommended as such a measure, while for industries series of gross product may be applied. Secondly, Peck and Weyant specify a "steady-state" version of the model by assuming constant growth rates for the electricity price, the price of competing fuel, income and the number of households.

The steady state version of the model is tested by Peck and Weyant over the period 1963 - 1982. These simulations give a reasonably good approximation to the actual growth in (electricity) consumption in the US. The strict steady-state assumptions are also modified by dividing the indicated time interval into sub-periods, assuming the growth rates to be constant within each of these intervals.

3.5 Energy demand and growth in the dwelling stock.

As argued earlier, in our opinion it is a weakness inherent in many of the energy demand studies surveyed above that, when applied to the residential sector, they do not specify explicitly the relation between energy use and *the dwelling stock*. In several studies, the capital concept is explicitly defined as "energy using equipment"(i.e. furnaces, stoves, appliances etc.). Some even suggest that the capacity of the capital equipment can be measured by the maximum or average energy use (Fisher and Kaysen (1962)). In particular with respect to the modeling of dynamic elements in energy demand, there are reasons to believe that costs of adjustment are mainly due to problems of changing the design of the building itself more than the costs of investing in new energy using equipment (even though the borderline between these two kinds of "costs" is not clearcut).

One study which takes explicitly into the account the relationship between energy use and the dwelling stock, is the analysis carried out by Roedseth and Strom (1976). This is a study of electricity demand in Norwegian households. Some elements in this model reflect features which are rather specific to the Norwegian energy market, where electricity plays a more important role and dual fuel systems are more common than in most other countries. Still, the study is an illustration of how the dwelling structure may be incorporated in an energy demand model. Some of the basic ideas are believed to be fruitful in analysing gas demand as well. The model is divided into three parts. In the first submodel, households' *total energy demand* (exclusive of fuels for transportation) is determined by a complete set of demand functions. These are derived from a translog indirect utility function, following the approach of Jorgenson (1974). Based on duality theory, this approach is very similar to the models described in chapter 2, and we therefore exclude details at this point. The second part of the model estimates *the electricity share* of total energy consumption in the household sector.1) Two relations are specified:

$$x_v = f_v(p_e, y, G) \tag{3.28}$$

$$x_{vo} = (1-\frac{A_e}{A})\, f_{ov}(p_e, y, G) \quad , \tag{3.29}$$

where

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1) The specification of (3.28) and (3.29) is based on the assumption that only electricity and fuel oil are used for heating purposes. This was a reasonable description of the Norwegian energy market in the 1960's and the early 1970's.

x_v is the share of total energy consumption used for heating,

x_{vo} is the use of fuel oil for heating purposes,

p_e is the price of electricity relative to the price of oil,

y is a real income variable,

G is the annual number of degree days,

A is the total number of dwellings, and

A_e is the number of dwellings heated with electricity.

Equation (3.28) determines the heating share of total energy consumption. (3.29) states that oil consumption relative to the total energy use for heating purposes is proportional to the stock of oil heated dwellings. The function $f_{vo}(\cdot)$ may thus be interpreted as expressing the short term utilization of the installed oil heating system. The motive for including the relative electricity price in this relation is the fact that many oil heated homes in Norway have *dual fuel capabilities,* allowing for a relatively costless switch from one fuel to another as response to changed relative prices.

From (3.28) and (3.29), Rodseth and Strom derive a relation explaining short term variations in the electricity share as

$$x_{ve} = 1 - (1-\frac{A_e}{A})\, f_o(p_e, y, G) \tag{3.30}$$

where $f_o(\cdot) = f_v(\cdot)\, f_{vo}(\cdot)$.[1]

The third part of the model aims at explaining *changes in the stock of dwellings* heated solely with electricity. This is done by postulating the following relation:

$$\frac{\Delta A_e(t)}{\Delta A(t)} = f_A(p_e, y) \tag{3.31}$$

where $\Delta A_e(t)$ and $\Delta A(t)$ are net additions to stocks. The authors stress that in this equation installation costs of various types of equipment should have been included in addition to fuel costs, but proper data were not available. Income is included in (3.31) since this variable is believed to influence the choice of fuel system in new buildings. *A long term equilibrium*

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[1] This second part of the Rodseth-Strom model should obviously be modified in order to be applicable for analysing gas demand. A simple extension of the model to include gas would be to specify a relation expressing the "general competition" (i.e. comprising all end uses) between gas and electricity in the household sector, i.e.

(*) $x_{ge} = f_{ge}(p_{ge}, y)$

where x_{ge} is the use of gas relative to the use of electricity, and p_{ge} is the corresponding price. The share of gas in total energy consumption, x_g, is then easily derived as:

(**) $x_g = (1 - x_v\ x_{vo})/x_{ge}$

situation is defined - and corresponding long term elasticities are calculated - when $\frac{\Delta A_e}{A_e} = \frac{\Delta A}{A}$.

The model of Rodseth and Strom is interesting because it is based explicitly on the fact that energy use is related to the dwelling stock. Furthermore, the model attempts to treat separately different end uses in the residential sector. However, the model was originally designed for analysing demand conditions in the Norwegian energy market, which distinguishes itself from most others, both because electricity has a large market share in total energy consumption and because dual fuel systems are widespread in the household sector. It may also be objected that the relation explaining fuel choices in the heating structure for dwellings is very simple. In the next chapter, an approach that treats this choice problem in a more fundamental way will be presented.

4 ENERGY USE AND INDIVIDUAL DATA: THE DISCRETE CHOICE APPROACH.

In the previous sections, all the suggested models were implicitly based on a specification in which energy/gas demand varied *continuously* with a set of explanatory factors such as prices and income. In this traditional *econometric approach* to the modeling of energy demand, relations are derived from assumptions of smooth and continuous consumer behaviour and are typically estimated on time series or pooled data. The consistency with economic theory and the statistical foundation are usually regarded as the main advantages of this procedure. On the other hand, it is clear that this methodology does not describe in detail how individuals respond to changes in factors influencing their behaviour. In particular, even when dynamic models are specified, the econometric models do not take into account *detailed information* on how energy demand is related to capital equipment and energy using appliances. At an *aggregate* level, i.e. when average figures and totals for energy use and other variables in a country are applied in the estimation, the continuity assumption embedded in the traditional model is often regarded as reasonable . Nevertheless, it is always a possibility that the procedure involves *aggregation errors.* While these models have proved to be rather reliable with respect to predictions when "things are running smoothly", they are probably poorly suited for analysing exogenous shocks and big changes in the choice conditions of the economic agents. Consequently, there is a need to supplement the traditional econometric approach with other methods explicitly taking into account detailed information on energy use and the stock of energy using equipment.

At the "microlevel", i.e. looking at an individual consumer, the specification of continuous relations describing energy demand is probably not the most appropriate. Because energy demand is closely related to indivisible capital goods, decisions on energy consumption may include choices between *a limited number of fuel systems.* In recent years, considerable attention has in the literature been paid to the question of how to model and estimate behavioural models which involve *discrete (often qualitative) variables.* The fact that choice variables are not quantitatively observable, may in itself create problems of applying traditional econometric methods such as e.g. Ordinary Least Square (OLS). Furthermore, even if the outcomes of the variables are countable, the fact that the choice alternatives are *limited* has the implication that the commonly made assumptions about the stochastic disturbances are no longer valid. OLS will no longer be efficient and the estimates may even be inconsistent1).

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1)The meaning of "limited choice alternatives" is of course ambiguous. It is not obvious what number of choice alternatives qualifies for the use of a discrete model and/or implies problems of the kind mentioned if a traditional procedure is followed. With respect to the modeling of gas demand this "evaluation" is, however, rather simple since a consumer usually can choose only between 3 or 4 different fuel systems.

When individual data on energy use and related factors are available, the application of a discrete choice model is certainly an attractive alternative to traditional econometric approaches. Below the main principles of the discrete choice procedure are sketched and discussed with respect to the modeling of natural gas demand.

4.1 Discrete choices and individual preferences.

The optimization problem faced by an individual consumer can in its simplest form be represented by

$$Max\ U(x_i, \alpha_i) \qquad (4.1)$$

$$\text{subject to } p'\, x_i \leq Y_i$$

where

x_i is a vector of commodities chosen by consumer i,

p is the commodity price vector,

Y_i is the income level for consumer i, and

α_i is a parameter characterizing the preferences of the consumer.

The solution to this problem is given by the individual demand functions

$$x_i = f\,(p, Y_i, \alpha_i) \quad . \qquad (4.2)$$

The vector of individual characteristics, α_i, is of course unobservable to the econometrician. When one attempts to fit equation (4.2) to a set of individual cross section data, the common ***implicit assumption*** is that the "individual preferences" are distributed randomly around an average parameter vector, $\overline{\alpha}$. The traditional ***stochastic*** specification of (4.2) is thus

$$x_i = \overline{f}\,(p, Y_i, \overline{\alpha}) + u_i \quad , \qquad (4.3)$$

where u_i is a stochastic random variable. The most common assumption to be made is that the stochastic residuals are normally distributed with zero mean and constant variance *irrespectively of which consumer is studied.* As will be shown in more detail in the next section, when choice alternatives are discrete, there are specific indications that these assumptions are no longer valid and that differences in individual preferences enter the model in a more fundamental way than is assumed in equation (4.3).

Intuitively, the problems created by differences in individual preferences can be indicated in the following way: For the sake of comparison, suppose that the consumer can make continuous choices with respect to gas consumption. In such a case it may be reasonable to assume that the observed variations in demand from consumers facing the same independent variables are mainly caused by errors of measurement, while unobservable differences in preferences only contribute to a minor extent. If this is the case, the "normal stochastic assumptions" need not be very problematic. On the other hand, when one operates within a model where the consumer has to choose between a limited number of fuel systems, variations in gas demand stem partly from the fact that consumers *enter into or leave the market*, respectively. This means that for individuals facing the same values for all the explanatory variables, some will be observed using a gas system, while others will be "out of the gas market", i.e. they have chosen another fuel system to serve their need. According to the specified theoretical relations, these differences in observed behaviour must have been caused by variations in the disturbance terms. When the dependent variable is discrete, it is far less likely that these differences in outcomes can be explained mainly by problems of measurement; the most reasonable factors "underlying " the disturbance terms in the discrete case are differences in preferences between consumers.

In contrast to the traditional econometric approach, *the discrete choice model* explicitely takes into account implications of unobservable variations in individual preferences when the number of choice alternatives is limited .

4.2 The discrete choice model: main features.

When presenting the principal features of the discrete choice approach, it is convenient to describe the consumer behaviour in terms of the *indirect utility function,* as this is directly related to the exogenous explanatory factors. Assume for simplicity that an individual for e.g. heating purposes can choose between *two choice alternatives, gas (1) or non-gas (0)* Let the utility level related to each alternative be denoted

$$U_j = V_j(z_j) + v_j \ , \quad j = 0,1 \ , \tag{4.4}$$

where

z_j is a vector of variables characterizing alternative j,

$V_j(z_j)$ is the indirect utility function for this alternative, and

v_j is a stochastic variable representing individual characteristics.

The consumer will obviously choose alternative 1, i.e. gas, if this yields higher utility than any other choice, i.e. if

$$U_1 > U_0 \quad . \tag{4.5}$$

Due to unobservable preference characteristics, there is a certain probability for this event to occur. A binary variable, y, related to this discrete choice problem may be defined which, without loss of generality, can be assumed to attain the value one when gas is chosen, zero otherwise. The choice probability for choosing gas can then be expressed as

$$P_1 = Pr(y=1) = Pr(U_1 > U_0) \quad . \tag{4.6}$$

Applying (4.4), we then have

$$P_1 = Pr(v_0 - v_1 < V_1 - V_0) = F(V_1 - V_0) \quad , \tag{4.7}$$

where F is the cumulative distribution function for the term $v_0 - v_1$. The explicit form of this probability model then depends on

- the form of the indirect utility function, and
- the assumed distribution of the disturbance terms.

In most theoretical and empirical works using discrete choice models, the indirect utility function, V, is specified as a linear relationship. The same assumption is adopted here. We furthermore apply z as a general symbol for the vector of independent variables included in $V_1 - V_0$ and β as the corresponding parameter vector, so that the choice probability can be written as

$$P_1 = F(\beta' z) \quad . \tag{4.8}$$

Some of the z-variables can be choice-specific, e.g. prices and costs, while others only characterize the individual in question, e.g. income and other socio-economic variables. If the latter type of variables appear in the utility function with the same coefficient, i.e. irrespectively of the choice made, they will obviously cancel out in the expression (4.8), otherwise the β's will equal the differences between the corresponding parameters in the indirect utility functions. Regarding the choice-specific characteristics, corresponding parameters are commonly assumed to be identical between alternatives. This means that a change in e.g. the price of natural gas and a similar change in the price of fuel oil have identical (absolute) impacts on the indirect utility levels (given the linear structure). This may be regarded as a rather strong assumption. The procedure may, however, be regarded as plausible when the underlying utility structure is of a socalled "Lancaster type"(Lancaster (1966)), i.e. in cases where the choices are between different "modes", producing various amounts of "services", and these services enter the utility function rather than the physical entities. In such cases, it may be noted that energy prices should be measured *per (common) energy unit.*

In order to obtain an explicit formulation of this probability model, a specific analytical form of the F-function must be chosen. In the literature, most attention has been paid to the following three models:

(i) The linear model: $F(w) = w$

(ii)) Probit model: $F(w) = \Phi(w)$,

where Φ is the cumulative normal distribution function.

(iii) Logit model: $F(w) = \dfrac{e^{w}}{1 + e^{w}}$.

where $w = \beta' z$

It may seem a bit strange to include a *linear* structure among the potential probability models. The linear functional form does not obey conditions of being a probability distribution, since no mechanism guaranties the value of the function to stay between 0 and 1. Still, because it yields a simple structure, it is some times recommended for preliminary studies and calculations (Amemiya (1981)). In contrast to the linear model, the *probit and the logit models* fulfill *globally* the properties of being probability functions. While the probit model assumes that the disturbances $v = (v_0, v_1)$ are normally distributed, the logit structure is a consequence of assuming the stochastic terms to be independently, identically *extreme value distributed* (type 1, McFadden(1973)). For discrete choice models derived from consumer behaviour, this is the specification most frequently applied.

In order to discuss briefly some econometric problems created by the discrete structure of this model, it is useful to write the relation between the the binary "outcome variable", y, and the probability expressed by (4.8) as

$$y = P_1 + \epsilon = F(\beta' z) + \epsilon \ . \tag{4.9}$$

This relation defines ϵ as the residual between the observed values of the binary variable and the theoretical probability for choosing this fuel alternative. When approaching the problem of estimating the discrete choice model, one simple procedure would be to carry out a regression directly on this equation. However, it is easy to demonstrate that this cannot be done straightforwardly by e.g. OLS: As mentioned, y is distributed as a binomial variable, with

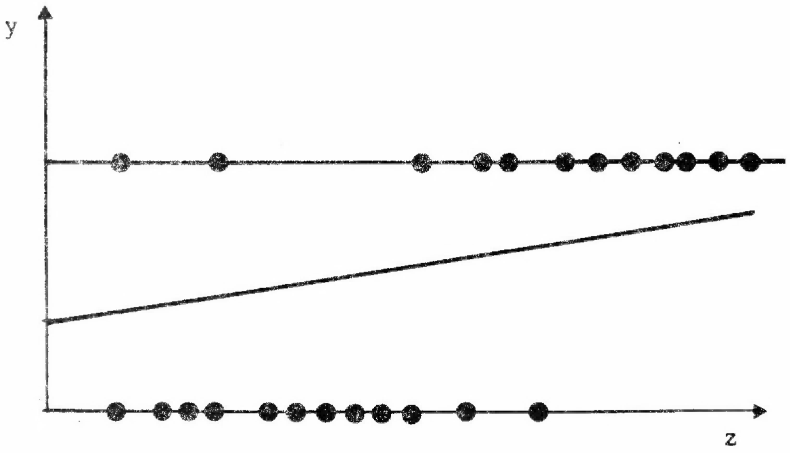
$$E(y) = P_1, \ \ var(y) = P_1(1 - P_1) \ \ , \tag{4.10}$$

and from this we easily derive

$$E(\epsilon) = 0 \ , \ var(\epsilon) = P_1(1 - P_1) \ . \tag{4.11}$$

From (4.11), it is seen that the variance of the residuals in (4.9) varies with the independent variables, z. This implies that if we try to fit a regression to (4.9), we have the case of *heteroscedastic disturbances,* creating well known problems when estimation techniques like OLS are used.

Figure 4.1: The regression line and discrete outcomes.



A simple illustration of the heteroscedasticity of this model when the probability structure is *linear,* is shown in figure 4.1. In this case, $E(y) = \beta' z$. Assuming for simplicity only one explanatory variable, the graph is a straight line as indicated in figure 4.1. With only two possible outcomes, observations of the binary variable will be located either on the abscissa axis or at the ordinate level of $y = 1$, as indicated in figure 4.1, where it is implicitly assumed that there is estimated an increasing relationship between the expected value and the independent variable. The figure clearly reveals that the expected "unexplained" deviations from this line must depend on the level of the z-variable. With the structure drawn in the figure, the variance will first increase and then diminish as the estimated probability approaches one.

The problem of heteroscedasticity embedded in the discrete choice model indicates that a proper estimation procedure should utilize explicitly the specified probability structure of the model. The most frequently applied procedure for estimating discrete choice models is *the maximum likelihood method.* Assume that the available sample contains observations on n individuals with their respective characteristics (z's). The probability for individual i to choose gas can thus be written as

$$P_{1i} = F(\beta' z_i) \quad ,i=1,\ldots,n. \tag{4.12}$$

A general expression of the probability that *a given outcome* (i.e. a specific value of y) for individual i is going to occur, is

$$Pr(y_i) = P_{1i}^{\,y_i}(1 - P_{1i})^{1-y_i} \tag{4.13}$$

The joint probability for the observed sample is accordingly given by the likelihood function

$$L = Pr(y_1, y_2, \ldots, y_n) = \prod_{i=1}^{n} P_{1i}^{\,y_i}(1 - P_{1i})^{1-y_i} \quad . \tag{4.14}$$

It can be shown that the log-likelihood operator, L, is globally concave, and therefore a solution which fulfills $\partial L / \partial \beta = 0$, is unique.

Discussions of methods of estimating discrete choice models commonly distinguish between two cases with respect to the nature of data (see e.g. Amemiya (1981) and Maddala (1983)):

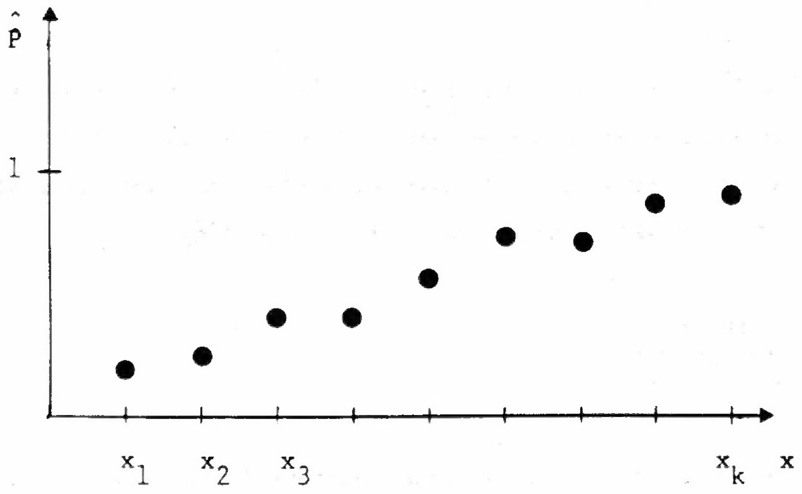
a) many observations per cell (grouped data), and

b) few observations per cell (strictly "individual" data).

If in the observed or available data several "individuals" have identical characteristics/independent variables (except for the unobserved features), the data are classified as grouped. When this is not an outstanding feature of the data, one has the situation with (strictly) individual data. The ML-estimator is obviously applicable in both cases. However, in a situation with grouped data another calculation method for discrete choice models has been suggested in the literature (originally in a series articles by J.Berkson (see e.g. Berkson (1953,1955)). This is called ***the mimimum chi-square (MIN-X^2-) method.*** One advantage of applying this procedure is that it is less demanding than ML with respect to computational techniques and equipment. In our case of modeling the demand for natural gas in Europe, a more fundamental advantage is that this method provides a possibility of estimating the parameters of the discrete choice model even if only average data for the countries included are available.

Amemiya (1981) uses the following figure (which could be compared with figure 4.1) to illustrate the computational advantages of a situation with many observations per cell

Figure 4.2: A situation with many observations per cell.



It is evident that it is easier to fit a regression line to a set of observations of observed frequencies than to observations on the binary variable, y, indicated in figure 4.1.

The mimimum chi-square ($MIN\text{-}X^2$-) method can be illustrated in a formal way by assuming that the vector of explanatory variables, z, is naturally grouped into K subsets of values, $z_{(1)}, z_{(2)}, \ldots, z_{(K)}$, where $K < n$. If furthermore n_k is the number of observations corresponding to $z_{(k)}$, and that among these m_k made the choice $y = 1$, *observed frequencies* are defined by

$$\hat{P}_k = \frac{m_k}{n_k} \tag{4.15}$$

The "theoretical counterpart" to the observed frequencies are obviously the probabilities given by (4.8), which may now be written as

$$P_k = Pr(y_i = 1) = F(\beta' z_{(k)}) \quad , \text{for} z_i = z_{(k)} \quad , k = 1,2,\ldots K. \tag{4.16}$$

The structural relation needed for the $MIN\text{-}X^2$-estimator to be used is arrived at by inverting relation (4.16) and expanding $F^{-1}(P_k)$ by a first-order Taylor series around $\hat{P}_k$. This yields the expression

$$F^{-1}(\hat{P}_k) \approx \beta' z_{(k)} + \frac{1}{f(F^{-1}(P_k))}(\hat{P}_k - P_k) = \beta' z_{(k)} + w_k \quad , \tag{4.17}$$

where f is the density function corresponding to F. The last equation in (4.17) simply defines w_k, which may be interpreted as a stochastic error term for this equation. $E(w_k) = 0$, since $E(\hat{P}_k) = P_k$. By again using the properties of a binomial distributed variable the variance of w_k can be expressed as (see e.g.Amemiya (1981), pp.1498)

$$\sigma_k{}^2 = var(w_k) = \frac{P_k(1 - P_k)}{n_k f^2 F^{-1}(P_k)} \quad . \tag{4.18}$$

The variance of w_k is thus a function of P_k, and therefore depends on the values of the independent variables, $z_{(k)}$. Again the problem of heteroscedasticity arises when trying to carry out an ordinary regression. The $MIN\text{-}X^2$-estimator is arrived at by applying *a weighted least square (WLS)* procedure to (4.17), i.e. dividing through this equation with σ_k, cf. that $var(w_k/\sigma_k) = 0$. The problem that σ_k in itself is unobservable is usually solved by substituting either $\hat{P}_k$ or $F(\hat{\beta}' x_k)$ for P_k in the regression (Amemiya (1981)).

As already mentioned, the case of grouped data and the specific estimation methods designed for such situations may be of particular relevance to the modeling of energy demand and fuel choices. If one limits oneself to analysing energy consumption in *one specific country,* truly "individual" data may be available from surveys and censuses. Since differences in incomes and other socio-economic characteristics generally exist between individual consumers, this may be regarded as a case of "few observations per cell". On the other hand, when attempting to model consumers' fuel choices in a region covering several countries, say in Western Europe, one may be faced with the problem that only average data on these decisions in the various countries exist1). However, in such cases a possible procedure is to proceed *as if* the situation was one with many observations per cell, i.e. assuming (implicitly) that consumer attributes/explanatory variables *vary between countries, but not within each country.* This obviously is a very strong assumption. However, international studies of energy use reveal that important differences exist *between countries* (see e.g. Schipper, Ketoff and Kahane (1985)). Based on these observations, one may assume that the "grouped data procedure" applied with average data still enables us to capture main trends and movements in energy consumption in the *the region as a whole.*

4.3 Multi-response models.

So far, it has been assumed that there are only two possible outcomes of the qualitative variable. In a model describing economic behaviour there may, however, obviously be several alternatives available for the agent. When discrete choice models are extended to such situations, additional questions arise. In the literature, a main distinction is drawn between *ordered models* and *unordered models.* Typical examples of the former are decisions on the number of children and the problem of choosing 1, 2 or 3 cars (given that these decisions may be related to a specific choice situation). However, with respect to energy demand, the choices between different fuels are hardly of this kind, i.e. no natural "ranking" of alternatives exists. Hence, in the following we will concentrate on examples of unordered models.

Let us now assume that the problem for the consumer is to choose between m alternatives. Relation (4.4) can in that case be extended to cover m different levels of indirect utility, and the probability for event i to occur is in general given by the expression

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1) The international database on energy use and dwelling characteristics at Lawrence Berkeley Laboratory is of this kind (see e.g. Olsen (1985)).

$$P_i = Pr\,(v_j < v_i + V_i - v_j \ \text{for} \mathit{all} j \neq i) = \int_{-\infty}^{\infty} \prod_{j \neq i} F(v_i + V_i - V_j)\, f\,(v_i)\, dv_i\,. \tag{4.19}$$

Again, the explicit form of this probabilistic model depends on the stochastic assumptions of the disturbance terms, v_j. By assuming that these are identically and independently extreme value distributed, McFadden (1973) arrived at *the multinomial logit model (MNL),* in which case

$$P_i = \frac{e^{V_i}}{\sum_{j=1}^{m} e^{V_j}} \quad . \tag{4.20}$$

As mentioned earlier in this section, the logit model has been very popular when studying and estimating discrete choice models, probably due to its simple structure and the fact that it has proved rather easy to solve computationally. However, in the multi-respons case the logit specification also involves a property that may be regarded as less advantageous. The MNL model obeys the socalled *Independence of Irrelevant Alternatives (IIA)* principle. The IIA principle states that the relative probability of alternative j to be chosen over i is independent of the availability of alternatives other than i and j. In the logit model outlined above, this is seen to be the case if the attributes of j, z_j, does not depend on the characteristics of any other alternative[1]. The IIA-property in the MNL model can in that case be seen from the expression

$$\log \frac{P_j}{P_i} = V_j - V_i \quad . \tag{4.21}$$

The IIA property implies certain restrictions between cross elasticities:

$$\frac{\partial \log P_i}{\partial \log z_{sj}} = \beta_s\, z_{sj}\, P_j \quad , \tag{4.22}$$

where z_{sj} is an arbitrary element in the vector z_j. From (4.22) it follows that the IIA property in the MNL model implies that the elasticity of the probability of choosing fuel i (P_i)

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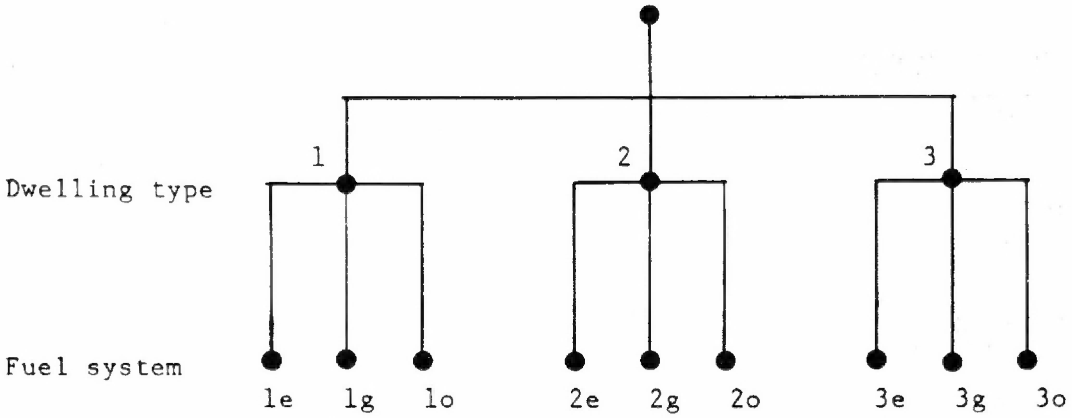
[1] It is important to note that the MNL model does not *necessarily* imply the IIA-property. The IIA will not be present in a MNL model if V_j depends on interactions between characteristics of different choice alternatives. For example with the respect to the specification of prices as explanatory variables, IIA follows if p_j is the only price variable among the attributes of alternative j, while it does not hold if e.g. V_j depends on p_j *relative to prices of other other alternatives.*

with respect to e.g. the price of fuel j is the same for all i.

The weakness of a model embedding the IIA property becomes evident when some of the choice alternatives by the consumer are regarded as "quite similar" compared to the other choice possibilities. The classical example is the choice between driving a car and taking a "red bus", in which case the availability of another bus alternative obviously should influence the relative probability of choosing between a car and the "red bus" alternative. Such "similarities" between choice alternatives typically arise because each alternative involves a *certain combination of features/outcomes,* combined with the fact that some of these features are regarded as more "fundamental" than others. This may also be a proper description when discussing factors determining *households' energy use.* Energy consumption is e.g. highly dependent on the choice of *type of dwelling,* which probably may be regarded as "more important" than the choice of fuel. Moreover, when analysing energy consumption by different *end uses,* it is presumably realistic to assume a certain sequence in these decisions. The consumers first choose the *space heating system,* and then secondly decide upon other end uses dependent on that "primary choice" (Goett and McFadden(1982)). In such cases, the use of the Luce/McFadden MNL model implying the IIA-property may clearly yield unrealistic result. A solution to this problem is *the nested multinomial logit model (NMNL)* suggested by McFadden (1978). Goett and McFadden (1982) have developed a very detailed framework for modeling consumer demand, using a NMNL approach for the modeling of fuel choices by different end uses. Ruderman (1985) presents a similar model for energy appliance choices in US households.

Figure 4.3: A "decision tree" for housing type - and space heating choices.



The principal features of this approach are illustrated in figure 4.3. There it is assumed that a consumer can choose between three types of housing units (i=1,2,3) and three kinds of fuels (j=electricity (e), natural gas (g), fuel oil (o)) for heating purposes.

The joint probability for choosing a given combination of dwelling/fuel system, P_{ij}, can be decomposed in the following way:

$$P_{ij} = P_{j/i} \cdot P_i \tag{4.23}$$

where $P_{j/i}$ is the probability of choosing fuel system j *conditioned* on the previous choice of dwelling type i, and P_i is the (unconditioned) probability of choosing dwelling type i. Consistent with the logic of the "decision tree" in figure 4.3, it is reasonable to assume that the utility of a specific combination of dwelling and fuel system, V_{ij}, is determined partly by attributes of the dwelling type alone, and partly by characteristics of the fuel system in the chosen housing unit. Using the notation of Maddala (1983), we can then write

$$V_{ij} = \beta' \, x_{ij} + \alpha' \, y_i \quad . \tag{4.24}$$

In the nested logit model each of the probabilities on the right hand side of (4.23) is specified with a standard MNL form. In the present case, the total model can therefore be described by the following system of equations:

$$P_{j/i} = \frac{e^{\beta' x_{ij}}}{\sum\limits_{j=e,g,o} e^{\beta' x_{ij}}} \tag{4.25}$$

$$P_i = \frac{e^{\alpha' y_i + \gamma' I_i}}{\sum\limits_{i=1,2,3} e^{\alpha' y_i + \gamma' I_i}} \quad . \tag{4.26}$$

The term

$$I_i = \log \sum_{j=e,g,o} e^{\beta' x_{ij}} \tag{4.27}$$

is denoted *the inclusive value* (McFadden (1978)), and can be interpreted as the "average desirability" of the different fuel system alternatives available given the choice of dwelling type i. When the coefficient γ of this inclusive value is 1, it is easy to see that the model

(4.25) and (4.26) degenerates to the standard joint MNL model.[1] When $0 < \gamma < 1$, the effects of the characteristics of the fuel system on the choice of dwelling type is reduced compared to the standard joint MNL model. The model treats the fuel systems within a specific housing type as more and more "similar" as γ approaches zero.

While the traditional MNL model follows from assumptions that the individual error terms are identically and independently extreme value distributed, McFadden(1978) has proposed *a generalized extreme value distribution (GEV),* and furthermore shown(see also Maddala (1983)) that the nested logit model can be derived as a special case of the GEV model[2].

A main point distinguishing the nested logit model from the traditional MNL model is that the former specification avoids the IIA-property. This can be seen by studying ratios between joint probabilities in the NMNL model. The similarities between alternatives within each group are reflected in the cross elasticities. From the fuel choice model above, a general expression for these magnitudes is given by

$$\frac{\partial \log P_{ij}}{\partial \log x_{skl}} = \begin{cases} -\gamma \beta x_{skl} P_{kl} , \text{ if } k \neq i \\ -\gamma \beta x_{skl} P_{kl} - (1-\gamma) \beta x_{skl} P_{k/l} , \text{ if } k = i \end{cases} , \tag{4.28}$$

where $i,k = (1,2,3)$, and $j,l = (e,g,o)$. From (4.28), it is seen that the cross elasticities (in absolute value) are larger between alternatives *within* each group (i.e. k=i) than *between* different types of dwellings.

Another advantage of applying a nested model is that the sequential structure simplifies the estimation procedure of the various parameters. In the indicated dwelling/fuel problem, the coefficient vector β can first be estimated from the subsystem (4.25), then the "inclusive values", I_i, are calculated and finally estimates on α can be obtained by inserting these values in (4.26) and applying a standard solution procedure.

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1)Inserting the expressions (4.25) and (4.26) in (4.23) when γ=1 yields

$$(*)\ P_{ij} = (e^{V_{ij}}) / (\sum_k \sum_l e^{V_{kl}}) ,$$

where the V's are given by relation (4.24). This is seen to be the standard MNL model (see also Maddala (1983) who, by starting out from (*), with $\gamma = 1$, derives the expressions for the related probabilities in (4.25) and (4.26).

2)This correspondence reveals that in the nested logit model the correlations within each group is taken care of by a single parameter. Amemiya (1981) has suggested other models which imply more complex correlation structures.

4.4 Some extensions of the traditional model.

The main strength of the discrete choice approach to the modeling of households energy demand is that it starts out from the level of the individual consumer and explicitly takes into account rigidities and limitations in substitution possibilities that may exist at this level. Accordingly, the procedure is particularly suited for incorporating information in the form of *micro data*. The presentation of the approach given above is rather brief, and is primarily meant as an indication of how the method can be used in energy demand modeling. More detailed "surveys" of models of discrete and qualitative variables, also including examples of application, can be found in Amemiya (1981), Maddala (1983), McFadden (1984) and Hanemann (1984). It should be emphasized that there may be problems of implementing and estimating a discrete choice model because it demands a rather sophisticated and detailed data base. Some problems of this kind are discussed in Olsen (1985).

A natural extension of the standard discrete choice model is to develop a *discrete/continuous choice model*. When applied to the natural gas market, a resonable specification may be that the choice of fuel system is discrete, while the decision on how to utilize a given system is a continuous one. In Goett and McFadden (1982), both these stages in consumers' energy demand behaviour are modeled. However, the two steps seem in this case to have been estimated separately. Another procedure, preferable to this, but also more demanding with respect to data, is to estimate the discrete and continuous decisions *jointly*. A clarifying presentation and a discussion of the general theoretical framework for a discrete/continuous model of consumer demand is given by Hanemann (1984). Among empirical applications of this theory to the energy market, is the study by Dubin and McFadden (1984), analysing simultaneously the decisions on electric appliance holdings and their utilization in US residential sector. In Dagsvik, Lorentsen, Olsen and Strom (1986), a discrete/continuous choice model for analysing European gas demand is presented. In this framework, the (discrete) fuel choice involves comparisons of indirect utility attached to the various fuel systems. Consistent with the theory of consumer behaviour, continuous equations describing gas use per household are derived from the indirect utility functions by using Roy's identity.

Another possible limitation to the standard discrete choice model is that it is *static* in nature. The relevance of a static model of course depends on the time horizon (i.e. whether it is "short run" or "long run"), on the data available and of course on the assumptions of how adaptions and changes in behaviour actually take place at the household level. Both the studies of Goett and McFadden (1982) and Ruderman (1985) use discrete models and cross section data to analyse energy appliance installations in *new homes*. In both these models it is therefore presupposed that appliance investments are irreversible, i.e. that the original chosen fuel system is not replaced until it is worn out (the point is explicitly stressed in both papers). This assumption *may* be regarded as particularly unfortunate in a demand model for natural gas. In this market, significant *conversions* from other fuel systems to gas have taken place and can, of course, also occur in the future. In particular, when observations for several years are applied, *a dynamic model* should probably be regarded as a more "ideal" framework for analysing gas demand. Based on a dynamic stochastic model suggested by Dagsvik (1983), a paper by Dagsvik, Lorentsen, Olsen and Strom (1986) specifies a dynamic extension of the traditional (static) MNL model. In this framework, choice probabilities are specified as in the standard model, but at each point of time the probabilities are *explicitly dependent on the choices previously made*. The model thus allows for conversion from one fuel system to another. The transition between different states over time is described as a Markov process, where the transition probabilities are functions of explanatory variables like fuel prices, income and socio-economic factors.

5 BLOCK RATE SCHEDULES AND NATURAL GAS AVAILABILITY

In this chapter, we focus on two important factors in natural gas demand analysis which have been neglected so far in the discussion of the various theoretical models, namely the existence of *decreasing rate schedules* for gas sales and *access to the distribution network*. A decreasing block rate schedule implies that for an individual consumer, the average price paid is reduced by increasing volumes purchased. This causes a simultanity problem when trying to estimate a demand function with a traditional average price included, since it can not be identified whether the price effect estimated in the "demand equation" refers to the slope of the true demand function or the slope of the rate schedule.

The availability of natural gas is obviously of great importance when undertaking studies of this market. Because the distribution network for natural gas has been extended to new areas in most European countries over the last two decades, the number of potential customers has changed considerably. Unless these changes in availability of natural gas is taken into account in an econometric analysis, biased estimates of price and in particular income elasticities should be expected.

In this chapter, we will briefly review some techniques that have been suggested to solve problems caused by decreasing block pricing and availability of supply.

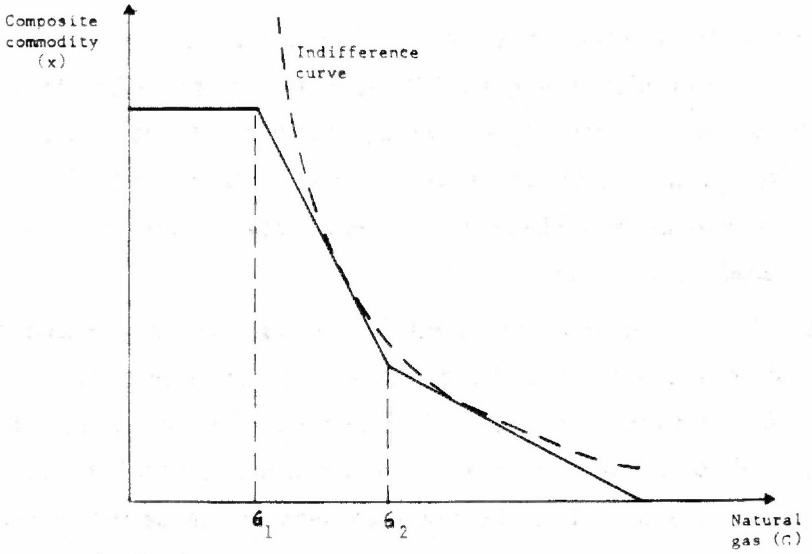
5.1 Declining block rate pricing.

A declining block rate schedule is a pricing schedule in which the unit price declines stepwise with the volume purchased by a customer. In addition, it often embodies a fixed charge for being hooked up to the network. As a consequence, the average unit price decreases with increasing amounts purchased. This situation violates the basic assumption of a competitive market in which individual consumers and producers are faced with exogenous prices. To illustrate the consequences of declining block rates on consumer demand, consider the following overly simplified situation (which follows the exposition in Taylor, Blattenberger and Rennhack (1982)): The individual consumer can choose between natural gas (G) and a composite commodity (X). For the latter, a fixed unit price (p_x) is paid. For natural gas, the consumer is faced by a fixed charge (p_0) and a price p_i per unit consumed in the i^{th} ($i=1,2,..$) block of the rate schedule, i.e. the price paid for volumes of gas between G_i and G_{i+1}. By assumption, $p_{i+1} < p_i$. For volumes less than G_1, the fixed charge is the only charge paid. The consumer's total budget is assumed to be fixed and is denoted by r. The budget constraint will then be of the form

$$p_0 + \sum_{i=1}^{j-1} p_i (G_{i+1} - G_i) + p_j (G - G_j) + p_x X = r \quad . \tag{5.1}$$

Figure 5.1 provides an illustration of this kind of budget constraint in the case of a two-block tariff structure.[1]

Figure 5.1: A budget constraint with decreasing block rate pricing.



From this figure, several distinctive features of the demand structure are revealed:

(i) Depending on the form of the preference structure, the equilibrium is either on the facet of the budget constraint with slope p_1 or at the segment corresponding to p_2. When changes in demand conditions (prices, income) occur, the optimal volume may "jump" from one segment to another. This means that the derived demand functions are *discontinuous* at some combinations of explanatory variables.

(ii) Because of the stepwise linearity of the budget constraint, demand functions cannot be derived in the traditional way from the first order conditions of the optimization problem.

(iii) From the figure it can also be easily demonstrated that the existence of *multiple equilibria* cannot be ruled out. This possibility is illustrated by the dotted indifference curve in figure 5.1, which is a tangent to two segments of the budget constraint. It is intuitively obvious that if the equilibrium is close to a situation with multiple equilibria, moderate changes in p_1 or p_2 can cause the equilibrium to change from one facet to another (cf. (i)).

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[1] Implicit in this illustration is an assumption that the fixed charge is paid even if no gas is consumed, i.e. p_0 should be interpreted as a hookup charge.

With the kind of budget constraint drawn in figure 1, it is clear that at least G_1 of gas will be consumed, since the fixed (hookup) charge in any case will have to be paid. This assumption can be modified if the individual's decision of being connected to the gas grid or not is also taken into account. This obviously introduces an element of *discrete choice* in the model: The individual has the choice of being hooked up to the gas grid - paying the charge p_0 - or to stay out of the gas market and thus avoiding these payments.[1] It should be noted that this kind of rate structure in the gas market constitutes a motive for applying a discrete choice approach to the modeling of gas demand even in a situation where dual fuel capability is installed by the households.

Let us then briefly review some solutions that have been suggested to solve the problems caused by decreasing block rates. In Blattenberger (1977) it is shown that when focus is on demand on an *aggregate level,* the discontinuity of demand functions for individual consumers tend to disappear in the aggregate demand relation. On the other hand, discontinuity problems caused by the existence of multi-facet budget constraints, may be particularly relevant in demand studies based on micro data.

It is shown by Gabor (1955-1956) that any multipart tariff can be replaced by an equivalent two-part tariff, the two parts being the marginal price prevailing for the actual volume consumed, and an "intramarginal premium". The latter is defined as the difference between what was actually paid for the total volume of gas purchased and what would have been paid if the marginal price was imposed on the whole amount. It can be shown that any change in this "intramarginal premium" (i.e. a change in either the fixed charge or any intramarginal price) gives rise to a pure income effect on the demand for natural gas. As opposed to this, a change in the marginal price yields both a substitution effect and an income effect (as in traditional demand analysis). Exploiting the distinction between the "intramarginal premium" and the marginal price suggests a procedure in which declining block rates are taken into account in the study of demand for natural gas. Based on information from the rate schedule, the "intramarginal premium" can be calculated, and used to construct adjusted figures for household's income. By using this adjusted income and data on marginal prices, a traditional approach to demand analysis is justified.

An alternative to this fixed charge/marginal price framework, is to employ ex post average prices in the calculations. As mentioned above, this causes a traditional simultanity problem, as volumes consumed and average prices are functionally related. The problem can only be solved by recognizing the relationship describing the rate schedule as a relation

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[1] Realistically, this requires a framework which includes more than one energy good, so that the consumer makes this decision because another energy carrier is evaluated as more beneficial. Thus, the models discussed in chapter 4 are the appropriate ones.

between the average price and the volume consumed. In the literature, two different procedures to assess this latter relationship are recommended. In Halvorsen (1975,1976), parameters in the average price equation are estimated by positing the parameters of this relation to be functions of supply side (cost) variables. The other methodology is to estimate the average price relation directly from detailed information of the rate schedule. In both cases, a two-stage estimation procedure is then applied to estimate the parameters in the actual demand equations.

5.2 Availability of natural gas.

Natural gas availability varies considerably between countries and also between different regions and sectors within countries. Over time, the general tendency is that the distribution network for gas in Western Europe has been extended to include as potential customers an increasing share of the total number of households and industrial users. Unless variations in natural gas availability is taken explicitly into account, estimated elasticities of demand could be seriously biased. In order to cope with this problem, Taylor, Blattenberger and Rennhack (1982) suggest dividing the total market into three parts:

i) customers to whom natural gas has been available for some time,
ii) customers to whom natural gas quite recently has become available, and finally
iii) customers to whom piped gas is not available.

Underlying the distinction between the first two categories, is a hypothesis that customers who for a long time have had the possibility of being hooked up to the gas grid, are expected to have adjusted to this situation. On the other hand, in areas to which the network quite recently has been extended, one should expect a *gradual conversion* to gas appliances. Taylor, Blattenberger and Rennhack emphasize that the real reason why it is important to treat the gas network saturation explicitly, is that the availability changes over time, combined with the fact that new hookups and conversions do not take place instantaneously.

In their formal model, the demand for natural gas is specified by a flow-adjustment relation of the kind discussed in chapter 3. As a starting point, they specify the demand relation in a *stable availability environment* as

$$\ln G_t^* = A_0 + A_1 \ln G_{t-1}^* + A_2 \ln p + A_3 r^* \tag{5.2}$$

where

G is the demand for natural gas,

p is a vector of fuel prices,

r_* is the income of potential gas customers, and

A_0, A_1, A_2, A_3 are coefficients.

The asterisks denote that the demand relation relates to a stable availability environment. Price variables are assumed to be the same throughout all regions. With respect to income, this is assumed to be homogeneous within each region. Thus, the income variable in relation (5.2) is simply calculated by multiplying total income in the region with a measure of gas availability, g. A general expression for the *change in the availability* is

$$\Delta g = \frac{g_t - g_{t-1}}{g_{t-1}} \tag{5.3}$$

In a situation with instantaneous conversion, the demand for natural gas would have developed over time according to

$$G_t = (1 + \Delta g_t) G_t^* \tag{5.4}$$

However, since capital costs are involved in this process, conversions can more realistically be assumed to occur at a slower rate than indicated by this equation. The following modification of (5.4) is therefore suggested:

$$G_t = (1 + \rho_t \Delta g_t) G_t^* \tag{5.5}$$

where $0 < \rho_t < 1$ by assumption. By relating the parameter ρ_t to income and a number of other exogenous variables and substituting this relation into (5.5), Taylor, Blattenberger and Rennhack (1982) arrive at the final equation to be estimated.

The framework just presented shows one possible way of incorporating gas network availability in a traditional econometric model. It is implicitly understood that the procedure is designed for applying *aggregate* data on gas consumption and other variables. As a consequence, the specification of gas network availability is rather ad hoc. In many respects, the incorporation of availability of gas is more straightforward - at least principally - in a *discrete choice model.* The reason is that such models are derived from the *micro level,* and suitable for applying data on factors influencing individual behaviour, i.e. also information on gas network availability. In the following, the main principles of how the availability of the gas grid can be introduced in a discrete choice framework are discussed.

Let us go back to the multi-response (MNL) model with 3 different fuel technologies available as discussed in the previous chapter. By convention, fuel 1 represents natural gas. Obviously, the probability of choosing a certain technology is dependent on whether natural gas is available or not. Define P_i as the probability of choosing fuel i *conditional on natural gas being available as one of the technologies.* Let us furthermore introduce binary variables y_i for each of the fuel choices, attaining the value one if technology i is chosen, zero otherwise (cf. the variable y in the binomial case). In a similar way, we introduce a binary variable, z, to represent the availability of natural gas. Within a MNL framework, the conditional probabilities, P_i, can then be expressed as follows:

$$P_i = Pr(y_i = 1 \mid z = 1) = \frac{e^{V_i}}{e^{V_1} + e^{V_2} + e^{V_3}} \quad , i = 1,2,3 \ . \tag{5.6}$$

Similarly, for households without access to the network for natural gas, the probabilities for choosing between the technologies 2 and 3 are given by

$$\bar{P}_i = Pr(y_i = 1 \mid z = 0) = \frac{e^{V_i}}{e^{V_2} + e^{V_3}} \quad , i = 2,3 \ . \tag{5.7}$$

As briefly discussed in the previous chapter, decisive for how such a model should be estimated is the kind of data which is available. In the present model, the type of information about gas network availability which is at hand is also of great importance. The most ideal situation is obtained when strictly individual data exist, *also with respect to the availability of natural gas for the different households.* By this we mean that for *each individual in the sample* there is information about the possibility of being connected to the gas grid. If one is only interested in the demand for natural gas (and not the demand for the other two energy carriers), an adjacent procedure is to restrict the observations to be included in the calculations to households living in "gas areas". The estimation of the model could then proceed in the way described in chapter 4. However, a more efficient procedure consists of utilizing the choices undertaken by households outside "gas areas"" as well, cf. that exactly the same parameters are included in the probabilities expressed by (5.6) and (5.7) respectively. Introducing the subscript j to indicate an individual consumer, the following extended ML-procedure can then be applied:

$$L = \prod_{j=1}^{N} \prod_{i=1}^{3} P_{ij}^{y_{ij} z_j} \, \overline{P}_{ij}^{y_{ij} (1-z_j)} \tag{5.8}$$

where N is the total number of observations.

Unfortunately, information on gas network availability is rarely of this kind. Instead, the typical information on gas network connection is *the number of households* in the sample for which natural gas is available as energy carrier. In the following, we will assume that this is the situation. Let the number of households living in "gas area" be denoted by N_g. Among these households, the number having chosen technology i is denoted by n_g^i. If these data on these variables had been available, it would be reasonable to suggest an estimation procedure analogues to the *MIN*-X^2- method described in chapter 4, based on estimates of observed frequencies of the following type

$$\hat{P}_i = \frac{n_g^i}{N_g} \quad , \tag{5.9}$$

and carry out a GLS estimation procedure on the relations

$$\hat{P}_i = P_i + \epsilon_i \quad , \tag{5.10}$$

where the ϵ_i's are stochastic residuals with zero means and known variance-covariance matrix [1]. The problem with applying this procedure is, however, that by assumption *only* n_g^1 *is "observable" in the sample.* For technologies 2 and 3, only figures for the number of households using these fuels *in the total population* are available. Let n^i denote the total number of households having chosen fuel i, where obviously $n^1 = n_g^1$. Since $P_i \neq \overline{P}_i$ for i= 2,3 (cf. the difference between (5.6) and (5.7)), there is in general no information in the sample enabling the estimation of $\hat{P}_2$ and $\hat{P}_3$. At first, it looks as though the estimation of all the parameters of the model must be based solely on the equation (5.10) for fuel 1, natural gas. Clearly, this places considerable restrictions on the calculations, and implies that far less efficient estimators are obtained compared to a situation where complete information of fuel choices *within* the "gas area" exists. However, it should be noted that within a MNL model, an additional set of relations can be utilized in the estimation procedure. As revealed in chapter 4, the IIA-property embedded in this probability structure, implies that the ratio of two choice probabilities are independent of which other alternatives are available for the

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[1] The individual observations which make up n_g^i are identically multinomially distributed with probability vector (P_1,P_2,P_3), and n_g^i is accordingly distributed similarly.

individual consumer. In the present case, this means that the ratio between n_g^2 and n_g^3 should be expected to equal the ratio between n^2 and n^3. Accordingly, *the latter could be applied as an estimate of* $(\frac{\hat{P}_2}{P_3})$, and the following relation could be added to the equations to be estimated:

$$\log (\frac{\hat{P}_2}{P_3}) = V_2 - V_3 + \epsilon \tag{5.11}$$

By including equation (5.11) in the set of equations to be estimated, the estimates of the coefficients will be improved. It should, however, be stressed that while the distribution of $\hat{P}_1$ is known to be multinomial, the distribution of the ratio $\log (\frac{\hat{P}_2}{P_3})$ is not so easy to derive. Thus, a GLS procedure may be more complicated to apply in this case than when all $\hat{P}$'s can be estimated separately.

In a discrete choice model *not* involving the IIA-property, summary information on gas network availability can not be treated consistently within the model in the way indicated above without dramatic loss of information. In such cases, an alternative way of taking into account data on natural gas availability, is to include this factor as *an explanatory variable* in the indirect utility functions of the consumers. This solution is chosen in the dynamic model for European residential gas demand developed in Dagsvik, Lorentsen, Olsen and Strom (1986).

Essential to any of the procedures briefly described above is access to a meaningful measure of natural gas availability in various regions. The ideal measure for availability is the number of customers in a region which *have access to piped natural gas* (actual plus potential customers) relative to the total number of customers. This kind of information may be hard or impossible to get at. In Taylor, Blattenberger and Rennhack (1982), the proportion of a region's population living in communities served by natural gas is used as a proxy for the availability of natural gas in the residential sector.

In most European countries, even this kind of information may be hard to collect. In some countries, the only indicator of the extension of the natural gas network is data on the number of households actually connected to the network. By using such a measure of availability, the (endogenous) decision taken by a household whether to hook up or not, is neglected.

The question of availability of natural gas to final consumers also involve another aspect, quite different from the one related to the potential of physical hookup to the network. Due to limitations of supply or capacity problems in transportation, deliveries to groups of consumers have in periods been curtailed by local distribution companies. If informations on such restrictions in supply exist, a possible solution to the problem can be obtained by correcting the actual figures of gas consumption, and base the estimation of the true demand function on these corrected figures rather than the observed (rationed) market point (see Murphy et. al (1981)). A precondition for this procedure to hold, is that the local utilities do not take advantage of the shortage by increasing the price of gas, but actually put into effect some sort of physical rationing. Since we in general are looking at governmentally regulated utilities, this assumption is probably acceptable.

In a work by M.A Fuss (1977b), consequences for factor demand of constraints on supply of natural gas and fuel oil are explored within the framework of a KLEM - model as outlined in section 2.1. The basic idea is that curtailment on supply can be incorporated in a cost minimizing framework of producer behaviour with the implication that the producer is faced with shadow prices rather than actual prices. Here we will avoid a lengthy formal exposition, and only report the general set-up of the procedure applied by Fuss.

Curtailment of supply of natural gas is introduced in the model as a constraint on the optimization problem. Formally, the introduction of curtailments both in Fuss (1977b) and Wood and Spierer (1984) is identical to the restricted cost function approach outlined in chapter 2. However, the interpretation of the model differs between the two cases. While in the model in chapter 2 capital was assumed to be fixed only in the short run while "flexible" in the long run, in the present context the restricted factor supply (of gas) is not related to any specific time perspective. In other words: in this model, the quasi-fixed factor relates to "states" with or without curtailments in natural gas supply, not to the difference between "short run" and "long run".

With a slightly more general notation than in chapter 2 first order conditions for cost minimization problem have the following form:

$$\nabla C_{P_1} = x_1(P_1, P^*, Q) \tag{5.12}$$

$$\nabla C_{P^*} = x^* \tag{5.13}$$

where

∇C is the gradient of the cost function,

P_1 is a vector of prices on unconstrained inputs,

P^* is a vector of shadow prices on restricted inputs,

x_1 is a vector of unconstrained inputs, and

x^* is a vector of available volumes of curtailed inputs.

Shadow prices on restrained inputs are obtained by inverting the set of demand functions (5.13), i.e.

$$P^* = h(x^*, P_1, Q) \tag{5.14}$$

Fuss (1977b) actually applies the model to analyse a situation where supplies are assumed to be unconstrained in the estimation period, but constrained in the simulation period. However, the method is equally applicable when the historical data are influenced by curtailments in supply. In such cases, the proper procedure is to estimate the restricted cost function directly, treating curtailed inputs as exogenous factors. As both the restricted and unrestricted cost function are equivalent (dual) representations of the technology and producer behaviour, the estimated (restricted) cost structure can be used when projecting future demand irrespectively of whether future gas supply will be curtailed or not.

6 CONCLUSIONS

The purpose of this paper is to discuss approaches and models suitable as tools for projecting natural gas demand. As has been reviewed in the previous chapters, a number of different econometric models and methodologies for energy demand analysis have been suggested in the literature. Which model to choose in a particular case, as e.g. as a representation of European natural gas demand, is not clearcut, and will depend on several issues. In general, there may be a trade-off between realism and complexity of the model framework on the one hand, and data availability and operational convenience on the other. However, with modern computational equipment, the latter argument need not be very strong. In the outset, it should also be stressed that it is not at all obvious that the same formal structure should be specified for all sectors. On the contrary, for each submodel, one should aim at including features that are specific for the demand structure of the industry or consumer group in question.

Regarding the structure of the different models, it is of central importance that they take into account possibilities of *substitution* between different energy carriers. Experiences during the last 15 years have clearly demonstrated how changes in relative fuel prices may motivate consumers to change the composition of their fuel choices and energy consumption. Another important feature of energy demand, stressed several times in previous chapters, is the interrelationships between energy use and the acquisition and utilization of *capital equipment*. In turn, this calls for a *dynamic model* including mechanisms describing how consumers adjust their stocks of capital goods, and thus tracing explicitly changes in energy use over time. Many of the dynamic models most frequently applied in econometric studies, are of the single-equation type where the adjustment process is specified very ad hoc, and relationships to other inputs or goods typically are not modeled. A much more satisfactory dynamic framework is the model suggested by Berndt, Fuss and Waverman (1980). This model is a multi-input framework of producer behaviour, where the optimization process explicitly takes into account adjustment costs of quasi-fixed factors. In the dynamic cost-of-adjustment model, the principle treatment of natural gas (energy) is similar to the representation of other "variable" inputs. The model may therefore be a particularly relevant as a description of technology and behaviour in *manufacturing industries,* where a significant part of the total energy consumption is related to industrial processes. In Gjeldsvik and Roland (1986), estimation of a dynamic cost-of-adjustment model for the manufacturing sector in Western Europe is reported.

For the residential sector, where natural gas (and energy in general) is used primarily for heating purposes, we have argued on several occasions that in the formal model, efforts should be made in relating energy and gas consumption explicitly to the *stock of dwellings* (the same argument may be used for the commercial sector). An attractive analytical framework which takes care of this interrelationship, is *a discrete choice model* discussed in some detail in chapter 4 above. Applied as a specification of residential energy demand, such a model takes into account the fact that the number of heating technologies available for a consumer is limited. A dynamic discrete-continuous choice model for energy use for space heating in the residential sector in Western Europe is suggested by Dagsvik, Lorentsen, Olsen and Strom (1986).

Finally, we would like to underline the importance of the specific features of the natural gas market pointed out in chapter 5, namely the presence of *declining block rate schedules* and the *access to the distribution network for natural gas.* Partly because of conceptual difficulties and problems of having available relevant data of these factors, it may not be an easy task to incorporate them explicitly in a formal framework. To improve the data situation at this point and to overcome the difficulties of representing these factors analytically may therefore serve as a recommendation for future research.

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Statistisk
Sentralbyrå

ISBN 82-537-2665-1
ISSN 0801-3845