2009/3
Documents

Statistics Norway

Statistisk sentralbyrå

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The New Keynesian Phillips Curve revisited

Statistics Norway/Research Department

Preface

The present paper is a considerably revised and extended version of Discussion Papers No. 500, Statistics Norway. The analyses now comprise VAR based tests of both the *exact* and the *inexact* form of the New Keynesian Phillips curve using European and US data.

Oslo, 26 January 2009

The New Keynesian Phillips Curve revisited

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January 26, 2009

Abstract

Recently, several authors have questioned the evidence claimed by Galí and Gertler (1999) and Galí, Gertler and López-Salido (2001) that a hybrid version of the New Keynesian Phillips Curve approximates European and US inflation dynamics quite well. We re-examine the evidence using the vector autoregressive framework and likelihood based methods, paying particular attention to the stationary and nonstationary, and possibly cointegrated, nature of variables involved. Our results show that the *exact* as well as the *inexact* form of the hybrid NKPC are clearly at odds with the European data. On the other hand, Galí and Gertler's (1999) finding that the *inexact* hybrid NKPC is a "good first approximation" to the US inflation dynamics seems less controversial. However, the assumption of the model that the stochastic term forms a sequence of innovations may be problematic as we find indication of autocorrelation in the estimated residuals. The *exact* form of the hydrid NKPC is firmly rejected by the US data.

Keywords: European and US inflation, the New Keynesian Phillips Curve, *exact* and *inexact* rational expectations, vector autoregressive models, likelihood based methods.

JEL classification: C23, C52, E12, E31

Acknowledgements: We are grateful to Jordi Galí for supplying us with the data set used in Galí and Gertler (1999) and Galí, Gertler and López-Salido (2001). We thank Michael Clements, David Hendry, Eilev S. Jansen, Søren Johansen, Ragnar Nymoen, Terje Skjerpen, the co-editor Michel Juillard and two anonymous referees for helpful comments and suggestions on an earlier draft. The usual disclaimer applies.

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1 Introduction

The New Keynesian Phillips Curve, henceforth NKPC, has become popular as a theory for understanding inflation dynamics and recent years have seen a growing body of research attempting to evaluate its empirical performance. However, the numerous attempts typically differ with respect to data used, sample period studied and econometric methods applied such that no consensus about the supportive evidence of the model is established in the literature, see e.g. Henry and Pagan (2004) and the references cited therein, Sbordone (2005), Rudd and Whelan (2007), Bjørnstad and Nymoen (2008) and Juillard *et al.* (2008). The influential studies by Galí and Gertler (1999), henceforth GG, and Galí, Gertler and López-Salido (2001), henceforth GGL-S, find strong evidence in favour of the model using both European and US post-war data within a conventional GMM framework. Specifically, these studies find that European and US inflation dynamics are consistent with a hybrid version of the NKPC that relates inflation to expected future inflation, lagged inflation and real marginal costs. Forward-looking behaviour plays a dominant role in explaining inflation.

Several authors have re-examined this evidence using the same data set as well as the GMM methodology and raised quite a few empirical issues that question the robustness of the results in GG and GGL-S. For instance, Mavroeidis (2006) demonstrates that the parameters in GG are weakly identified and that US inflation dynamics are consistent with both forward-looking and backward-looking behaviour, whereas real marginal costs appear to be an irrelevant determinant of inflation.[1] Similarly, Rudd and Whelan (2005) argue that the upward bias of the forward-looking estimates may be large when estimating the structural form of the NKPC, as GG do, rather than the corresponding closed form solution of the model. Bårdsen *et al.* (2004) also show by the encompassing principle that the estimates in GGL-S most likely are biased in favour of a significant role for expected future inflation because that variable is found negligible in respecified models where variables from the instrument set directly and significantly cause inflation. Galí *et al.* (2005) answer some of the critics and maintain their conclusion about the importance of the forward-looking behaviour in explaining inflation dynamics. Interestingly, Rudd and Whelan (2007) cannot reject the hypothesis that inflation and real marginal costs are completely unrelated when using GG's original data, but with *revised* labour share data as proxy for real marginal costs. In itself, this finding suggests that the empirical evidence in GG is not robust to revisions to data.

Recently, Fanelli (2008) and Kurmann (2007) have re-examined the empirical evidence in GG and GGL-S further by means of likelihood based methods. Fanelli (2008) proposes a two step procedure that consists of specifying agents' expectations with vector autoregressive (VAR) models, possibly cointegrated, and deriving the

[1]The issue of weak identification of forward-looking models estimated by GMM is thoroughly discussed in Mavroeidis (2004, 2005) and Nason and Smith (2008) among others.

cross-equation restrictions that the NKPC imposes on the VAR. Unlike GGL-S, Fanelli (2008) concludes that the hybrid NKPC based on the same data set is far from being a good candidate in explaining European inflation dynamics. By using a reversed engineering technique, Kurmann (2007) shows how the cross-equation restrictions can be expressed as constraints on the VAR coefficients of the forcing variable of the NKPC. In line with GG's findings, Kurmann (2007) concludes that the hybrid NKPC based on the same US data cannot be rejected by a conventional likelihood ratio test.[2]

In this paper, we revisit the NKPC using the same data set as in GG and GGL-S, but within the VAR framework and likelihood based methods, paying particular attention to the cases where variables involved are either stationary or nonstationary, and possibly cointegrated. By using a different econometric methodology, we shed further light on the empirical evidence in GG and GGL-S. Also, the simultaneous treatment of the two data sets by the same methodology enables us to pinpoint empirical differences which are not so evident from the previous studies by Fanelli (2008) and Kurmann (2007). We extend the analyses in the two latter studies by focusing on both baseline (only forward-looking behaviour) and hybrid models as well as on *exact* in the sense of Hansen and Sargent (1991) and *inexact* formulations, with the latter formulation allowing for a stochastic term in the NKPC. The *exact* and *inexact* formulation of the NKPC are not nested, however. Also, whereas the rational expectations (RE) restrictions entailed by the former have a simple form, the restrictions entailed by the latter are much more complicated due to higher order polynominals in the unconstrained VAR coefficients. As a consequence. the numerical treatment of the two sets of restrictions is quite different and thus makes a comparison of the likelihood estimates of the two formulations interesting. We follow the reversed engineering approach by Kurmann (2007), albeit modified to suit our context, in the case of the *inexact* NKPC. In contrast to Fanelli (2008) and Kurmann (2007), we also test formally the assumption of the *inexact* NKPC that the stochastic term is a sequence of innovations with no autocorrelation properties present. Finally, and unlike the previous studies, we express the likelihood as a function of all parameters involved, which makes an explicit inspection of the profile likelihood possible. As such, we are able to highlight any problems with weak identification that may come out of the estimation.

Our empirical findings suggest that a reduced rank VAR is a well specified model in terms of well-behaved residuals in the case of European data. We confirm Fanelli's (2008) finding that the *inexact* hybrid NKPC is clearly rejected by a likelihood ratio test. Additionally, we find that the likelihood surface is characterised by

[2] Juselius (2008) also uses VAR models and likelihood based methods to evaluate the NKPC, albeit only the *exact* version, with European and US data. However, Juselius (2008) considers an extended information set that permits testing of the forward-looking IS curve and the NKPC jointly and investigates a different sample period than that of GG and GGL-S. Nevertheless, Juselius (2008) finds that the inflation dynamics of the NKPC is not at all evident in the data.

a rather sharp ridge, confirming the criticism of Mavroeidis (2005) among others on the subject of poor identification of the parameters of the model. When fitting the *exact* hybrid NKPC as well as the *exact* and *inexact* version of the baseline NKPC, we maintain the conclusion that the NKPC is severely at odds with the European data. When using the US data, we find that a full rank VAR produces a model with no serious misspecification. We confirm Kurmann's (2007) finding that the *inexact* hybrid NKPC is not rejected by a likelihood ratio test. Also, the point estimates of the structural parameters that come out of the likelihood estimation are by and large in line with the estimates in GG, supporting the claim that the NKPC is a "good first approximation" to the US inflation dynamics. However, the assumption of the model that the stochastic term forms a sequence of innovations may be problematic as we find indication of autocorrelation in the estimated residuals. We also obtain mixed supportive results when fitting the *inexact* form of the baseline NKPC, while the *exact* form of the baseline as well as the hybrid NKPC is firmly rejected by the US data.

The rest of the paper is organised as follows: Section 2 briefly outlines the NKPC models estimated by GG and GGL-S. Section 3 presents the VAR approach and the likelihood based methods used in the present paper, while Sections 4 and 5 report and contrast empirical results for the European and US data with those of GGL-S and GG, respectively. Section 6 concludes. Details about the methodology used and contour plots of the likelihood estimation are provided in Appendix 1 and 2, respectively.

2 The NKPC model

As explained by Roberts (1995), there are several routes from a theoretical set up of firm's pricing behaviour that lead to the NKPC model, including the linear quadratic adjustment cost model of Rotemberg (1982) and the models of staggered contracts developed by Taylor (1979, 1980) and Calvo (1983). GG and GGL-S estimate two distinct versions of the NKPC model based on Calvo's model of staggered nominal pricing in an imperfectly competitive environment with firms producing differentiated products. The first version, referred to as the baseline model, assumes only forward-looking price setters, whereas the second version, referred to as the hybrid model, combines both forward-looking and backward-looking behaviour. Following GG and GGL-S, the baseline model is a linear RE model and may be formulated as (lower case letters denote logs of variables involved):

(1) $$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t + u_t,$$

where π_t denotes inflation in period t, mc_t represents real marginal costs in period t, $E_t\pi_{t+1}$ denotes the conditional expectations given the information at time t of inflation in period $t+1$ and u_t is a stochastic error term, which we discuss in Section

3. The slope parameter $\lambda = \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta[1+\alpha(\varepsilon-1)]}$ depends on the degree of price stickiness (θ), the subjective discount rate (β), the curvature of the underlying production function (α) and the elasticity of demand (ε). GG and GGL-S derive the hybrid version of (1) by allowing a fraction (ω) of the firms to use a backward-looking rule of thumb in their price decisions based on lagged inflation as a predictor. Again, following GG and GGL-S, the hybrid version of (1) reads as

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \tilde{\lambda} mc_t + u_t, \tag{2}$$

where

$$\begin{aligned} \gamma_f &= \beta\theta\phi^{-1}, \\ \gamma_b &= \omega\phi^{-1} \text{and} \\ \tilde{\lambda} &= \frac{(1-\omega)(1-\theta)(1-\beta\theta)(1-\alpha)}{\phi[1+\alpha(\varepsilon-1)]}, \end{aligned}$$

with $\phi = \theta + \omega[1-\theta(1-\beta)]$. We notice that the hybrid model nests the baseline model as a special case when there are no backward-looking firms present (i.e., $\omega = 0$). Accordingly, if the baseline model is true, then $\gamma_f = \beta$, $\gamma_b = 0$ and $\tilde{\lambda} = \lambda$. In general, the parameter spaces $0 \leq \beta \leq 1$, $0 \leq \lambda$ and $0 \leq \gamma_f, \gamma_b \leq 1$, $0 \leq \tilde{\lambda}$ are required to give an admissible economic interpretation of an estimated baseline and hybrid NKPC, respectively.

Using quarterly data of the growth in the GDP deflator as a measure of inflation and the labour income share as a proxy for real marginal costs over the sample period $1960Q1 - 1997Q4$ and $1970Q1 - 1998Q2$ in the US and European case, respectively, GG and GGL-S estimate both (1) and (2) by means of GMM.[3] Their empirical findings may be summarised as follows: (i) forward-looking behaviour is dominant as indicated by a wide range of estimates of γ_f broadly speaking being twice as large as the estimates of γ_b. (ii) because the coefficient γ_b is found to be statistically different from zero, the baseline model is rejected by the data. (iii) the labour income share drives inflation as suggested by a positive and significant estimate of $\tilde{\lambda}$. In contrast, alternative forcing variables such as output gap measures do not perform well. Consequently, both GG and GGL-S conclude that the hybrid version of the NKPC can be used as a "good first approximation" to both European and US inflation dynamics.

3 Econometric issues

The basic idea behind the procedures in this paper is to start out with a well specified VAR and test, using a likelihood ratio test, the implications of the NKPC on the

[3] For comparison, GGL-S also present results for US over the shorter sample period $1970Q1 - 1998Q2$.

coefficients of the VAR. To construct a likelihood test, we need to work out the maximum likelihood estimator of the coefficients, both with and without the RE restrictions imposed. We thus start with a well specified two-dimensional VAR of order k having the form

$$X_t = A_1 X_{t-1} + \cdots + A_k X_{t-k} + \Phi D_t + \epsilon_t, t = k+1, \ldots, T, \tag{3}$$

where $X_t = (\pi_t, mc_t)'$, D_t represents constants and $\epsilon_{k+1}, \ldots, \epsilon_T$ are independent Gaussian variables with expectation zero and (unrestricted) covariance matrix Ω. The initial observations of $X_1, \ldots, X_k$ are kept fixed.

The way the likelihood ratio test is constructed in our context depends on the time series involved being stationary, i.e., $I(0)$, or nonstationary, i.e., $I(1)$. The two situations correspond to whether the impact matrix $-\Pi = I - A_1 - \cdots - A_k$ has full or reduced rank. In the following we shall focus on the situation where the rank is full, corresponding to X_t being stationary, but we indicate how the testing procedure can be modified to suit the reduced rank case.

Another relevant econometric issue when deriving the RE restrictions on the VAR concerns whether the NKPC is specified in its *exact* ($u_t = 0$) or *inexact* form ($u_t \neq 0$). Absence of an unobserved disturbance term is a restrictive and nontrivial assumption and there are numerous justifications for why such a term should be included in the model, see e.g. Rotemberg and Woodford (1999) and Sbordone (2005). Kurmann (2007) and Fanelli (2008) are empirical examples where the *inexact* NKPC is dealt with. For the sake of comparison with these and other related studies, we study the *exact* as well as the *inexact* NKPC in this paper. As we shall demonstrate below, the *exact* version is algebraically less involved and produces much simpler RE restrictions on the VAR than what follows from the *inexact* version. Hence, the numerical treatment of the *exact* model is also much simpler than the *inexact* model. We refer to Appendix 1 for some details about the numerical optimising procedures used here.

3.1 Exact rational expectations

It is essential how the conditional expectations in (1) and (2) are represented. We proceed with the well established practice of spelling out the implications of the NKPC on model (3), see e.g. Baillie (1989) and Hansen and Sargent (1991). Using vector notation the *exact* baseline model may be written in compact form as

$$c_1' E_t(X_{t+1}) + c_0' X_t + c_{-1}' X_{t-1} = 0, \tag{4}$$

where $c_1 = (\beta, 0)'$, $c_0 = (-1, \lambda)'$ and $c_{-1} = (0, 0)'$. $E_t(X_{t+1})$ denotes the conditional expectations of X_{t+1} given $X_{k+1}, ..., X_t$ in (3). In the *exact* hybrid model $c_1 = (\gamma_f, 0)'$, $c_0 = (-1, \tilde{\lambda})'$ and $c_{-1} = (\gamma_b, 0)'$. We notice that (4) contains restrictions involving the conditional expected value of the observations *one-step* ahead and the present and lagged observed values. Having specified the information set, the conditional expectation $E_t(X_{t+1})$ can be worked out and the RE restrictions on the VAR stated explicitly. Expressing (3) at time $t+1$ and taking conditional expectations imply, after pre-multiplying with c_1', that

$$c_1' E_t(X_{t+1}) - c_1' A_1 X_t - \cdots - c_1' A_k X_{t-k+1} - c_1' \Phi D_{t+1} = 0. \tag{5}$$

Hence, equating (4) and (5) implies that the following restrictions on the coefficient of the model (3) must be satisfied:

$$-c_1' A_1 = c_0',\ -c_1' A_2 = c_{-1}',\ -c_1' A_j = 0,\ j = 3, \ldots, k \text{ and } c_1' \Phi D_{t+1} = 0. \tag{6}$$

For fixed values of the parameters $\psi = (\beta, \lambda)$ or $\psi = (\gamma_f, \gamma_b, \tilde{\lambda})$, the restrictions may be tested by a Wald, Lagrange multiplier or likelihood ratio test. We shall employ the latter, which is particularly useful as it is independent of the specific parameterisation used. In our case this means that the outcome of the tests is the same irrespective of whether the *exact* hybrid model is parameterised by $\gamma_f, \gamma_b, \tilde{\lambda}$ or any three of $\alpha, \varepsilon, \omega, \theta, \beta, \tilde{\lambda}$ as long as three of them are fixed. To see how the likelihood ratio test can be carried out, notice that if we pre-multiply by the non-singular matrix $(c_{1\perp}, c_1)$, where $c_{1\perp}$ is a matrix with columns orthogonal to the columns in c_1 and take the restrictions (6) into account, the model (3) decomposes into two parts

$$\begin{aligned} c_{1\perp}' X_t &= c_{1\perp}' A_1 X_{t-1} + \cdots + c_{1\perp}' A_k X_{t-k} + c_{1\perp}' D_t + c_{1\perp}' \epsilon_t && (7) \\ c_1' X_t &= -c_0' X_{t-1} - c_{-1}' X_{t-2} + c_1' \epsilon_t. && (8) \end{aligned}$$

The error terms $c_{1\perp}' \epsilon_t$ and $c_1' \epsilon_t$ are correlated, so the two parts cannot be estimated separately. But conditioning on $c_{1\perp}' X_t$ and the past, the model (3) can be expressed as the product of a conditional part and a marginal part, where the marginal part is given by (8). From standard results of a multivariate Gaussian distribution it follows that the error term in the conditional model is $v_t = c_{1\perp}' \epsilon_t - c_{1\perp}' \Omega (c_1' \Omega c_1)^{-1} c_1' \epsilon_t$. Thus, the conditional model is expressed as

$$c_{1\perp}' X_t = \rho(c_1' X_t - c_0' X_{t-1} - c_{-1}' X_{t-2}) + c_{1\perp}' A_1 X_{t-1} + \cdots + c_{1\perp}' A_k X_{t-k} + c_{1\perp}' D_t + v_t, \tag{9}$$

where $\rho = c_{1\perp}' \Omega (c_1' \Omega c_1)^{-1}$. In the marginal model there is only an unknown variance. The unknown parameters in the conditional model can be found by regressing $c_{1\perp}' X_t$ on $c_1' X_t, X_{t-1}, \ldots, X_{t-k}$ and $c_{1\perp}' D_t$. If SSC denotes the mean sum of squares from this regression and SSM denotes $\frac{1}{T-k} \sum_{t=k+1}^{T} (c_1' X_t + c_0' X_{t-1} + c_{-1}' X_{t-2})^2$, the maximal value of the likelihood when the restrictions in (6) are imposed, apart from a constant, is given by

$$L_{H,max}(\psi)^{-(T-k)/2} = \frac{SSC \times SSM}{det(c_1' c_1) det(c_{1\perp}' c_{1\perp})}. \tag{10}$$

For fixed values of ψ the likelihood ratio test of the restrictions in (6) compares the value in (10) with the corresponding value where the RE restrictions are not imposed. As is well known, the value of -2 log of the likelihood ratio statistic is asymptotically χ^2-distributed, where the number of degrees of freedom equals the difference between the number of parameters in the unrestricted and the restricted model. In the case where the parameters ψ are unknown, the expression $L_{\max}(\psi)$ from (10) can be considered as a profile or concentrated likelihood for ψ. The maximum likelihood estimates can be found as the values which maximise the profile likelihood and the profile likelihood can be studied as a function of ψ. Such a property can be very useful because often the parameters in ψ are the quantities of prime interest, as in the present paper where the coefficient of the forward-looking term in the NKPC is essential for the economic interpretation of the model. A likelihood ratio test can still be carried out as earlier. However, the additional loss in degrees of freedom equals the number of freely varying parameters in ψ that are estimated. For parameter vectors ψ of moderate dimension maximising (10) represents no particular problem and may be done using a standard numerical optimising procedure.

All the arguments above carry over to the nonstationary $I(1)$ case. The additional feature that has to be accounted for is the reduced rank of the impact matrix $-\Pi$. This is most easily done by reparameterising (3) as a VAR in equilibrium correction form where the impact matrix is explicitly involved. A *necessary* condition for the empirical success of the *exact* NKPC in the nonstationary case is that inflation must be cointegrated with real marginal costs. We may see this by reformulating the restrictions in (6) to read as follows:

$$\begin{aligned} c_1'\Pi &= -(c_0 + ... + c_{-k+1})', \\ c_1'A_2 &= -c_{-1}',\ c_1'A_j = c_{-j+1},\ j = 3, \ldots, k \text{ and } c_1'\Phi D_{t+1} = 0, \end{aligned} \tag{11}$$

where $c_{-2} = ... = c_{-k+2} = 0$. Because the impact matrix $-\Pi$ has reduced rank, we notice that the first part of (11) entails restrictions on the adjustment parameters as well as the parameters describing the cointegration space. Hence, if $c_1'\Pi = d' = -(c_1 + ... + c_{-k+1})'$ is satisfied, then d' belongs to the cointegration space. Because (11) contains several additional restrictions, it is apparent that d' belonging to the cointegration space is only a *necessary* and not a sufficient condition for the *exact* NKPC to hold empirically. For the *exact* form of the baseline and the hybrid model we have that $d = (1 - \beta, -\lambda)$ and $d = (1 - \gamma_f - \gamma_b, -\widetilde{\lambda})$, respectively. We shall employ the testing procedure suggested by Johansen and Swensen (1999, 2004) in the nonstationary case.[4]

It is clear from the above discussion, that prior information about the cointegration rank is useful when evaluating the empirical performance of the NKPC,

[4] The procedure suggested by Johansen and Swensen (1999, 2004) is also applied in an empirical investigation of the *exact* NKPC and the forward-looking behaviour of exporters for a small open economy, see Boug *et al.* (2006a, 2006b).

either by standard likelihood ratio tests for stationary VAR models or for VAR models with $I(1)$ processes as described in Johansen and Swensen (1999, 2004).

3.2 Inexact rational expectations

We now turn to the *inexact* form of the NKPC and its implications for the testing procedure and the RE restrictions on the VAR, both in the stationary and nonstationary case. Again, we start out with a well specified bivariate VAR representing the reference model which the *inexact* RE restrictions are tested against. Due to the presence of the unobserved disturbance term in the NKPC, the restrictions in (4) now take the form

$$c_1' E_t(X_{t+1}) + c_0' X_t + c_{-1}' X_{t-1} + u_t = 0, \tag{12}$$

where the content of the different c matrices are as previously defined and u_t is assumed, like in Kurmann (2007), being a sequence of innovations, i.e., $E_t(u_{t+1}) = 0$. The form in (12) has two important implications for the RE restrictions that differ from those in (4). First, rewrite (12) at time $t+1$ to get

$$c_1' E_{t+1}(X_{t+2}) + c_0' X_{t+1} + c_{-1}' X_t + u_{t+1} = 0. \tag{13}$$

Using the law of iterated expectations, and the assumption that $E_t(u_{t+1}) = 0$, implies that

$$c_1' E_t(X_{t+2}) + c_0' E_t(X_{t+1}) + c_{-1}' X_t = 0. \tag{14}$$

We notice that (14) involves not only the conditional expected value of the observations *one-step* ahead as in (4), but also the conditional expected value of the observations *two-steps* ahead in addition to the present observed values. Second, the innovations must be uncorrelated because

$$E(u_t u_{t+1}) = E[u_t E_t(u_{t+1})] = E(u_t \cdot 0) = 0. \tag{15}$$

Again, once the information set is specified, the conditional expectations $E_t(X_{t+1})$ and $E_t(X_{t+2})$ in (14) can be worked out and the RE restrictions on the VAR stated explicitly. Following the usual procedure, see e.g. Fanelli (2008) and Kurmann (2007), (5) may be rewritten at time $t+1$ as

$$c_1' E_{t+1}(X_{t+2}) = c_1' A_1 X_{t+1} + \cdots + c_1' A_k X_{t-k+2} + c_1' \Phi D_{t+2}. \tag{16}$$

Thus,

$$c_1' E_t(X_{t+2}) = c_1' E_t[E_{t+1}(X_{t+2})] = \tag{17}$$

$$c_1' E_t(A_1 X_{t+1} + A_2 X_t + \cdots + A_k X_{t-k+2} + \Phi D_{t+2}) =$$

$$
\begin{gathered}
c_1' A_1 E_t(X_{t+1}) + c_1' A_2 X_t + \cdots + c_1' A_k X_{t-k+2} + c_1' \Phi D_{t+2} = \\
c_1'(A_1^2 + A_2) X_t + c_1'(A_1 A_2 + A_3) X_{t-1} + \cdots + \\
c_1'(A_1 A_{k-1} + A_k) X_{t-k+2} + c_1' A_1 A_k X_{t-k+1} + c_1'(A_1 \Phi D_{t+1} + \Phi D_{t+2}).
\end{gathered}
$$

The result of inserting (5) and (17) in (14) is an equation in $X_t, \ldots, X_{t-k+1}, D_{t+2}$ and D_{t+1}, which is identically zero. Hence, the following restrictions on the coefficients of the VAR must be satisfied in the *inexact* NKPC case:

(18)
$$
\begin{aligned}
c_1'(A_1^2 + A_2) + c_0' A_1 + c_{-1}' &= 0 \\
c_1'(A_1 A_2 + A_3) + c_0' A_2 &= 0 \\
&\vdots \\
c_1'(A_1 A_{k-1} + A_k) + c_0' A_{k-1} &= 0 \\
c_1' A_1 A_k + c_0' A_k &= 0
\end{aligned}
$$

and

(19)
$$
c_1'(A_1 \Phi D_{t+1} + \Phi D_{t+2}) + c_0' \Phi D_{t+1} = 0, t = k+1, \ldots.
$$

We observe that the restrictions in (18) and (19) are more complicated, and thus numerically more involved, than those in (6). Generally, the restrictions implied by (14) may be tested by maximizing the likelihood with respect to the parameters in $A_1, \cdots, A_k$ and Φ satisfying the restrictions (18) and (19). However, the complicated nature of (18) and (19) implies a rather difficult constrained optimization problem.

Kurmann (2007) proposed a reversed engineering approach and was able to find explicit solutions to the restrictions in (18) in the NKPC situation. Briefly speaking, the reversed engineering approach in our case involves expressing the parameters of the marginal cost equation from the VAR as a function of the parameters of the inflation equation from the VAR as well as the coefficients of the NKPC (γ_f, γ_b and $\tilde{\lambda}$).

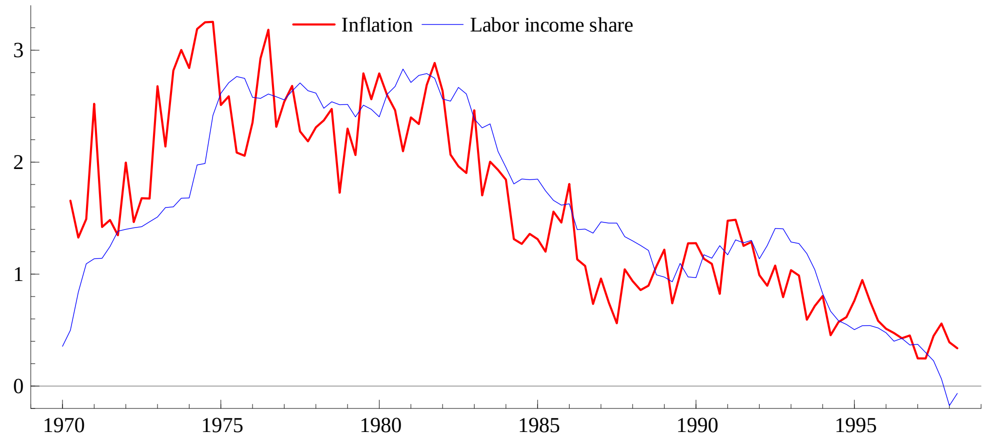
Imposing the restrictions in this way solves the optimization problem uniquely, apart from constraints on the non-stochastic parameters, in the stationary case. In the nonstationary situation, we must in addition impose a restriction ensuring that the impact matrix $-\Pi$ is singular. We thus ends up with an unconstrained maximization problem in $2k$ and $2k-1$ parameters in the stationary and nonstationary case, respectively, in addition to γ_f, γ_b and $\tilde{\lambda}$. The restrictions in (19) are a bit more problematic than those in (18). When the deterministic terms of the reference VAR are only *unrestricted* constants, (19) has a simple expression. However, the case when the VAR includes *restricted* constants is more complicated. We refer to Appendix 1 for some details on these issues and how the reversed engineering approach is handled in the present study.

A reasonable strategy for examining the *inexact* RE restrictions seems to be as follows: First we fit VAR models to the data imposing (18) and (19). Then, we evaluate the change in the maximal value of the likelihood. If this change is not deemed too large, we proceed to estimate the innovation term and test its time series properties with respect to autocorrelation. Thus, both implications of the *inexact* NKPC are tested.

4 European inflation dynamics

In this section, we evaluate the baseline and the hybrid NKPC using the likelihood based methods described above applying the same European data set as in GGL-S. As previously mentioned, the data are quarterly time series covering the sample period $1970Q1 - 1998Q2$. Inflation is measured as the quarterly percentage change in the GDP deflator, whereas real marginal costs are proxied by labour income share constructed as the ratio of compensation to employees to nominal GDP. Figure 1 displays the measures of inflation and real marginal costs.[5]

Figure 1: The European data



It is evident that the two time series move quite closely together during both the high and low inflation periods. Importantly, both inflation and the labour income share exhibit a clear downward trend with no apparent mean reverting property, at least from the mid 1970s, suggesting that π_t and mc_t are nonstationary $I(1)$ series. Therefore, a reduced rank VAR is a candidate as an empirical model. We pursue this hypothesis by fitting the bivariate VAR model (3) to the data with an *unrestricted*

[5]In Figure 1, the scale of the labour income share is adjusted to match that of inflation.

constant.[6] Thus, $D_t = 1$ and $\Phi = \mu$. Initial modelling suggests that $k = 5$ is the appropriate choice of lag length to arrive at a well-specified model in terms of well-behaved residuals, as indicated by standard diagnostic tests.[7] Then, we apply the cointegration rank test to the model, see Johansen (1995, p. 167).[8] The test results are reported in Table 1.

Table 1: Tests for cointegration rank. European data

r	λ_i	λ_{trace}	λ^a_{trace}
$r = 0$	0.261	33.07 [0.000]**	30.01 [0.000]**
$r \leq 1$	0.003	0.40 [0.528]	0.36 [0.548]

Notes: r denotes the cointegration rank and λ_i are the eigenvalues from the reduced rank regression, see Johansen (1995). The λ_{trace} and λ^a_{trace} are the trace statistics without and with degress of freedom adjustments, respectively. The p-values in square brackets, which are reported in PcGive, are based on the approximations to the asymptotic distributions derived by Doornik (1998). The asterisk ** denotes rejection of the null hypothesis at the 1 per cent significance level.

We observe that the rank should be set to one, indicating existence of a cointegration relationship between inflation and real marginal costs. Apart from the variance-covariances the reduced rank model contains 21 parameters and the value of the 2 log likelihood is 428.94. Also, likelihood ratio tests clearly reject the hypothesis that inflation and real marginal costs are stationary or long run excludable with rank equal to unity. Accordingly, the necessary condition of cointegration for the empirical success of the NKPC model is met in the non-stationary case of the European data. However, it remains to test formally the RE restrictions entailed by the NKPC in a cointegrated VAR by means of the procedures explained in Section 3. The maximum likelihood estimates of the *exact* and the *inexact* version of the baseline and the hybrid NKPC are presented in Table 2.

The *exact* baseline model consists of 13 parameters corresponding to 8 degrees of freedom when reducing them by the number of estimated structural parameters. However, an identificantion problem arises as the cointegration vector, which is

[6]In fact, a *restricted* constant may be imposed, i.e., taking $\mu = \mathrm{a}\kappa_0$ where a are the so-called adjustment coefficients. The likelihood ratio test statistic is 0.55 with one degree of freedom when comparing a VAR with an *unrestricted* constant and a VAR with a *restricted* constant imposed. The reason why the latter model is not used as a starting point is technical. Imposing the restriction at the same time as a reduced rank in the *inexact* RE-hypothesis, implies more complicated restrictions on the coefficients, see the explanation at the end of Appendix 1. Because the two models are so similar, we do not expect that choosing the alternative, more complicated strategy would have made any difference.

[7]Noticeably, the instrument set used in GGL-S includes among other variables five lags of inflation. Fanelli (2008) also concludes that the choice of $k = 5$ describes the dynamics of the bivariate VAR sufficently well. Results from the diagnostic tests, here and below, are available from the authors upon request.

[8]The rank test is performed using PcGive 10.3, see Hendry and Doornik (2001) and Doornik and Hendry (2001).

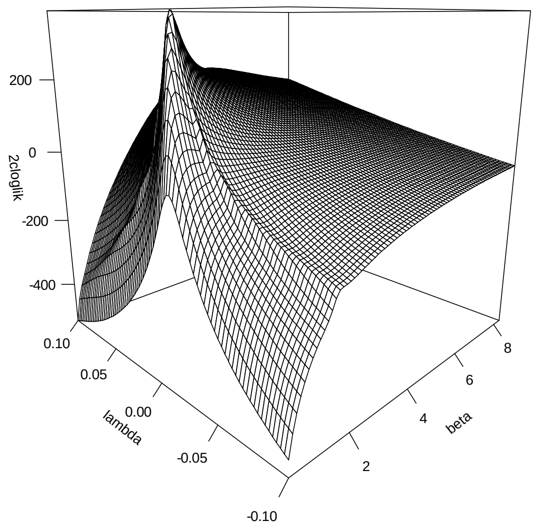
Table 2: Maximum likelihood estimates of the NKPC. European data

	Unrestricted				Restricted			
Model	$2\log L$	β,γ_f	γ_b	λ,$\widetilde{\lambda}$	$2\log L$	β,γ_f	γ_b	λ,$\widetilde{\lambda}$
Baseline, exact	380.15	1.06	–	0.001	374.94	1.0	–	0.00
Hybrid, exact	418.91	1.38	−0.38	−0.000	378.01	1.0	0.01	0.00
Baseline, inexact	375.17	1.01	–	0.000	371.02	1.0	–	0.00
Hybrid, inexact	383.04	2.80	−1.66	0.004	376.92	1.0	0.03	0.00

Notes: Maximal value of the likelihood of the reduced rank VAR with the RE restrictions imposed.

super-consistently estimated, has the form $(1-\beta,\lambda)$ and can be considered as fixed for inference based on asymptotic approximations. That the structural parameters are weakly identified in the present context is evident from Figure 2, which displays the surface of the concentrated log likelihood for the *exact* baseline model.[9]. The identification problem shows up as a sharp ridge in the surface. Hence, it is difficult to distinguish empirically between different combinations of parameter values, particularly on or close to the ridge, producing the largest value of the likelihood.

Figure 2: Surface of the concentrated log likelihood for the *exact* baseline NKPC. European data



Because there is only one free parameter in the cointegration vector β and λ can

[9]Contour plots corresponding to Figure 2 and the other surface plots presented below are provided in Appendix 2.

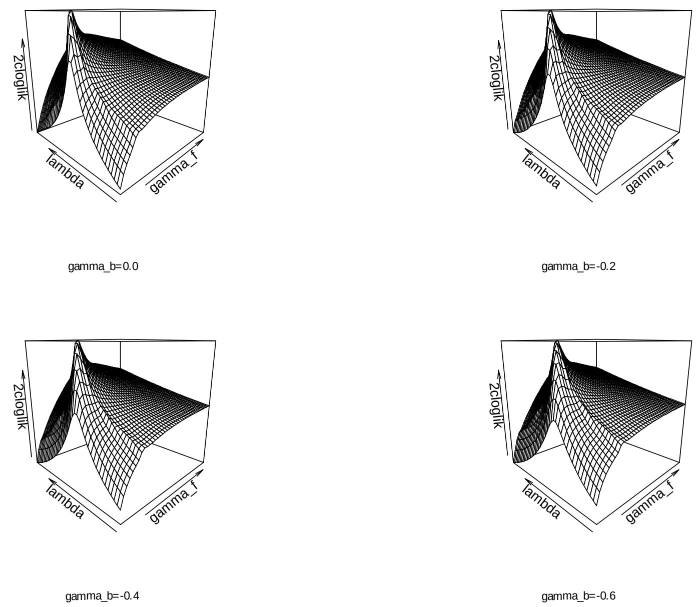
be expected to be functionally linked, at least asymptotically. As explained below (10), a correctly sized likelihood ratio test should then compare the profile likelihood ratio statistic with the critical value corresponding to 9 rather than 8 degrees of freedom. Comparing $428.94 - 380.15 = 48.79$ to a χ^2_9 distribution implies a strong rejection of the *exact* baseline model at commonly used significance levels. Moreover, the estimates for (β, λ) equals $(1.06, 0.001)$ and is outside the region of having an admissible economic interpretation. When the region for admissible parameters is constrained to $0 \leq \beta \leq 1, 0 \leq \lambda$, the estimates are on the boundary. The likelihood ratio test in such situations has a non-standard asymptotic distribution, see e.g. Gouerieroux and Monfort (1995), and can be represented as a weighted sum of independent χ^2-distributions. The critical values computed from this distribution are always smaller than those computed from a standard χ^2-distribution with degrees of freedom equal to the difference between the total number of parameters in the a priori model and the number of unconstrained parameters under the null hypothesis. Hence, if using the standard χ^2-distribution entails rejection of the null hypothesis, so does using the more complicated distribution taking the constraints into account. But the likelihood ratio statistic $428.94 - 374.94 = 54.00$ is larger than all commonly used quantiles from a χ^2_{10} distribution.[10]

The method used for the baseline model can also be employed in the *exact* form of the hybrid NKPC. Because the model is more versatile, we expect a better empirical fit. From the second line in Table 2 we see that this is indeed the case. If the parameters are estimated without restrictions, the maximal value of the likelihood is not so far from the reference VAR, $428.94 - 418.91 = 10.03$. However, the identification problem still remains. In this case the cointegration vector is $(1 - \gamma_f - \gamma_b, \widetilde{\lambda})$. Thus, as the sample length increases, we expect a stronger functionally dependence between γ_f, γ_b and $\widetilde{\lambda}$. Figure 3 shows the surface of the concentrated log likelihood of the *exact* hybrid model as functions of γ_f and $\widetilde{\lambda}$ for different values of γ_b.

We notice that the surface plots are rather similar over a wide range of values of γ_b. The contour plot of the concentrated log likelihood, for γ_b corresponding to the maximal value of the likelihood, reveals a huge spike at the maximum, see Figure 8 in Appendix 2. The level curves corresponding to the likelihood value of 400 enclose small regions while the maximum is 418.91, so in addition there exist local minima. Taking the lack of identification into account the degrees of freedom is 9 also in this case, which means that the p-value of the likelihood ratio test is 0.35. Once again, the estimates of the structural parameters do not have an admissible economic interpretation. When the parameter values are restricted to the admissible economic parameter space, i.e. $0 \leq \gamma_f, \gamma_b \leq 1, 0 \leq \widetilde{\lambda}$, the estimates are on the boundary. By the same argument outlined above, the critical values of the

[10] The number of degrees of freedom is calculated as follows: $10 = 21 - (12 - 1)$ = total number of parameters $-$ (effective number of parameters $-$ number of constrained effective parameters).

Figure 3: Surface of the concentrated log likelihood for the *exact* hybrid NKPC. European data



likelihood ratio test based on the complicated χ^2-distribution are smaller than the values from a standard χ^2_{10} distribution. Nevertheless, the likelihood ratio statistic $428.94 - 378.01 = 50.93$ is larger than all commonly used quantiles. Thus, we find no support for the *exact* NKPC in the European data, irrespective of whether the baseline or the hybrid model is examined.

We now turn to the *inexact* version of the NKPC. The RE restrictions on the VAR, as explained in Section 3, have a different and more complicated form than with the *exact* case. Using a modification of the reversed engineering approach, the *inexact* hybrid model can be expressed with 13 parameters. The results of fitting the *inexact* version of the baseline as well as the hybrid model with and without admissible restrictions on the parameters imposed are displayed in the two last lines of Table 2. A preliminary inspection of surface plots of the log likelihood shows that the identification problem is still present. Hence, there are less free parameters under the null hypothesis than it may appear. But the values of the likelihood ratio statistics are so large that both the baseline and the hybrid model are strongly rejected. The same picture is evident when the economically, interpretable parameter restrictions are imposed on the NKPC.

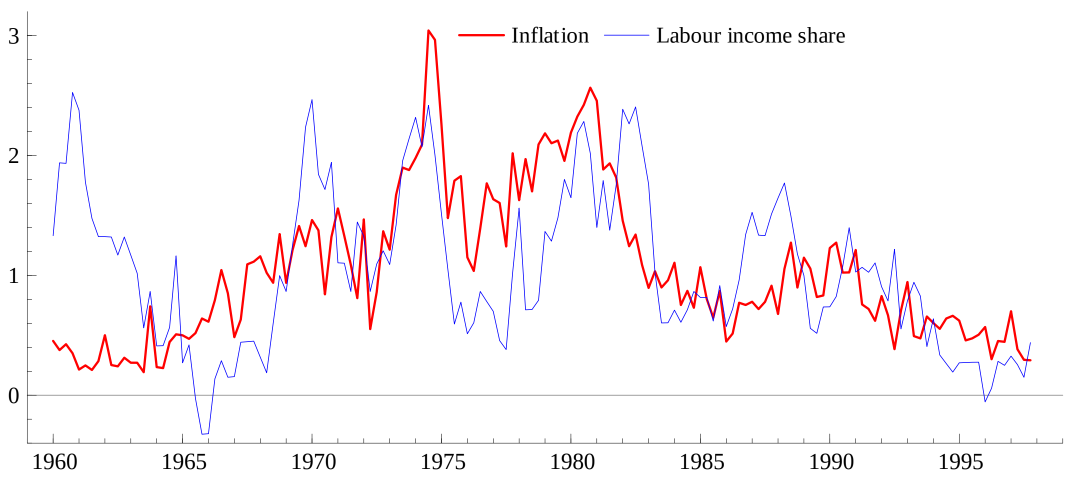
The clear meassage from these findings is that the evidence in GGL-S that the NKPC with a dominant role for forward-looking behaviour does reasonably well in describing European data must be considered fragile. We conclude in line with

Fanelli (2008), who used a more restrictive approach, that the restrictions deduced from the NKPC are strongly at odds with the European data. An important lesson of the findings here is that although the necessary condition of cointegration between inflation and real marginal costs seems to hold, the overall restrictions implied by the NKPC model are formally rejected by the data according to likelihood ratio based inference. GGL-S, on the other hand, use a less formal method by just looking at the significance status of the inflation and real marginal costs terms to claim the empirical success of the model. As noted by Juselius (2008), what GGL-S essentially do is to only estimate the cointegration relationship between inflation and real marginal costs without really testing the RE restrictions implied by the NKPC model as such.

5 US inflation dynamics

We now present estimates of the NKPC models and evaluate their empirical performance by likelihood based methods applying the same US data set as in GG. As in the previous section, inflation is measured as the quarterly percentage change in the GDP deflator, whereas real marginal costs are proxied by labour income share. The data now covers the sample period $1960Q1 - 1997Q4$. Figure 4 displays the measures of inflation and real marginal costs in the case of US data.[11]

Figure 4: The US data



As in the European case, there is a close co-movement between inflation and real marginal costs. However, contrary to the nonstationarity status of the European data, both US inflation and real marginal costs fluctuate around their respective

[11] In Figure 4, the scale of the labour income share is adjusted to match that of inflation.

means with no apparent trending behaviour. We may then suspect, as will be verified formally below, that the US series are better described by a stationary VAR than a nonstationary one. Specifying an unrestricted VAR in $X_t = (\pi_t, mc_t)'$, we find that $k = 4$ produces a model with no serious misspecification.[12] Indeed, the equation for inflation suffers from non-normal residuals, which is due to some large outliers in the mid 1970s. These outliers may be mopped up by impulse dummies, but doing so does not alter the results from the cointegration analysis, which we now turn to. Table 3 reports the findings from applying the cointegration rank test to the US data based on the unrestricted bivariate VAR of order four.

Table 3: Tests for cointegration rank. US data

r	λ_i	λ_{trace}	λ^a_{trace}
$r = 0$	0.084	16.87 [0.029]*	15.98 [0.041]*
$r \leq 1$	0.023	3.52 [0.061]	3.34 [0.068]

Notes: r denotes the cointegration rank and λ_i are the eigenvalues from the full rank regression, see Johansen (1995). The λ_{trace} and λ^a_{trace} are the trace statistics without and with degress of freedom adjustments, respectively. The p-values in square brackets, which are reported in PcGive, are based on the approximations to the asymptotic distributions derived by Doornik (1998). The asterisk * denotes rejection of the null hypothesis at the 5 per cent significance level.

We notice that the hypothesis of no cointegration can be rejected at the 5 per cent significance level, while the hypothesis of at most one cointegrating relationship between inflation and real marginal costs can be rejected at the 6 per cent level. Therefore, we conclude that the impact matrix has full rank so that a stationary VAR in levels seems to fit the US data reasonably well. Apart from the variance-covariances the full rank model contains 18 parameters and the value of the 2 log likelihood is in this case 495.02. The maximum likelihood estimates of both the *exact* and the *inexact* version of the baseline and the hybrid NKPC are displayed in Table 4.

Overall, the decrease in the maximal value of the likelihood by imposing the *exact* RE restrictions entailed by the NKPC is somewhat less with the US data than with the European data. The maximal value of the 2 log likelihood is estimated to 468.22 in the baseline case. The number of restrictions is 7, so the model is strongly rejected also in the US case. The point estimates of β (1.03) and λ (0.003) are on the boundary of having admissible, economic interpretaions.

Estimating the *exact* hybrid model yields a maximal value of the 2 log likelihood of 475.26. Hence, when the coefficient of the lagged inflation is also estimated, implying loss of one degree of freedom, the NKPC is still strongly rejected. The

[12]The chosen model specification is identical to the one in Table 6 in Mavroeidis (2006). GG also include four lags of inflation in the instrument set underlying the GMM estimation of the NKPC. Reducing the bivariate VAR by $k = 3$, as in Kurmann (2007), gives a model with severe autocorrelation and heteroscedasticity in the estimated error terms.

Table 4: Maximum likelihood estimates of the NKPC. US data

	Unrestricted				Restricted			
Model	$2\log L$	β,γ_f	γ_b	λ,$\widetilde{\lambda}$	$2\log L$	β,γ_f	γ_b	λ,$\widetilde{\lambda}$
Baseline, exact	468.22	1.03	–	0.003	466.08	1.0	–	0.000
Hybrid, exact	475.26	1.31	–0.28	–0.004	466.73	1.0	0.0	0.001
Baseline, inexact	485.53	1.00	–	0.021	–	–	–	–
		(0.03)	–	(0.009)				
Hybrid, inexact	486.14	0.89	0.11	0.020	–	–	–	–
		(0.09)	(0.08)	(0.007)				

Notes: Maximal value of the likelihood of the full rank VAR with the RE restrictions imposed. Standard errors in brackets.

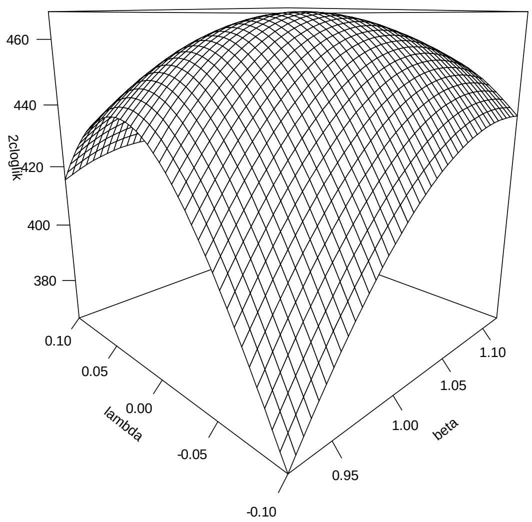
point estimates of γ_f (1.31), γ_b (-0.28) and $\widetilde{\lambda}$ (-0.004), corresponding to the maximum, all violate the admissible parameter space. Restricting parameter values to the admissible space entails a further decrease in the maximal value of the likelihood. Even if we take into account the necessary modifications of the asymptotic distributions, the *exact* hybrid model is rejected. Specifically, in the hybrid model the modified asymptotic distribution has critical values smaller than those of the χ^2_9 distribution, whose 99 percentile is 21.67.

Also for the US data we provide surface and contour plots of the concentrated log likelihood for the *exact* version of the baseline as well as the hybrid model. Figures 5 and 6 clearly show that the surface now looks much more quadratic than what is the case with the European data, indicating that the models do not suffer from the weak identification problem. Such feature of the US data is not surprising because no cointegration vector is super-consistently estimated in this case.

As noted in the introduction, Kurmann (2007) finds that the *inexact* hybrid NKPC fits the US data well. Again, we apply the reversed engineering approach modified to handle the optimization problems in our context. From the third and fourth line in Table 4 we notice that the likelihood ratio test for both the baseline and the hybrid model provides indication that the fit is satisfactory. The corresponding p-values are 0.22 and 0.18 with degrees of freedom equal to 7 and 6, respectively. Another indication of the good fit of the *inexact* version of the NKPC is the admissible estimates that come out of the profile likelihood. Because the *inexact* version of the NKPC appears to be reasonable, we have included estimates of the standard errors, obtained from inverting the numerical Hessians.

To conclude whether the *inexact* NKPC explains the data well we also check if the stochastic terms can be considered as innovations. The estimated coefficients of the matrices $A_1, \cdots, A_4$ and μ for the *inexact* baseline and hybrid model are displayed in Table 5. Using these estimates, $E_t(X_{t+1})$ and hence u_t of (12), can be estimated. There is indication of autocorrelation in the estimated innovations. For the baseline model the Box-Pierce statistic computed from the 21 first autocorrelations is 19.95, whereas the Box-Ljung statistic is 21.37. With $21 - 12 = 9$ degrees

Figure 5: Surface of the concentrated log likelihood for the *exact* baseline NKPC. US data



of freedom the p-values are 0.018 and 0.011, respectively. Similarly, for the hybrid model the Box-Pierce statistic is 20.58 and the Box-Ljung statistic is 22.13 with p-values of 0.015 and 0.008, respectively.

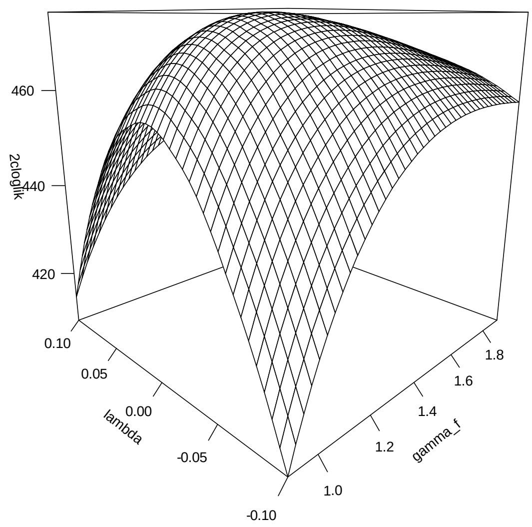
Table 5: Maximum likelihood estimates of the matrices $A_1, ..., A_4$ and μ of the *inexact* NKPC. US data

A_1		A_2		A_3		A_4		μ
0.6149	0.0718	0.2354	−0.0573	0.1008	−0.0235	0.0351	0.0000	−0.1183
0.0002	0.9166	−0.1154	0.0169	0.0375	−0.0851	0.1453	−0.0046	0.7897
0.6450	0.0693	0.1950	−0.0498	0.1064	−0.0259	0.0463	−0.0054	0.0375
−0.0498	0.9177	−0.1446	0.0231	0.0514	−0.0778	0.2447	−0.0284	−0.2161

Notes: Baseline model (upper panel), hybrid model (lower panel).

In line with Kurmann (2007), our likelihood based estimates coincide by and large with GG's GMM estimates, confirming the claim that forward-looking behaviour is an important feature of the price setting among US firms. However, and unlike Kurmann (2007) who did not consider the issue, we do not find evidence that the stochastic terms are innovations as assumed in the model. Hence, our findings using the US data are somewhat mixed with respect to the empirical success of the *inexact* NKPC.

Figure 6: Surface of the concentrated log likelihood for the *exact* hybrid NKPC. US data



6 Conclusions

In this paper, we revisit the evidence in GG and GGL-S that both European and US inflation dynamics are in line with the NKPC model. To this end, we use the same data set and likelihood based methods for testing the RE restrictions through VAR models. We extend recent VAR based likelihood analyses of the NKPC in four ways. First, we treat the two data sets by the same methodology, paying particular attention to the stationary as well as the nonstationary, and possibly cointegrated, nature of variables involved. Second, we focus not only on the *inexact* form, but also on the *exact* form of the baseline as well as the hybrid NKPC. Third, we test formally the assumption of the *inexact* model that the stochastic term forms a sequence of innovations with no autocorrelation properties present. Fourth, we express the likelihood as a function of all parameters involved, which makes an explicit inspection of the likelihood surface, and thus also the potential weak identification problem, possible.

Our empirical findings using the European data contradict the evidence in GGL-S, irrespective of which version of the NKPC is studied. We do find though, in line with GGL-S, that the inclusion of a backward-looking term improves the empirical fit. However, the increase in the value of the likelihood is not large enough to yield a well-specified model. We also show that the likelihood surface has a rather sharp ridge, indicating that the NKPC may be weakly identified as criticised by others using alternative methods. On the other hand, we confirm by and large GG's finding that the *inexact* baseline or hybrid NKPC are good candidates in explaining US inflation dynamics. However, the assumption of the models that the stochastic term forms a sequence of innovations may be problematic as we find indication of autocorrelation in the estimated residuals. The *exact* form of the baseline as well as the hybrid model is firmly rejected by likelihood ratio tests using US data.

At the outset, testing the NKPC by means of VAR reduced forms is restricitive as most modern DSGE models imply reduced form dynamics are vector ARMA processes. Also, VAR based tests of the NKPC may be more restrictive than their GMM counterparts, which make no assumptions about the nature of the reduced form dynamics. That said, the transparancy of the VAR approach used in this paper is an advantage compared to the analysis in GG and GGL-S, which forces the assumption of rational expectations in the GMM estimation by a somewhat arbitrary choice of instruments. As stressed by Galí *et al.* (2005), however, it is vital for likelihood based methods to work well that the overall structure of the underlying model is correctly specified. Generally, VAR models by themselves are based on few assumptions about the data generating process, but it can be a challenge to find a congruent empirical specification. For systems with several variables such a task may be quite demanding or even impossible due to the large number of parameters involved. This potential problem is less pressing in the present context as the VARs involve only two variables sufficient enough to arrive at *well-specified* underlying models. Therefore, we think it is worth trying to explain what may be the main reasons for the clear rejection of the NKPC in the European case.

As emphasised in the text, the first part of the restrictions in the reduced rank VAR involves the long-run parameters and the cointegration relationship between inflation and real marginal costs. The rejection of the NKPC model is therefore mainly due to the restrictions imposed on the short run parameters. A reasonable interpretation is thus that the hybrid model with *only* one lag does not properly reflect the rather complex dynamic structure of European inflation, as indicated by the well-specified fifth order VAR. It is interesting that GGL-S begin their paper by showing that the traditional Phillips curve (with no forward-looking behaviour) with five lags of inflation does a reasonable good job of characterising inflation. Because the value of the likelihood increases when allowing for lagged inflation, expanding the NKPC further in this direction may be helpful to make the model perform better in terms of explaining European inflation persistence. Likewise, alternative measures of inflation not based on the GDP deflator may be needed in order to improve the empirical fit of the NKPC. The deflator for the non-farm business sector is probably a more relevant proxy for actual prices set by firms in the economy than the GDP deflator, which is essentially a residual in the national accounts following modern practice of double deflating. Similarly, alternative proxies for marginal costs may deliver empirically robust specifications of the NKPC. For instance, McAdam and Willman (2004) suggest using a CES production function with factor specific technological change as a basis for a marginal cost measure. Doing so has, however, been beyond the scope of the present paper and is left for future reasearch.

References

[36] Baillie, R.T. (1989): Econometric tests of rationality and market efficiency, Econometric Reviews 8, 151-186.

[36] Bjørnstad, R. and R. Nymoen (2008): The New Keynesian Phillips Curve Tested on OECD Panel Data. Economics: The Open-Access, Open-Assessment E-Journal, Vol. 2, 2008-23, 1-18.

[36] Boug, P., Å. Cappelen and A.R. Swensen (2006a): Expectations and Regime Robustness in Price Formation: Evidence from Vector Autoregressive Models and Recursive Methods, Empirical Economics 31, 821-845.

[36] Boug, P., Å. Cappelen and A.R. Swensen (2006b): The New Keynesian Phillips Curve for a Small Open Economy, Discussion Papers 460, Statistics Norway.

[36] Bårdsen, G., E.S. Jansen and R. Nymoen (2004): Econometric Evaluation of the New Keynesian Phillips Curve, Oxford Bulletin of Economics and Statistics 66, Supplement, 671-686.

[36] Calvo, G.A. (1983): Staggered Prices in a Utility Maximising Framework, Journal of Monetary Economics 12, 383-398.

[36] Doornik, J.A. (1998): Approximations to the Asymptotic Distribution of Cointegration Tests, Journal of Economic Surveys 12, 573-593.

[36] Doornik, J.A. and D.F. Hendry (2001): Empirical Econometric Modelling using PcGive Volume I, Timberlake Consultants Ltd, London.

[36] Fanelli, L. (2008): Testing the New Keynesian Phillips Curve through Vector Autoregressive Models: Results from the Euro area, Oxford Bulletin of Economics and Statistics 70, 53-66.

[36] Galí, J. and M. Gertler (1999): Inflation Dynamics: A Structural Econometric Analysis, Journal of Monetary Economics 44, 195-222.

[36] Galí, J., M. Gertler and J.D. López-Salido (2001): European Inflation Dynamics, European Economic Review 45, 1237-1270.

[36] Galí, J., M. Gertler and J.D. López-Salido (2005): Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve, Journal of Monetary Economics 52, 1107-1118.

[36] Gouerieroux, C. and A. Monfort (1995): Statistics and Econometric Models, Cambridge University Press, Cambridge.

[36] Hansen, L.P. and T.J. Sargent (1991): Exact linear rational expectations models: Specification and estimation. In: Hansen, L.P., Sargent T.J. (eds.): Rational Expectations Econometrics. Westview Press, Boulder.

[36] Hendry, D.F. and J.A. Doornik (2001): Modelling Dynamic Systems using PcGive Volume II, Timberlake Consultants Ltd, London.

[36] Henry, B.S.G. and A.R. Pagan (2004): The econometrics of the new Keynesian policy model: Introduction, Oxford Bulletin of Economics and Statistics 66, Supplement, 581-607.

[36] Johansen, S. (1995): Likelihood-Based Inference in Cointegrated Vector Autoregressive Models, Oxford University Press, New York, (2.ed. 1996).

[36] Johansen, S. and A.R. Swensen (1999): Testing Exact Rational Expectations in Cointegrated Vector Autoregressive Models, Journal of Econometrics 93, 73-91.

[36] Johansen, S. and A.R. Swensen (2004): More on Testing Exact Rational Expectations in Cointegrated Vector Autoregressive Models: Restricted Constant and Linear Term, Econometrics Journal 7, 389-397.

[36] Juillard, M., O. Kamenik, M. Kumhof and D. Laxton (2008): Optimal price setting and inflation inertia in a rational expectation model, Journal of Economic Dynamics & Control 32, 2584-2621.

[36] Juselius, M. (2008): Testing the New Keynesian Model on U.S. and Euro Area data, Economics: The Open-Access, Open-Assessment E-Journal, Vol. 2, 2008-24, 1-26.

[36] Kurmann, A. (2007): VAR-based estimation of Euler equations with an application to New Keynesian pricing, Journal of Economic Dynamics & Control 31, 767-796.

[36] Mavroeidis, S. (2004): Weak Identification of Forward-Looking Models in Monetary Economics, Oxford Bulletin of Economics and Statistics 66, Supplement, 609-635.

[36] Mavroeidis, S. (2005): Identification Issues in Forward-looking Models Estimated by GMM, with an Application to the Phillips Curve, Journal of Money, Credit, and Banking 37, 421-448.

[36] Mavroeidis, S. (2006): Testing the New Keynesian Phillips Curve without assuming identification, Brown Economics Working Paper No. 2006-13, Brown University. Available at SSRN: http://ssrn.com/abstract=905261.

[36] McAdam, P. and A. Willman (2004): Supply, Factor Shares and Inflation Persistence: Re-examining Euro Area New Keynesian Phillips Curves, Oxford Bulletin of Economics and Statistics 66, Supplement, 637-670.

[36] Nason, J.M. and G.W. Smith (2008): Identifying the New Keynesian Phillips Curve, Journal of Applied Econometrics 23, 525-551.

[36] R Development Core Team (2006): R: A language and environment for statistical computing, R Foundation for Statistical Computing, Vienna, Austria.

[36] Roberts, J.M. (1995): New-Keynesian Economics and the Phillips Curve, Journal of Money, Credit, and Banking 27, 975-984.

[36] Rotemberg, J.J. (1982): Sticky Prices in the United States, Journal of Political Economy 90, 1187-1211.

[36] Rotemberg, J.J. and M. Woodford (1999): The cyclical behavior of prices and costs. In Taylor, J.B. and M. Woodford (Eds.): Handbook of Macroeconomics, vol. 1B. North-Holland, Amsterdam.

[36] Rudd, J. and K. Whelan (2005): New Tests of the New Keynesian Phillips Curve, Journal of Monetary Economics 52, 1167-1181.

[36] Rudd, J. and K. Whelan (2007): Modeling Inflation Dynamics: A Critical Review of Recent Research, Journal of Money, Credit and Banking, Supplement to Vol. 39, 155-170.

[36] Sbordone, A.M. (2005): Do Expected Future Marginal Costs Drive Inflation Dynamics? Journal of Monetary Economics 52, 1183-1197.

[36] Taylor, J.B. (1979): Staggered wage setting in a macro model, American Economic Review 69, 108-113.

[36] Taylor, J.B. (1980): Aggregate dynamics and staggered contracts, Journal of Political Economy 88, 1-23.

Appendix 1

Computational details

The estimation of the *exact* as well as the *inexact* NKPC consists of numerical maximization concentrated likelihoods using iterative methods. The computational burden in the two cases are as explained in the text quite different. Whereas it is possible to find a profile likelihood depending only of the coefficients in the NKPC in the *exact* case, the profile likelihood in the *inexact* case will also include roughly half of the parameters of $A_1, \ldots, A_k$. To perform the numerical optimization the procedure "optim" in the statistical package R was used, see http://www.r-project.org/ and R Development Core Team (2006). It contains several options, notably the quasi-Newton BFGS (Broyden, Fletcher, Goldfarb and Shanno), the Nelder-Mead method and a method based on simulated annealing. Running the options sequentially seems to be quite useful and helpful in finding suitable starting values. Also the functions to be optimized appear to be smooth enough, so numerical derivatives are sufficient to determine the gradient and Hessian matrix in the BFGS method. But all these iterative methods provide no guarantee finding the global maximum and not ending up in a local one. This problem becomes of course more pressing when many variables are involved, and is thus an argument in favour of looking for simpler formulations. We consider bivariate VAR as dictated from the NKPC model.

The reversed engineering approach

As explained in the text, we apply the reversed engineering approach, proposed by Kurmann (2007), in order to conduct the numerical optimization problem involved when the *inexact* NKPC is examined. Here we provide some details about the approach and how it is modified to suit our context using the hybrid NKPC as an example. We first recall that in the hybrid model the dimension $p = 2$, $c_1' = (\gamma_f, 0)$, $c_0' = (-1, \widetilde{\lambda})$ and $c_{-1}' = (\gamma_b, 0)$. Letting

$$A_i = \begin{pmatrix} a_{11,i} & a_{12,i} \\ a_{21,i} & a_{22,i} \end{pmatrix}, i = 1, \ldots, k, \tag{20}$$

from the VAR, we follow Kurmann (2007) and express the parameters of the labor share equation explicitly in terms of the parameters of both the inflation equation and the NKPC equation to read as

$$
\begin{aligned}
(21) \qquad a_{21,1} &= \frac{a_{11,1} - \gamma_f(a_{11,1}^2 + a_{11,2}) - \gamma_b}{\gamma_f a_{12,1} + \widetilde{\lambda}} \\
a_{22,1} &= \frac{a_{12,1} - \gamma_f(a_{11,1}a_{12,1} + a_{12,2})}{\gamma_f a_{12,1} + \widetilde{\lambda}} \\
a_{21,2} &= \frac{a_{11,2} - \gamma_f(a_{11,1}a_{11,2} + a_{11,3})}{\gamma_f a_{12,1} + \widetilde{\lambda}} \\
a_{22,2} &= \frac{a_{12,2} - \gamma_f(a_{11,1}a_{12,2} + a_{12,3})}{\gamma_f a_{12,1} + \widetilde{\lambda}} \\
&\vdots \\
a_{21,k-1} &= \frac{a_{11,k-1} - \gamma_f(a_{11,1}a_{11,k-1} + a_{11,k})}{\gamma_f a_{12,1} + \widetilde{\lambda}} \\
a_{22,k-1} &= \frac{a_{12,k-1} - \gamma_f(a_{11,1}a_{12,k-1} + a_{12,k})}{\gamma_f a_{12,1} + \widetilde{\lambda}} \\
a_{21,k} &= \frac{a_{11,k} - \gamma_f a_{11,1}a_{11,k}}{\gamma_f a_{12,1} + \widetilde{\lambda}} \\
a_{22,k} &= \frac{a_{12,k} - \gamma_f a_{11,1}a_{12,k}}{\gamma_f a_{12,1} + \widetilde{\lambda}}.
\end{aligned}
$$

To avoid uninteresting cases we consider only situations where $\gamma_f a_{12,1} + \widetilde{\lambda} \neq 0$. Thus, ignoring the deterministic terms for the moment, the likelihood for the stationary case can be expressed as a function of the $2k+3$ parameters $a_{11,1}, a_{12,1}, \cdots, a_{11,k}, a_{12,k}$ and γ_f, γ_b and $\widetilde{\lambda}$. In the nonstationary case the impact matrix $\Pi = A_1 + \cdots + A_k - I$ must have reduced rank. Because $p = 2$, this means that Π, if it is not equal to zero, must satisfy the additional restriction

$$
(22) \qquad \det(\Pi) = \det \begin{pmatrix} \sum_{i=1}^k a_{11,i} - 1 & \sum_{i=1}^k a_{12,i} \\ \sum_{i=1}^k a_{21,i} & \sum_{i=1}^k a_{22,i} - 1 \end{pmatrix} = 0.
$$

Summing the expressions in (21) yields

$$
\begin{aligned}
(23) \qquad \sum_{i=1}^k a_{21,i} &= \frac{(\sum_{i=1}^k a_{11,i})(1 - \gamma_f a_{11,1}) - \gamma_b - \gamma_f(\sum_{i>1}^k a_{11,i})}{\gamma_f a_{12,1} + \widetilde{\lambda}} \\
(24) \qquad \sum_{i=1}^k a_{22,i} - 1 &= \frac{(\sum_{i=1}^k a_{12,i})(1 - \gamma_f a_{11,1}) - \widetilde{\lambda} - \gamma_f(\sum_{i=1}^k a_{12,i})}{\gamma_f a_{12,1} + \widetilde{\lambda}}.
\end{aligned}
$$

After some algebraic manipulations the restriction in (22) may be expressed as

$$a_{11,k} = \frac{(\gamma_f + \gamma_b - 1)}{\tilde{\lambda}} \sum_{i=1}^{k} a_{12,i} - \sum_{i=1}^{k-1} a_{11,i} + 1. \tag{25}$$

The likelihood for the $I(1)$ case is therefore a function of the $2k+2$ parameters $a_{11,1}, a_{12,1}, \cdots, a_{11,k-1}, a_{12,k-1}, a_{12,k}$ and γ_f, γ_b and $\tilde{\lambda}$.

Finally, consider the coefficient of the non-stochastic terms, Φ. These coefficients must satisfy $c_1'(A_1\Phi D_{t+1} + \Phi D_{t+2}) + c_0'\Phi D_{t+1} = 0, t = k+1, \ldots,$ that is the restrictions in (19). Because only constants are involved, $\Phi = (\mu_1, \mu_2)'$ and the restriction reduces to $(c_1'(A_1 + I) + c_0')\Phi = 0$. Using coordinates this may be written $(\gamma_f(a_{11,1}+1) - 1)\mu_1 + (\gamma_f a_{12,1} + \tilde{\lambda})\mu_2 = 0$, or $\mu_2 = -\frac{\gamma_f(a_{11,1}+1)-1}{\gamma_f a_{12,1}+\tilde{\lambda}}\mu_1$. In the case where the time series are $I(1)$ and the constant is restricted, $\Pi = (\mathrm{a}_1,\mathrm{a}_2)'(b_1, b_2)$ and $\Phi = (\mathrm{a}_1,\mathrm{a}_2)'\kappa_0$. Then $[(\gamma_f(a_{11,1}+1) - 1)\mathrm{a}_1 + (\gamma_f a_{12,1} + \tilde{\lambda})\mathrm{a}_2]\kappa_0 = 0$, which is more difficult to express explicitly.

Appendix 2

Figure 7: Contour of the concentrated log likelihood for the *exact* baseline NKPC. European data

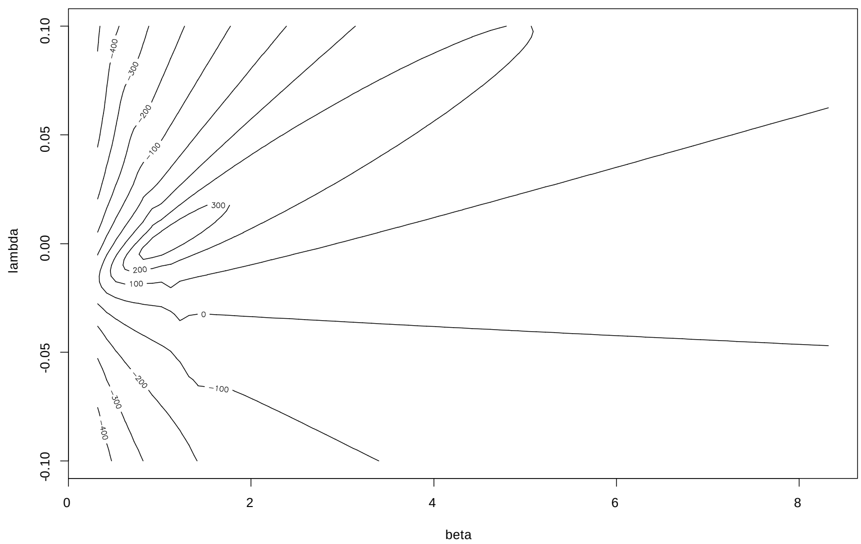


Figure 8: Contour of the concentrated log likelihood for the *exact* hybrid NKPC. European data

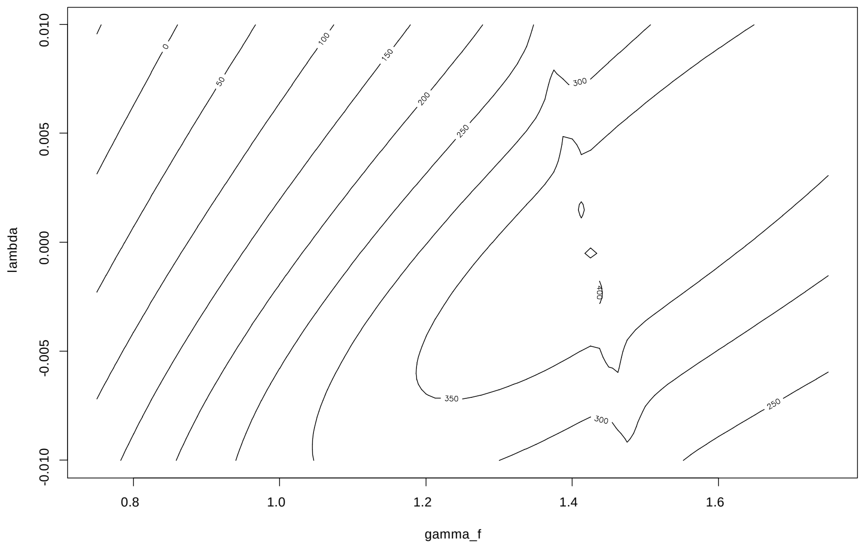


Figure 9: Contour of the concentrated log likelihood for the *exact* baseline NKPC. US data

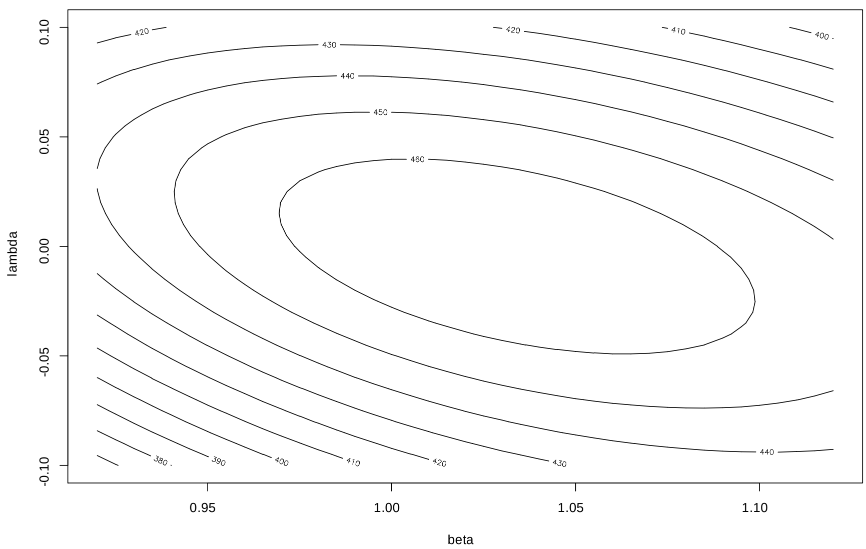


Figure 10: Contour of the concentrated log likelihood for the *exact* hybrid NKPC. US data