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Documents

Is there Something Rotten in this State of Benchmark?

A Note on the Ability of Numerical Models to Capture Welfare Effects due to Existing Tax Wedges

1. Introduction

The purpose of this note is to examine how applied models are able to capture welfare effects of resource reallocations when tax wedges prevail. The motivation for such an examination is that most applied models adopt volume and price indexes from the National Accounts (NA) or a Social Accounting Matrix (SAM) where all prices, such as market prices and producer prices, are normalised to unity in the base year to which the model is calibrated. One might then suspect that such a normalisation implies that the model will ignore the tax wedges that prevail in the base year, thereby destroying its ability to address welfare effects of reallocations of resources. This note shows that this suspicion is not well founded.

In order to explain the reason most clearly, I employ a model framework neglecting everything which is not necessary in order to illustrate the point. In section 2, I start by identifying how taxes create well known distortive wedges between marginal costs and marginal utilities. Section 3 shows how an applied model calibrated to NA/SAM define those concepts introduced in the theoretical framework. The basic problem is how to decompose value concepts into a price component and a volume component. In the theoretical literature, this problem is non-existent because quantities are supposed to be measured in physical units. However, at any operational level of aggregation, most variables are aggregates and volumes and prices have to be measured by indexes.

I must admit that there would have been much more important to issue this note, had the conclusion been the opposite of what it is. Is there any reason to rationalise that there is no reason for model users to change their ordinary way of forming data sets and calibration procedures as far as the principle ability to capture tax wedges is concerned ? Isn't it a little too much to suspect most numerical model analyses of welfare gains from reallocation to be based on irrelevant operationalisation and measurement of fundamental variables? My answer to such objections to this note, is simply that I have personally found the issues which it addresses hard to understand, despite several years of experience both with constructing, applying and interpreting results from numerical CGE models. During my attempts to answer the question posed in the title, I discussed the subject with several other persons with the same type of modelling experience. As far as I was able to comprehend, no one had any precise convincing argument for answering yes or no to my question. Indeed, I got the impression that, on average, the consciousness about how theoretical concepts are made operational and measured in official statistics could and should be brought to a higher level. This note may represent a small pedagogical contribution to this end.

2. The basic framework

I consider the simplest possible economy which allows the illustration of the point. Production takes place in two industries each producing one single consumption good, C_1 and C_2. Production requires homogenous labour, L, as the only factor of production. The production functions, $f_i(L_i)$, exhibit the standard regularity conditions. Labour is perfectly mobile between the two industries. Labour is inelastically supplied, and the labour and product market clears. Formally, the production possibilities are constrained by the following equations:

(1) $L = L_1 + L_2$

(2) $C_i = f_i(L_i)$

Welfare is given by a standard utility function

(3) $U = U(C_1, C_2)$

and is determined by (1), (2) and (3) once the employment in one industry, say L_1, is known. Welfare effects due to reallocation of labour are calculated by changing L_1. The welfare effect of a marginal reallocation becomes

(4) $$dU = \left(1 - \frac{u_2 f_2'}{u_1 f_1'}\right) u_1 f_1' dL_1 \equiv \alpha u_1 f_1' dL_1 .$$

The factor α includes the marginal rates of substitution and transformation, thus, capturing the effects of reallocation on welfare. It will play an important role in the subsequent discussion. When consumers and producers are price takers, α can be measured by the relevant prices. Optimal allocation of consumption expenditure implies that

(5) $$\frac{u_2}{u_1} = \frac{P_2^C}{P_1^C}$$

where P_i^C is the consumer price of good i.

The wage rates may differ between industries due to industry specific effective taxation of labour. Let W be the net-of-tax wage rate received by the workers. It is related to the wage rate paid by producers in industry i, W_i, by

$$(6) \quad W_i = \left(1 + t_i^W\right) W \ , i = 1,2$$

Profit maximising producer behaviour implies that

$$(7) \frac{f_2'}{f_1'} = \frac{W_2}{W_1} \frac{P_1^P}{P_2^P}$$

where P_i^P is the producer price of good i. Finally, indirect taxation creates wedges between the producer prices and the corresponding consumer prices:

$$(8) \quad P_i^C = \left(1 + t_i^C\right) P_i^P \ , i = 1,2$$

Inserting (5) - (8) into the expression for α in (4) yields

$$(9) \quad \alpha = 1 - \frac{\tau_2^C}{\tau_1^C} \frac{\tau_2^W}{\tau_1^W}$$

where $\tau_i^j \equiv 1 + t_i^j$, i =1,2 and j = W, C has been introduced for the sake of a more compact notation. $\tau_i^C \tau_i^W$ represents the total taxation imposed on the production and consumption of good i. The producer prices are seen to be irrelevant for the magnitude of α, since these prices represent an intermediate transaction in the transformation of labour input to welfare. The first best optimum is obtained when α=0, corresponding to uniform or offsetting taxes that do not affect the relative prices.

For the purpose of this paper, it is not necessary to extend the basic model any further. In particular, it is not necessary to determine the change in L_1 in (4). Whatever the reason for such a change, the important thing is that the presence of non-uniform taxes implies a change in welfare due to the reallocation of labour. The reallocation of labour from industry *2* into *1*, will increase welfare if

$\tau_1^C \tau_1^W > \tau_2^C \tau_2^W$, i.e. if the good 1 is more heavily taxed than good 2 when the joint effect of taxes on consumption and production is taken into account.

The issue in this paper is to examine to what extent empirical (applied) models are able to capture α adequately. The reason why this may be a problem is that most of the variables in such models are aggregates. Consequently, volumes measured in physical units and prices per physical unit do not exist. Instead, volumes and prices are measured by indexes which may be defined in several ways. The conventions followed in the NA and SAM represent one alternative which is widely accepted as the conceptual and empirical foundation for calibration of applied models.

3. Will models calibrated to the NA or SAM capture the welfare effect from reallocations?

In the applied model, the two goods are aggregates of several commodities. Let N_i be the set of commodities aggregated into good i, i = 1,2. In the NA and SAM, the consumption volume of good i, $C_i^{C(0)}$, is defined as consumption of commodities belonging to the aggregate good i evaluated in fixed base year consumer prices:

$$(10)\quad C_i^{C(0)} \equiv \sum_{j \in N_i} P_j^C(0) C_j \ , i = 1,2$$

where C_j is the volume of commodity j measured in physical units, $P_j^C(0)$ is the consumer price per physical unit of commodity j in the base year 0 to which the model is calibrated. In the following, the base year represents the initial situation prior to any policy reform that generates reallocations. Neither C_j nor $P_j^C(0)$ are model variables, but are used in order to construct the aggregate model concepts. The utility function is now defined over the aggregate goods in the applied model:

$$(11)\quad U = U\left(C_1^{C(0)}, C_2^{C(0)}\right).$$

and the marginal rate of substitution is equal to the ratio between the corresponding consumer price indexes.

Turning to the production side, the relevant output price concept is the producer price. Let $C_i^{P(0)}$ denote the production of the aggregate good i measured in producer prices per physical unit in the base year. This value equals the basic value of gross production in the NA and SAM. $C_i^{P(0)}$ is related to the volumes and producer prices of the detailed commodities in the aggregate in the same way as described in (10):

$$(12) \quad C_i^{P(0)} \equiv \sum_{j \in N_i} P_j^P(0) C_j \ , i = 1,2$$

where $P_j^P(0)$ is the base year producer price per physical unit of commodity j. The production functions in the applied model take the form

$$(13) \quad C_i^{P(0)} = f_i(L_i) \ , i = 1,2$$

In the theoretical framework laid out in the previous section, (2) implicitly assumes product market equilibrium. In the empirical model the counterpart to this equilibrium condition must equate consumption and production when both variables are measured in the same prices. Most models measures commodity flows in basic values, and in this case we have:

$$(14) \quad \lambda_i^C C_i^{C(0)} = C_i^{P(0)} \ , i = 1,2$$

where the parameter λ_i^C is defined as

$$(15) \quad \lambda_i^C \equiv 1\Big/\left(1 + t_i^C(0)\right) \ , i = 1,2$$

where $t_i^C(0)$ is the aggregate tax rate levied on consumption of good i in the base year. This rate is found by dividing the base year value of consumption in consumer prices by the corresponding producer value. Thus, the l.h.s. of (14) transforms the market value of consumption into the corresponding basic value by netting out indirect taxes from the market prices.

Consumer prices and producer prices per physical unit is related as described in (8). However, the NA and SAM normalises both consumer and producer price indexes to unity in the base year. Thus, in the model, the value and volume of a given composite good are identical in the base year. Accordingly, the price indexes in the applied model only capture how prices change relative to the base year. This normalisation implies that the model equations linking producer price indexes and consumer price indexes must take the form

$$(16)\quad P_i^C = \lambda_i^C\left(1 + t_i^C(0)T_i^C\right)P_i^P$$

where T_i^C is an exogenous variable which is used to simulate changes in the consumption taxes levied on good i. $T_i^C = 1$ in the base year. Multiplying with the parameter ensures that $P_i^C(0) = P_i^P(0) = 1$.

I will assume that information about the input and supply of labour is available in physical units, e.g. man hours. Wage rates per man hour are also assumed to be available, and industry specific taxes on labour are modelled as in (7). Thus, for homogenous labour the volume and price concepts are obvious and indexes are not needed for the representation of this variable in the applied model. This will help to emphasise potential problems regarding the definition of volume and price indexes for heterogeneous goods when the model is intended to capture welfare gains from reallocations.

The applied model will include behavioural equations being analogue to (5) and (6) above where consumer and producer prices are represented by the indexes introduced in (14). As pointed out in the previous section, the producer prices are irrelevant for measuring the distortive effect on relative prices caused by taxes. What matters is how the model captures the product of the marginal rate of substitution and the marginal rate of transformation which determines the factor α introduced above. However, we must also take into account that the generation of utility from a marginal increase in L_i goes through three stages in the empirical model. Firstly, labour produces more of good i measured in basic values. Secondly, this increment is magnified to a larger increase in the corresponding market value of good i. Thirdly, utility increases when the market value of good i is consumed. The second step in this process was not necessary to specify in the theoretical framework. The model will now calculate the following value of α:

$$(17)\quad \alpha = 1 - \frac{P_2^C \dfrac{\partial C_2^{C(0)}}{\partial C_2^{P(0)}} \dfrac{\left(1+t_2^W\right)W}{P_2^P}}{P_1^C \dfrac{\partial C_1^{C(0)}}{\partial C_1^{P(0)}} \dfrac{\left(1+t_1^W\right)W}{P_1^P}} = 1 - \frac{\left(1+t_2^C(0)T_2^C\right)}{\left(1+t_1^C(0)T_1^C\right)} \frac{\tau_2^W}{\tau_1^W}.$$

The model simulation of reallocations induced by a reform, is supposed to have the base year as reference point. The discussion in section 2 showed that the base year value of α is especially important when studying the welfare effects. According to the definitions above, this value reduces to

$$(18) \quad \alpha(0) = 1 - \frac{\tau_2^C}{\tau_1^C}\frac{\tau_2^W}{\tau_1^W}$$

which is identical to (9). It can therefore be concluded that the calibrated model indeed captures the distortions of relative prices due to taxes levied on transactions measured in physical units as well as on those measured in fixed prices.

4. Conclusions

This paper has used a simplified framework to demonstrate that the price and volume indexes available in the NA or SAM can be used directly for calibration when a theoretical model structure is to be quantified. The important provision is that the product market equilibrium conditions measures all supplies and demands in the same price set, i.e. in equivalent units. The model will then capture tax wedges although all prices are normalised to unity in the base year of the model simulations.

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