# Taxation of the rich and the cost of raising tax revenue

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**Abstract:**
Taxation of capital income and wealth designed to redistribute from the rich may harm small open economies with a globalized capital market as investments are distorted. This study shows that raising tax revenue by taxing wealth is less costly than by taxing labor income within a simplified model framework designed for modest levels of taxes on capital income and wealth. The explanation is that a recidence based tax on wealth collects tax revenue from wealthy investors without distorting investments. The study also shows that raising tax revenue by increasing the tax rate on capital income marginally above the foreign tax level is less costly than by increasing the tax rate on labor income even though foreign investments is distorted. An assessment of these results together with other empirical and theoretical studies uncover that the cost of taxing capital income and wealth is likely to increase with the level of these taxes, however.

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## Sammendrag

Piketty (2013) anbefalte en global formuesskatt for å omfordele inntekt. Sentral skattelitteratur støtter imidlertid ikke en slik anbefaling, se Auerbach og Hassett (2015). En årlig formuesskatt er overflødig siden det er mer hensiktsmessig å skattlegge kapitalinntekter og diverse overføringer av formue ifølge Boadway og Pestieau (2019). Beskatning av bedriftsoverskudd i åpne økonomier med et globalisert kapitalmarked hemmer dessuten næringsinvesteringene. Gordon (1986) og Razin og Sadka (1991) viser at overskudd ikke bør beskattes i slike tilfeller. Beskatning av bedriftsoverskudd hemmer også næringsinvesteringer i deler av økonomien som ikke opererer i et globalisert kapitalmarked. Investeringseffekten blir imidlertid tilnærmet nøytralisert hvis selskapsskattesatsen er på linje med kapitalinntektsskattesatsen, og risikofri avkastning trekkes fra i en eventuell skatt på aksjonærer, se Sørensen, 2005a og 2022. Formue og kapitalinntekter (inntil normalavkastning) i åpne økonomier med et delvis globalisert kapitalmarked bør ikke skattlegges ifølge resultatene ovenfor.

Denne studien bidrar ved å beregne velferdskostnaden av å drive inn skatteinntektene ved å skattlegge kapitalinntekter, formue og arbeidsinntekter i en liten åpen økonomi med et delvis globalisert kapitalmarked. Studien viser at skattefinansieringskostnaden er lavere for skatt på formue enn for beskatning av arbeidsinntekt. Forklaringen er at skatt på formue innebærer å ta fra de rike uten å hemme insentivene til å investere i næringslivet. Studien viser også at skattefinansieringskostnaden forbundet med at selskaps- og kapitalinntektsskatten settes marginalt høyere enn tilsvarende skattenivå i utlandet er lavere enn skattefinansierings-kostnaden forbundet med en marginal økning i skatten på arbeidsinntekt. En evaluering av disse resultatene sammen med resultater fra tidligere empiriske og teoretiske studier avdekker at kostnadene ved skattlegging av kapitalinntekter og formue sannsynligvis vil øke med nivået på disse skattene. Ønsket om å skattlegge formue styrkes imidlertid også av evalueringen. Skatteutvalget 2022 anbefalte på sin side å ersatte deler av formuesskatten med skatt på arv.

Resultatene i denne studien er utledet i et enkelt modellrammeverk designet for moderate skattesateser på kapitalinntekter og formue. Rammeverket utelukker skatteomgåelse ved at velstående investorer flytter til utlandet, eller skatteunndragelse ved at formue overføres til skatteparadiser. Langsiktige virkninger av et svakere vern om privat eiendomsrett er også utelatt.

# 1. Introduction

Piketty (2013) recommended a global wealth tax to redistribute income as the rich are getting richer in many countries. However, this policy advice conflicts with results within the literature on taxation, see Auerbach and Hassett (2015). An annual net wealth taxes are unnecessary since objectives can be better achieved by suitably designed taxes on capital income and transfers of wealth, see Boadway and Pestieau (2019). Also, taxation of corporate profit distorts business investments in open economies with a globalized capital market as pre-tax return requirements are increased to preserve profits. A zero tax on corporate profit is optimal according to Gordon (1986) and Razin and Sadka (1991) as taxation of immobile labor raise tax revenue at a lower welfare cost. Taxation of corporate profit also distorts business investments within the non-globalized part of the economy. However, such distortions are approximately neutralized if the corporate tax is aligned with the capital income tax, and a risk-free rate of return is deducted from the tax on shareholders, see Sørensen, 2005a and 2022. Hence, both capital income below the risk-free return and wealth should not be taxed within open economies with a partially globalized capital market according to these results. Indeed, only a few countries tax wealth today, and competition among countries to attract capital explains falling corporate tax rates within OECD countries, see Devereux et al. (2008).

The aim of the present study is to investigate whether such lenient taxation of wealth and capital income raise tax revenue at the lowest possible welfare cost. The study contributes by calculating the welfare cost of raising tax revenue by taxing capital income, wealth, and labor income within a small open economy with a partially globalized capital market with both domestic and foreign investors. The study shows that taxation of wealth is less costly in terms of welfare than taxation of labor income even when the supply of labor is fixed. This finding supports the recommendation by Piketty and contradicts the conclusion in Boadway and Pestieau (2019). The explanation is that a residence-based tax on wealth collects tax revenue from domestic investors without distorting their business investments. Investments by foreigners is not distorted as they are exempt from the wealth tax. Also, the welfare weight attached to domestic investors is smaller than the welfare weight attached to workers. Hence, taxation of wealth is less costly. The study also shows that introducing a marginal tax on all sources of capital income is less costly than taxation of labor income. The explanation is that the aligned tax on corporate and capital income collects tax revenue from domestic investors in a non-distorting manner but distorts

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investments by foreign corporations. The benefit of collecting tax revenue from domestic investors is larger when the capital income tax rate is marginal, however. Hence, this finding contradicts the conclusions in Gordon (1986) and Razin and Sadka (1991).

Results are derived within a simplified model framework designed for modest levels of taxes on capital income and wealth. The model framework excludes tax avoidance as investors move abroad, tax evasion as wealth and profit is transferred to tax havens, distortions in the allocation of consumption over time, and impacts of weaker private property rights. The cost of raising tax revenue is likely to increase with tax levels on both wealth and capital income due to such impacts. Results are therefore assessed within a context which incorporates such excluded features.

## 2. The literature

Piketty (2013) shows that the rich are getting richer in many countries. The concentration of wealth is also increasing, see Saez and Zucman (2016). A higher return on wealth for investors with higher levels of wealth contributes to this development, see Fagereng et al. (2016). At the same time, the tax on capital income has fallen in the EU because of tax competition in a globalized capital market according to Devereux et al. (2008). Only a few countries have chosen to tax wealth today. The present study examines whether such lenient taxation of capital and wealth raise tax revenue at the lowest possible cost. The study shows that taxation of wealth is less costly in terms of welfare than taxation of labor income even when the supply of labor is fixed. Hence, this finding supports the recommendation by Piketty.

Equity income from the corporate sector is subject to both corporation tax and to personal taxes on dividends and capital gains within many countries. Advocates of the 'old view' have stressed the need to relieve such double taxation to avoid distortions in corporate investments, see Harberger (1962). Advocates of the 'new view' have pointed out that retained earnings are the main source of equity finance, and that double taxation is not a serious problem when effective tax rates on capital gains on shares are modest, see King (1974). The view that personal taxes on capital does not distort corporate investments is further strengthened by international integration of capital markets. Boadway and Bruce (1992) shows that personal taxes on capital does not distort corporate investments when incentives to invest is determined by foreign

investors' required return on shares. However, a tax on corporate profit in a small open economy with a globalized capital market implies a higher rate of return requirement, which leads to lower investments and less capital per worker, which in turn lowers wages. Such a connection between taxation of corporate profit, wages and prices is largely confirmed by empirical studies, see Fuest et al. (2018) and Baker et al. (2020). A low corporate tax can both stimulate investments and reduce profit shifting to low-tax countries according to de Mooij and Ederveen (2008). If capital is perfectly mobile, capital should as mentioned not be taxed even if other countries tax capital according to Gordon (1986). Harmonization of taxes between countries does not change this result, see Razin and Sadka (1991). Note that taxation of corporate profit does not distort investment decisions when all costs are deducted from the tax base, see Sandmo (1974). Such costs include returns on both equity and debt, as well as depreciation allowances which correspond to true depreciation. Taxation distorts investment decisions when a share of the investment is financed with equity and returns on equity is non-deductible, however.

Both personal and corporate taxes on equity income can distort the required return on shares which are not traded in the international stock market, however. Sørensen (2005b) shows that shareholder taxation will be neutral with respect to investments in stocks if well diversified shareholders are granted a deduction for a risk-free rate of return, see also Sørensen (2005a). Lindhe and Södersten (2012) however argue that such shareholder taxation is likely to leave the distortions caused by the corporate income tax unaffected, and to distort shareholders' portfolio decisions. Sørensen (2022) on the other hand argues that distortions caused by the corporate income tax is approximately neutralized by such shareholder taxation when the corporate tax rate is aligned with the capital income tax rate. Hence, lenient taxation of corporate and capital income is required to minimize distortions in business investments when the globalized and non-globalized part of the economy is taxed with the same rate. The present study contributes by showing that such aligned capital income taxation might be desirable even though foreign investments are distorted. The study shows that raising tax revenue by increasing the tax rate on capital income marginally above the foreign tax level is less costly than by increasing the tax rate on labor income within a small open economy with a partially globalized capital market. Lenient tax rules which allow for profit shifting is another desirable way to tax capital and stimulate foreign direct investments (FDI) according to Hong and Smart (2010).

However, such tax differentiation between foreign and domestic investors is often prevented by international cooperation to minimize tax evasion, see Slemrod and Wilson (2009).

Piketty's policy advice is as metioned in conflict with results within the literature on taxation, see Auerbach and Hassett (2015) and Boadway and Pestieau (2019). Guvenen et al. (2019) on the other hand shows that a wealth tax of 2-3 percent provides a significant welfare gain in the United States as wealth is allocated from low-productive to high-productive investors. One may however argue that rates of returns are likely to converge as the labor effort of high-skilled investors is restricted, as investors may hire high-skilled investment advisors, and as investment projects are ultimately picked from the same pool of proposals/ ideas. The welfare gain in Guvenen et al. (2019) is eliminated in this case. The present study assumes perfect competition with equal investors who can borrow/save at a given interest rate. The study shows that taxation of wealth is less costly than taxation of labor income or capital income when the welfare weight attached to domestic investors is sufficiently small. Several theoretical studies show that a uniform wealth tax does not distort domestic investors' incentive to invest in this case, see e.g. Bjertnæs (2023), and Bjerksund and Schjelderup, 2019 and 2022. These studies support the finding in the present study.

The model framework does not consider that an increased wealth tax can lead to more rich people avoiding taxes by moving, see Agrawal et al. (2020), or evading taxes by placing wealth in tax havens, see Alstadsæter et al. (2019). However, several of those who avoid/ evade tax in this way continues their business. The model framework also excludes distortions in the allocation of consumption over time due to taxation of capital income and wealth. Chamley (1986) and Judd (1985), on the other hand, argue that the required tax revenue should be generated solely through taxing labor income in the long run, and hence that the capital income tax should be zero. Capital income taxes impose an exponentially growing tax burden on consumption. This is incompatible with standard Ramsey principles where tax distortions are smoothed out over time, see Judd (1999). Jacobs and Rusu (2018) argue that taxes on capital income are redundant for the same reasons why commodity taxes are redundant within optimized solutions. Such taxes are redundant when they cannot alleviate distortions from taxing labor income. Their optimal tax on capital income is zero even though a fixed rate of return prevents that investment are distorted. Diamond and Spinnewijn (2011), on the other hand, show that a tax on savings for high-income

households, and/or a subsidy on savings for low-income households, increases welfare when job skills and the propensity to save vary between individuals. The explanation is that such taxation leaves room for redistribution without adversely affecting individuals' choice of job. Atkinson and Sandmo (1980) show cases where tax on capital income increases welfare in overlapping generation models when generation-specific lump-sum taxes are excluded. Andersen (2020) finds that capital income should be taxed in small open economies with heterogeneous individuals and overlapping generations. Even though the points above are not included in the analytical framework, they are included in the discussion of the cost of raising tax revenue.

# 3. The model framework

A model framework is constructed to calculate the overall welfare cost of raising tax revenue by taxing capital income, wealth, and labor income within a small open economy. Foreign direct investments are undertaken by foreign corporations with a fixed after-tax rate of return requirement. Domestic investments are based on optimized investment decisions of a representative domestic investor. The economy also consists of $L$ working individuals which receives a wage, $w$, for their fixed supply of labor. The welfare cost of raising tax revenue by taxing capital income and wealth is compared with the welfare cost of non-distorting taxes on labor income to illuminate on the cost of taxing the rich.

## 3.1. Traded goods

It is assumed that patents or brands are required to produce the traded good, and that such patents and brands are owned by foreign investors/ multinationals only. These foreign investors decide where to locate their production/investments abroad or within the small open economy. Production of the traded good, $X_1$, is given by a Cobb-Douglas production function using capital, $K_1$, and labor, $L_1$, to simplify calculations.

(1) $X_1 = L_1^{\propto} K_1^{1-\propto}$

The traded good is both consumed and invested. Firms producing the traded good are price takers due to competition in the global market. It is assumed that firms operate in the interest of their owners. The fixed price, $P_1$, which is also the price of the investment good, is identical and normalized to unity in all periods. Depreciation of capital and adjustment costs are set equal to

zero to simplify calculations without losing main insights. These assumptions exclude capital gains connected with liquidation of capital. A foreign investor allocates a fixed amount, $K^*$, between the small open economy and abroad to maximize the after-tax cash flow to the investor. These amounts are invested as equity. The optimization problem of the foreign investor is

(2) $\max\limits_{K_1, L_1} P_1(K^* - K_1)(1 + r) + [(P_1 L_1^\propto K_1^{1-\propto} - wL_1)(1 - t_r) + P_1 K_1]$

The first expression within the objective function equals the cash flow to foreign investors from investments abroad. The amount invested, $P_1(K^* - K_1)$, generates a fixed after-tax return equal to the interest rate abroad, $r$. The foreign tax rate is set equal to zero to simplify the model framework. The second expression equals the after-tax cash flow to the foreign investor from investments within the small open economy. The expression equals sales revenues, $P_1 L_1^\propto K_1^{1-\propto}$, minus wage costs, $wL_1$, net of corporate taxes, $1 - t_r$, plus the value of the remaining real capital, $P_1 K_1$. Hence, the profit is taxed within the small open economy. The impact of allowing for profit shifting is discussed in a later section. First order conditions imply that

(3) $P_1(1 - \alpha)L_1^\propto K_1^{-\propto} = P_1 \dfrac{r}{(1-t_r)}$

and

(4) $P_1 \alpha L_1^{\propto-1} K_1^{1-\propto} = w.$

Equation (3) implies that the producer value of the marginal revenue product of capital equals the rate of return requirement per unit of capital, $\dfrac{r}{(1-t_r)}$. Equation (4) implies that the marginal revenue product of labor equals the wage rate, $w$. Equation (3) implies that

(5) $\dfrac{K_1}{L_1} = \left(\dfrac{r}{(1-t_r)(1-\alpha)}\right)^{-\frac{1}{\propto}}.$

Hence, the capital-labor ratio is given by the fixed rate of return on foreign investments, $r$, and the corporate tax rate, $t_r$. Implementing equation (4) into equation (3) implies that

$$(6) \quad w = \alpha \left( \frac{r}{(1-t_r)(1-\alpha)} \right)^{\frac{(\alpha-1)}{\alpha}}.$$

Equation (6) shows that the endogenous wage rate, $w$, is determined by the exogenous return on foreign investments, $r$, the corporate tax rate, $t_r$, and the traded-good price.

## 3.2. Non-traded goods

The non-traded good is sold in a perfectly competitive domestic market at a price $P_2$ by firms which are owned by a representative domestic investor. The domestic investor is the sole owner of such firms because of a lower pretax rate of return requirement than foreign investors. Production of the non-traded good, $X_2$, is given by a Cobb-Douglas production function using capital, $K_2$, and labor, $L_2$.

$$(7) \quad X_2 = L_2^\alpha K_2^{1-\alpha}$$

The fixed initial wealth of the domestic investor, $M_0$, is invested in bonds, $B_0$, at a price identical to the price og the traded good, and in shares, $K_2$, at a price identical to the traded good. Investments in bonds could be interpreted as bank deposits.

$$(8) \quad M_0 = B_0 + K_2$$

The equity of non-traded goods firms, which equals the investment in shares, are invested as real capital, $K_2$. The share value of the company equals the value of the invested capital due to free entry. The after-tax wealth one period later in the case where the investor does not consume any of the wealth, $M_1$, equals.

$$(9) \quad M_1 = B_0[1 + r(1 - t_r)][1 - t_k] + [(P_2 L_2^\alpha K_2^{1-\alpha} - wL_2)(1 - t_r) + K_2][1 - t_k]$$

The first expression on the right-hand side of equation (9), $B_0[1 + r(1 - t_r)][1 - t_k]$, equals the after-tax wealth from investments in bonds. The interest rate net of capital income tax amounts to $r(1 - t_r)$. The wealth net of capital income tax is multiplied with one minus the wealth tax, $(1 - t_k)$, to obtain the after-tax wealth. The second expression on the right-hand side of

equation (9) consists of after-tax profits, $(P_2 L_2^{\propto} K_2^{1-\propto} - wL_2)(1 - t_r)$ and revenues from liquidation of the capital, $K_2$. This expression is multiplied by one minus the wealth tax rate.

It is assumed that a fixed amount, $M_0$, of the after-tax wealth the following period, $M_1$, is invested. The remaining after-tax wealth is consumed, i.e. the after-tax return is consumed. Assuming a constant taxable wealth each future period is consistent with empirical studies of the Norwegian wealth tax, see Bjørkli and Arntsen (2021) and Ring (2020). The investor is assumed to maximize the utility derived from consuming the after-tax return on wealth invested. Hence, the investor is maximizing consumption possibilities with respect to investments in stocks and bonds. Consumption possibilities, $M_c$, are given as

$$(10) M_c = B_0\big(1 + r(1 - t_r)\big)(1 - t_k) + [(P_2 L_2^{\propto} K_2^{1-\propto} - wL_2)(1 - t_r) + K_2](1 - t_k) + B_0 r(1 - t_r)t_k + (P_2 L_2^{\propto} K_2^{1-\propto} - wL_2)(1 - t_r)t_k - M_0.$$

The consumption possibility equals the after-tax wealth one period later in the case where the investor does not consume any of the wealth, $M_1$, minus the fixed amount invested the following period, $M_0$, plus the reduction in wealth tax payments as some of the wealth is consumed, $B_0 r(1 - t_r)t_k$ plus $(P_2 L_2^{\propto} K_2^{1-\propto} - wL_2)(1 - t_r)t_k$. Note that such wealth tax payments are incorporated into the expression for the after-tax wealth one period later, $M_1$. Equation (8) into (10) implies that

$$(11) M_c = B_0\big(1 + r(1 - t_r)\big) + [(P_2 L_2^{\propto} K_2^{1-\propto} - wL_2)(1 - t_r) + K_2] - (1 + t_k)M_0.$$

Maximizing consumption possibilities, equation (11), with respect to labor effort, $L_2$, and investments in stocks, $K_2$, and bonds, $B_0$, and taking account of equation (8), leads to the following first order conditions,

$$(12) P_2(1 - \alpha)L_2^{\propto} K_2^{-\propto} = r$$

and

$$(13) P_2 \alpha L_2^{\propto - 1} K_2^{1-\propto} = w.$$

Equation (12) implies that the marginal revenue product of capital equals the rate of return requirement per unit of capital, $r$, which is fixed. Equation (13) implies that the marginal revenue product of labor equals the wage rate, $w$. Taxation of capital income and wealth do not affect the rate of return requirement for domestic investors. The explanation is that investments in stocks, and hence, real capital are taxed with the same rates as investments in financial capital. Hence, the tax burden is unchanged when investments are reallocated towards real capital[1]. Equation (12) and (13) implies that

$$(14)\frac{K_2}{L_2} = \frac{(1-\propto)}{\propto}\frac{w}{r}.$$

A given $w$ and $r$ determines the ratio between input factors, $\frac{K_2}{L_2}$. Equation (12) and (13) also implies that

$$(15)P_2 = \frac{r}{(1-\propto)}\left(\frac{(1-\propto)}{\propto}\frac{w}{r}\right)^{\propto}.$$

Hence, the endogenous price of the domestic good, $P_2$, is determined by the exogenous price on traded goods, the interest rate, $r$, and the wage rate, $w$, which was determined by the return on foreign investments, $r$, the corporate tax rate, $t_r$, and the traded-good price.

Implementing the first order conditions, equation (12) and (13), and equation (8) into equation (10) implies that

$$(16)M_c = M_0(r(1 - t_r) - t_k).$$

Equation (16) shows that wealth tax payments amount to $M_0 t_k$, and that capital income tax payments amount to $M_0 r t_r$. Hence, the allocation of stocks and bonds does not matter for the consumption possibilities the following period as the rate of return is identical for these investments.

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[1] Maximizing the after-tax wealth, $M_1$, with respect to stocks and bonds leads to the same rate of return requirement.

### 3.3. Consumers

The economy consists of $L$ working individuals which receives a wage, $w$, for their fixed supply of labor. Their after-tax wage, $w(1 - t_l)$, is consumed. The representative investor does not work. The after-tax return on his/her investments is consumed. The utility of both the domestic investor and workers are given by

$$(17) U = C_1{}^a C_2{}^{1-a}.$$

Each worker's consumption of the traded and the non-traded good are denoted $C_1$ and $C_2$, respectively. The investor's consumption of the traded and the non-traded good are denoted $C_1^I$ and $C_2^I$, respectively. The budget constraint for each worker is given by

$$(18) w(1 - t_l) = C_1 + P_2 C_2.$$

The budget constraint of the domestic investor is given by

$$(19) M_c = C_1^I + P_2 C_2^I.$$

Each worker maximize utility, equation (17), given the budget constraint, equation (18). First order conditions imply that

$$(20) C_1 = w(1 - t_l) a$$

And

$$(21) C_2 = \frac{w(1 - t_l)(1 - a)}{P_2}.$$

The investor is maximizing utility, equation (17), given the budget constraint, equation (19). First order conditions imply that

$$(22) C_1^I = M_c a$$

And

$$(23) C_2^I = \frac{M_c(1-a)}{P_2}.$$

Indirect utility for each worker, $v$, is found by inserting equation (20) and (21) into equation (17).

$$(24) v = (w(1-t_l)a)^a \left(\frac{w(1-t_l)(1-a)}{P_2}\right)^{1-a}$$

Indirect utility for the investor, $v_I$, is found by inserting equation (22) and (23) into equation (17).

$$(25) v_I = (M_c a)^a \left(\frac{(M_c)(1-a)}{P_2}\right)^{1-a}$$

## 3.4. Market equilibrium

It is assumed that supply equals demand within each market. There is no government demand for the non-traded good and for labor. This approach is chosen to focus on the cost of raising tax revenue due to taxation of capital income and wealth. The supply of the non-traded good equals demand, i.e.

$$(26) X_2 = LC_2 + C_2^I.$$

The supply of labor equals demand for labor, i.e.

$$(27) L = L_1 + L_2.$$

The equilibrium condition for the traded-good is not required to calculate the welfare cost of raising tax revenue. Hence, government consumption of the traded-good is not included as a variable within the model framework. A balanced government budget is required for the economy to be in a steady state solution, however. This condition is satisfied if tax revenue generated is spent on public consumption of the traded-good.

## 3.5. The welfare function

The social welfare function, W, is given by the sum of utility for workers plus the utility of the representative investor multiplied with a welfare weight, $\gamma$.

$$(28) W = L(w(1-t_l)a)^a \left(\frac{w(1-t_l)(1-a)}{P_2}\right)^{1-a} + \gamma(M_c a)^a \left(\frac{(M_c)(1-a)}{P_2}\right)^{1-a}$$

The welfare gain of public spending is excluded from the model framework to focus on the cost of raising tax revenue. The following assumptions are implemented to simplify calculations: $\alpha = a = 0.5$. Assumptions regarding production and utility functions simplify calculations without losing main insights. Appendix A shows that these assumptions imply that the welfare function is given as

$$(29) W = L \left(\frac{0.125(1-t_r)(1-t_l)}{r}\right)^{0.5} \left(\frac{0.125(1-t_r)^{0.5}(1-t_l)}{r}\right)^{0.5} + \gamma\left(0.5M_0(r(1-t_r)-t_k)\right)^{0.5} \left(\frac{0.5M_0(r(1-t_r)-t_k)}{(1-t_r)^{0.5}}\right)^{0.5}.$$

## 3.6. The government tax revenue

Total tax revenue paid to the government, *T*, is given by

$$(30) T = t_l wL + t_r rM_0 + t_r \frac{r}{(1-t_r)}K_1 + t_k M_0.$$

The first term on the right-hand side of equation (30), $t_l wL$, is the tax paid on labor income. The second term, $t_r rM_0$, is the capital income tax paid by the domestic investor. The third term, $t_r \frac{r}{(1-t_r)}K_1$, equals corporate taxes paid by firms owned by foreign investors. The fourth term, $t_k M_0$, is the wealth tax paid by the domestic investor. Appendix B shows that equation (30) can be transformed to equation (31).

$$(31) T = t_l \frac{0.25(1-t_r)L}{r} + t_r rM_0 + t_r \frac{0.25(1-t_r)L}{r} - t_r \frac{0.0625(1-t_r)L(1-t_l)}{r} - t_r 0.25M_0(r(1-t_r)-t_k) + t_k M_0$$

# 4. The welfare cost of raising tax revenue

The welfare cost of raising tax revenue is found by calculating the welfare cost of a marginal tax increase divided by the increase in tax revenue due to the marginal tax increase. The welfare cost of a marginal increase in the labor income tax rate is found by taking the derivative of $W$, given by equation (29), with respect to the labor income tax rate.

$$(32) \frac{\partial W}{\partial t_l} = -L0.5 \left( \frac{0.125(1-t_r)(1-t_l)}{r} \right)^{-0.5} \left( \frac{0.125(1-t_r)^{0.5}(1-t_l)}{r} \right)^{0.5} \frac{0.125(1-t_r)}{r} -$$
$$L0.5 \left( \frac{0.125(1-t_r)(1-t_l)}{r} \right)^{0.5} \left( \frac{0.125(1-t_r)^{0.5}(1-t_l)}{r} \right)^{-0.5} \frac{0.125(1-t_r)^{0.5}}{r}$$

The change in tax revenue due to a marginal increase in the labor income tax rate is found by taking the derivative of $T$, given by equation (31), with respect to the labor income tax rate.

$$(33) \frac{\partial T}{\partial t_l} = \frac{0.25(1-t_r)L}{r} + \frac{0.0625 t_r L(1-t_r)}{r}$$

It is assumed that $t_r = 0$ to evaluate the welfare impacts when the capital income tax rate is zero. The domestic corporate tax rate equals the foreign tax rate in this case. Hence, this assumption is appropriate for countries with identical corporate tax levels. The assumption implies that

$$(34) \frac{\frac{\partial W}{\partial t_l}}{\frac{\partial T}{\partial t_l}} = -0.5.$$

The marginal utility of income for workers equals 0.5 when $t_r = 0$. Hence, a welfare increase of 0.5 measured in monetary units amounts to one. The welfare cost of raising tax revenue with the labor income tax measured in monetary units therefore equals one in this case. The explanation is that the tax increase on labor income does not alter the pre-tax wage rate or the price on the non-traded good as the rate of return requirement of the foreign investor is not affected. The tax increase on labor income consequently lowers the after-tax income of workers. The labor income tax collects tax revenue from workers without distorting the economy as the supply of labor is fixed. Hence, the cost of raising tax revenue with the labor income tax equals one. Tax revenue

generated is spent on government consumption of the traded-good. Such consumption is as mentioned not included as a variable within the model framework.

The welfare cost of a marginal increase in the wealth tax rate is found by taking the derivative of $W$, given by equation (29), with respect to the wealth tax rate.

$$(35) \frac{\partial W}{\partial t_k} = \gamma \left[ -0.5 \big(0.5 M_0 (r(1 - t_r) - t_k)\big)^{-0.5} \left( \frac{0.5 M_0 (r(1-t_r)-t_k)}{(1-t_r)^{0,5}} \right)^{0.5} M_0 0.5 - 0.5 \big(0.5 M_0 (r(1 - t_r) - t_k)\big)^{0.5} \left( \frac{0.5 M_0 (r(1-t_r)-t_k)}{(1-t_r)^{0.5}} \right)^{-0.5} \frac{M_0 0.5}{(1-t_r)^{0.5}} \right]$$

The change in tax revenue due to a marginal increase in the wealth tax rate is found by taking the derivative of $T$, given by equation (31), with respect to the wealth tax rate.

$$(36) \frac{\partial T}{\partial t_k} = M_0 + 0.25 t_r M_0$$

Assuming that $t_r = 0$ implies that

$$(37) \frac{\frac{\partial W}{\partial t_k}}{\frac{\partial T}{\partial t_k}} = -\gamma 0.5.$$

The marginal utility of income for investors equals 0.5 when $t_r = 0$. Hence, the welfare cost of raising tax revenue with the wealth tax measured in monetary units equals the welfare weight of the investor, $\gamma$, in this case. The explanation is that an increase in the wealth tax collects tax revenue from the domestic investor without distorting the economy. Domestic investment incentives are not distorted as the wealth tax is levied on all types on wealth. FDI is not distorted as the rate of return requirement of foreign investors is not affected. An unchanged rate of return requirement for foreign investors implies that the wage rate and the price of the non-traded good is unaffected. Hence, the cost of raising tax revenue by increasing the wealth tax equals the welfare weight attached to the utility of the domestic investor.

It follows from equation (34) and (37) that the cost of raising tax revenue with the wealth tax is lower than the cost of collecting tax revenue with the labor income tax if the welfare weight

attached to the utility of the investor is below one, i.e. below the welfare weight attached to the utility of workers. Equation (37) also uncover that the welfare cost of raising tax revenue with the wealth tax equals zero if the welfare weight attached to investors equals zero. Jacobs et al. (2017) find small and even negative welfare weights attached to the rich in the Netherlands. This is also found in France, see Bourguignon and Spadaro (2012). UK and Irish social welfare weights are on the other hand not much lower for top incomes than for average income according to Bargain and Keane (2010). Negative externalities from conspicuous consumption of wealthy domestic investors as well as rivalry for status justify modest welfare weights for such investors, however.

The welfare cost of a marginal capital income tax increase is found by taking the derivative of $W$, given by equation (29), with respect to the capital income tax rate.

$$(38)\frac{\partial W}{\partial t_r} = -L0.5\left(\frac{0.125(1-t_r)(1-t_l)}{r}\right)^{-0.5}\left(\frac{0.125(1-t_r)^{0.5}(1-t_l)}{r}\right)^{0.5}\frac{0.125(1-t_l)}{r} -$$
$$L0.5\left(\frac{0.125(1-t_r)(1-t_l)}{r}\right)^{0.5}\left(\frac{0.125(1-t_r)^{0.5}(1-t_l)}{r}\right)^{-0.5}\frac{0.125(1-t_r)^{-0.5}0.5(1-t_l)}{r} + \gamma\left[-0.5\big(0.5M_0(r(1-t_r)-\right.$$
$$t_k)\big)^{-0.5}\left(\frac{0.5M_0(r(1-t_r)-t_k)}{(1-t_r)^{0.5}}\right)^{0.5}M_0r0.5 + 0.5\big(0.5M_0(r(1-t_r)-$$
$$\left.t_k)\big)^{0.5}\left(\frac{0.5M_0(r(1-t_r)-t_k)}{(1-t_r)^{0.5}}\right)^{-0.5}\left[\frac{-M_00.5r(1-t_r)^{0.5}+0.5(1-t_r)^{-0.5}M_00.5(r(1-t_r)-t_k)}{(1-t_r)}\right]\right]$$

The marginal change in tax revenue due to a marginal increase in the capital income tax rate is found by taking the derivative of $T$ in equation (31) with respect to the capital income tax rate.

$$(39)\frac{\partial T}{\partial t_r} = \frac{-0.25t_lL}{r} + rM_0 + \frac{0.25(1-t_r)L}{r} - \frac{0.25t_rL}{r} - \frac{0.0625(1-t_r)L(1-t_l)}{r} + \frac{0.0625t_rL(1-t_l)}{r} - 0.25M_0(r(1-t_r)-$$
$$t_k) + 0.25t_rM_0r$$

Assuming that $t_r = 0$ to evaluate the welfare impacts when the initial capital income tax rate equals zero, i.e. when the corporate tax rate equals the foreign tax rate, implies that

$$(40)\frac{\frac{\partial W}{\partial t_r}}{\frac{\partial T}{\partial t_r}} = \frac{-0.5\left[\frac{L(1-t_l)}{r}+\gamma M_0\left(4r+\frac{t_k}{0.75}\right)\right]}{\left[\frac{L(1-t_l)}{r}+M_0\left(4r+\frac{t_k}{0.75}\right)\right]}.$$

Equation (40) shows that the welfare cost of raising tax revenue by implementing a marginal tax on capital income is approaching -0.5 if the wealth of the domestic investor, $M_0$, is approaching

zero. Hence, the cost of raising tax revenue by introducing a marginal tax on capital income is approaching the cost of raising tax revenue by increasing the tax on labor income in this case. This result is consistent with the view that small open economies should not tax capital income. The explanation is that the marginal capital income tax collects tax revenue from foreign investors in the traded goods sector. The rate of return requirement for foreign investors is consequently increased, and FDI is reduced. The wage rate is reduced as the rate of return requirement is increased. The price of the non-traded good is reduced due to the reduction in the wage rate. The utility of workers is reduced even though the price of the non-traded good is reduced. Hence, the capital income tax collects tax revenue by distorting FDI, and the tax burden is shifted on to workers. The cost of raising tax revenue by a marginal tax increase on capital income, given that the initial capital income tax rate equals zero, is approaching the cost of raising tax revenue by increasing the tax on labor income in this case.

The wealth of domestic investors is typically positive and sizable, however. Equation (40) shows that the welfare cost of raising tax revenue by introducing a marginal tax on capital income is approaching zero if the welfare weight attached to the utility of the domestic investor is approaching zero and the wealth of the domestic investor is sufficiently large. The cost of raising tax revenue by increasing the tax on labor income exceeds the cost of raising tax revenue by introducing a marginal tax on capital income in this case. Hence, this result contradicts the view that taxing labor income is less costly than taxing capital income within small open economies. The explanation is that the capital income tax collects tax revenue from domestic investors without distorting investments. Domestic investments are not distorted as the tax is levied on the return on all types of investments. Hence, a zero welfare weight attached to the utility of the domestic investor combined with a sufficiently large level of wealth implies that the cost of collecting tax revenue is approaching zero.

## 5. Discussion

Results above show that a sufficiently small welfare weight attached to domestic investors implies that raising tax revenue by taxing wealth is less costly than by increasing the tax rate on capital income marginally above the foreign tax level, and that raising tax revenue by increasing the tax rate on capital income marginally above the foreign tax level is less costly than by increasing the tax rate on labor income. These results hold within a simplified model framework

designed for modest levels of taxes on capital income and wealth. The welfare cost of taxing both capital income and wealth is however likely to increase with the level of these taxes due to distortions that are not incorporated into the model framework of the present study. Empirical studies shed light on tax levels which trigger such distortions, and hence, facilitates a discussion of the design of the tax system.

The point of departure for the discussion is that labor income taxation is part of a welfare maximizing tax system. This approach is chosen mainly because the literature shows that non-linear taxation of labor income is part of a welfare maximizing tax system, and because tax revenue generated by taxation of capital is way below levels required to finance government spending within developed countries. This point of departure implies that the welfare cost of raising tax revenue by a marginal increase in the tax rate on capital income and wealth should be equal to the welfare cost of raising tax revenue with a marginal increase in the tax rate on labor income.

Empirical studies find that the Norwegian wealth tax, which was below one percent until 2022 but which differs between types of capital, had a modest impact on taxable wealth, see Bjørkli and Arntsen (2021) and Thoresen et al. (2022), or even increased taxable wealth, see Ring (2020). Empirical studies of the wealth tax in Norway also reveal marginal positive effects on employment in companies with liquid owners, while the effects are negative in small companies with illiquid owners, see Bjørneby et al. (2022) and Berzins et al. (2019). Several wealthy investors moved abroad in 2022 however when taxation of dividend and wealth increased slightly. Many of these investors continued their business from abroad, however. Hence, loss of tax revenue seems to be the main adverse impact at these tax levels. Assuming that loss of tax revenue due to avoidance is the only adverse impact of the wealth tax, and that the welfare weight of domestic investors equals zero, implies that the wealth tax should be increased to a level where tax revenue generated is maximized, i.e. where the increase in tax revenue due to a marginal increase in the wealth tax is approaching zero. Why? Because the welfare cost of a marginal increase in the wealth tax equals zero in this case. Hence, this wealth tax level is required to implement a solution where the cost of raising tax revenue by increasing the tax on wealth equals the cost of raising tax revenue by increasing the tax on labor income. One may argue that the magnitude of such tax avoidance is determined by the total tax burden on domestic

investors, and that an exit-tax may alleviate such avoidance. Results within the present study shows that a switch from capital income taxation to taxation of wealth is desirable in this case. Why? Because welfare increases as the cost of raising tax revenue is reduced. One may also argue that an inheritance tax collects tax revenue from domestic investors without distorting investments, and that distortions in the allocation of consumption over time is more modest than with a wealth tax. An inheritance tax may trigger other forms of avoidance, however.  The inheritance tax should be increased to a level where the cost of raising more tax revenue equals the cost of raising more tax revenue with the labor income tax in this case.

Raising tax revenue by taxing wealth is less costly than raising tax revenue by increasing the tax rate on capital income marginally above the foreign tax level according to results above. The cost of taxing capital income is exaggerated however as important welfare gains are excluded from the calculations. A source-based corporate tax is justified by the occurrence of pure profit and foreign ownership, see Huizinga and Nielsen (1997), by providing incentives for human capital investments, see Nilsen and Sørensen (1997), by a desire to prevent tax avoidance by reporting labor income as capital income, see Gordon and Mac Kie-Mason (1994), as well as by a desire to redistribute among generations. Hence, arguments presented above suggest that the cost of raising tax revenue by taxing capital income is below the cost of raising tax revenue by taxing wealth. One may also argue that taxation of capital income combined with lenient rules which allow for profit shifting collects tax revenue from domestic investors without distorting investments by foreign or domestic investors. Impacts of taxing capital income resembles impacts of taxing wealth within the model framework of the present study in this case. Rules which allow for such profit shifting is often prevented by international cooperation to minimize tax evasion, however. Distortions in the allocation of consumption over time is also excluded from the model framework. Hence, the welfare cost of raising tax revenue by taxing capital income will increases with the level of the tax rate due to such distortions.

Extremely high tax rates on wealth and capital income combined with inflation and a low rate of return imply that private investors gradually lose their entire wealth. Confiscation of wealth translates to a weakening of private property rights. Both Acemoglu and Robinson (2012) and Hall and Jones (1999) argue that market economies with strong protection of private property rights have realized prosperity for nations, while nations with weak property rights have failed.

Preventing political power and resources from ending up within a limited elite is also important for societies to succeed, according to Acemoglu and Robinson (2012). An attempt to solve this problem with an extremely high wealth tax can be destructive to society, however. Hence, the welfare cost of raising tax revenue with extremely high tax rates on wealth and capital income are extremely large according to this argument.

# 6. Conclusion

This study calculates the welfare cost of raising tax revenue by taxing capital income, wealth, and labor income within a simplified model framework designed for modest levels of taxes on capital income and wealth in a small open economy with a partially globalized capital market. The study shows that taxation of both capital income and wealth is less costly in terms of welfare than taxation of labor income within this model framework when the welfare weight attached to wealthy domestic investors is sufficiently small.

The model framework adopted excludes tax avoidance as wealthy investors move abroad, or tax evasion as wealth is transferred to tax havens. Impacts of weaker private property rights are also excluded. An assessment of these results together results in other empirical and theoretical studies uncover that the cost of taxing capital income and wealth is likely to increase with the level of these taxes. The case in favor of taxing wealth is also strengthened by the assessment, however. Implementing substantial taxes on capital income and wealth is however likely to harm the economy as taxes are avoided and private property rights are weakened.

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# Appendix A.

Assuming that $\alpha = a = 0.5$ together with equation (6) implies that

(a1) $w = \frac{0.25(1-t_r)}{r}$.

These assumptions together with equation (a1) and (15) implies that

(a2) $P_2 = (1 - t_r)^{0.5}$.

Hence, these assumptions together with equation (a1), (a2), (16) and the welfare function given by equation (28) implies that the welfare is given as

(a3) $W = L \left( \frac{0.125(1-t_r)(1-t_l)}{r} \right)^{0.5} \left( \frac{0.125(1-t_r)^{0.5}(1-t_l)}{r} \right)^{0.5} + \gamma \left( 0.5 M_0 (r(1 - t_r) - t_k) \right)^{0.5} \left( \frac{0.5 M_0 (r(1-t_r) - t_k)}{(1-t_r)^{0.5}} \right)^{0.5}$.

# Appendix B.

Equation (7) and (26) implies that

(a4) $L_2^\alpha K_2^{1-\alpha} = LC_2 + C_2^I$

Equation (14) into (a4) implies that

(a5) $L_2 = \dfrac{LC_2 + C_2^I}{\left(\frac{(1-\alpha)}{\alpha}\frac{w}{rP_1}\right)^{1-\alpha}}$

Equation (21), (23) and (15) into (a5) implies that

(a6) $L_2 = L \propto (1 - t_l)(1 - a) + \dfrac{\propto(M_c)(1-a)}{w}$

Equation (5) and (27) implies that

(a7) $K_1 = \left(\dfrac{r}{(1-t_r)(1-\alpha)}\right)^{-\frac{1}{\alpha}}(L - L_2)$

Equation (a1), (a6) and (a7), and $\alpha = a = 0.5$ implies that

(a8) $t_r \dfrac{r}{(1-t_r)} K_1 = \dfrac{0.25 t_r(1-t_r)}{r}\left[L - 0.25L(1 - t_l) - \dfrac{(M_c)r}{(1-t_r)}\right]$

Equation (a1), (a8), (16) and (30) implies that

(a9) $T = t_l \dfrac{0.25(1-t_r)L}{r} + t_r r M_0 + t_r \dfrac{0.25(1-t_r)L}{r} - t_r \dfrac{0.0625(1-t_r)L(1-t_l)}{r} - t_r 0.25 M_0(r(1 - t_r) - t_k) + t_k M_0$