ARTIKLER 108

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ESTIMATED FROM HOUSEHOLD BUDGET DATA FROM THE YEARS 1967 AND 1973

By Erik Biørn

SAMMENLIKNING AV KONSUMUTGIFTSFUNKSJONER

ESTIMERT PÅ GRUNNLAG AV HUSHOLDNINGSDATA FRA ÅRENE 1967 OG 1973

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PREFACE

This paper is concerned with comparing consumer expenditure functions estimated from household budget data from two different years. The data base is the Surveys of Consumer Expenditure of 1967 and 1973 carried out by the Central Bureau of Statistics. Some methodological problems are also discussed.

The analysis was initiated when working with updating and reestimation of one of the tax incidence models of the Bureau, but has also interest in a wider context.

Central Bureau of Statistics, Oslo, 16 June 1978

Petter Jakob Bjerve

FORORD

I denne artikkelen sammenlignes konsumutgiftsfunksjoner (Engelfunksjoner) estimert på grunnlag av data fra Statistisk Sentralbyrås forbruksundersøkelser for årene 1967 og 1973. Noen metodeproblemer diskuteres også.

Analysen har sitt utspring i arbeidet med å tallfeste og oppdatere konsumkoeffisienter i en av Byråets skatteinsidensmodeller, men har også interesse i en større sammenheng.

Statistisk Sentralbyrå, Oslo, 16. juni 1978

Petter Jakob Bjerve

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INNHOLD

1. Introduction

Useful tools in analysing consumption data at the micro (household) level are expenditure functions (Engel functions), describing in a condensed form the dependence of the composition of consumption on income (total consumption expenditure), the number of household members, and various demographic and socio-economic background variables. As prices are usually excluded from the list of explanatory variables, the data most often used to obtain inference on consumer expenditure functions are household reports collected during a fairly short period, e.g., one year.

Estimation of expenditure functions may be of interest by itself, since it represents a convenient data reduction. More important, such functions are building blocks in several econometric models. For instance, a model developed in the Central Bureau of Statistics for analysing the incidence of simultaneous changes in direct and indirect taxes, has a complete set of estimated consumer expenditure functions as one of its main components. (A formal presentation of the model is given in Biørn and Garaas [3].)

The present study was initiated when working with updating and reestimation of this model. The methods and results, however, also seem interesting in a wider perspective. The basic problem may be formulated in the following way: Suppose there exist two complete sets of expenditure functions: one estimated from data collected in the year 0, the other based on data from the year 1. (The word 'complete' indicates that the functions cover all consumption commodities.) These functions might be regarded as two entirely separated equation systems. However, economic theory tells us that both data sets should be considered as generated by one common system of consumer demand functions with prices as specified arguments. This raises the following general question: How should we rationally compare the two sets of estimated expenditure functions in order to detect similarities and differences in the structure of consumption in the two years?

The data base of the study is two Norwegian household budget surveys, relating to the years 1967 and 1973 respectively. The theoretical model, including its stochastic properties, is set out in chapter 2. Chapter 3 contains a brief description of the data and some remarks on the problem of estimation. Empirical results are given in chapter 4; the two systems of estimated expenditure functions are compared in four different ways. Finally, chapter 5 briefly summarizes the findings.

2. Theoretical background

Our point of departure is the static theory of consumer demand. With reference to this theory, we shall briefly state some general properties of a complete set of expenditure functions (section 2.1), and give some remarks on its stochastic structure (section 2.2). Then we shall discuss the parametric specification to be used in the paper (section 2.3).

2.1. The expenditure functions: general properties

Assume that the consumption goods are divided into N groups, and let x_i denote the quantity consumed of the i'th group, p_i its price (index) and y total consumption expenditure,

$$(1) \quad \sum_{i=1}^{N} p_i x_i = y.$$

The unit of analysis is the household. The demand function of the i'th good can be written as

$$(2) \quad x_i = f_i(y, p_1, \ldots, p_N) + v_i \quad (i = 1, \ldots, N),$$

where v_i is a stochastic disturbance, and $f_1, \ldots, f_N$ are functions supposed to satisfy the "adding-up condition"

$$(3) \quad \sum_{i=1}^{N} p_i f_i(y, p_1, \ldots, p_N) = y,$$

and the "homogeneity conditions"

$$(4) \quad f_i(\lambda y, \lambda p_1, \ldots, \lambda p_N) = f_i(y, p_1, \ldots, p_n) \quad (i = 1, \ldots, N)$$

for all admissible values of $p_1, \ldots, p_N, y$, and λ.

As is well-known, conditions (3) and (4) are ingredients of the orthodox theory of consumer demand - the static theory of the utility-maximising consumer who takes prices and total consumption expenditure as exogenously given. For reasons given in section 2.3, we shall, however, neglect the conditions of symmetry of the Slutsky substitution matrix which are also elements in this theory.

From (1) - (3) we obtain

(5) $\Sigma p_i v_i = 0.$

When all prices are constants, the expenditure allocated to the i'th good,

(6) $c_i = p_i x_i,$

is a function of y with disturbance $p_i v_i$. If in particular p_i takes the value p_i^T $(i = 1,\ldots,N)$, then

(7) $c_i = F_i^T(y) + u_i^T \quad (i=1,\ldots,N),$

where

(8) $F_i^T(y) = p_i^T f_i(y, p_1^T,\ldots, p_N^T)$

is the expenditure function of the i'th good corresponding to the price vector $(p_1^T,\ldots,p_N^T)$, and

(9) $u_i^T = p_i^T v_i$

is its disturbance. Owing to (4), $F_i^T(y)$ is homogeneous of degree one in total expenditure and prices. Moreover, in view of (3) and (5), we have

(10) $\sum_i F_i^T(y) = y$ (identically in y),

(11) $\sum_i u_i^T = 0.$

2.2. Remarks on the structure of the disturbances

Suppose there exist observations on $c_1,\ldots, c_N$, and $y(=\Sigma c_i)$ from two samples of households, collected in period no. 0 (the year 1967), and no. 1 (the year 1973) respectively. We assume that all households observed in period no. T have been confronted with the same price vector $(p_1^T,\ldots, p_N^T)$ $(T = 0, 1)$. Possible price variations within the two periods are assumed to be random (relative to the sample variation of total expenditure); thus they may be absorbed by the disturbances.

Letting the subscript t indicate the number of the household report (in each sample), we make the following assumptions:

(12) $E(u_{it}^{T}) = 0$ for all i and t; and T = 0, 1,

(13) $E(u_{it}^{T}u_{js}^{T}) = \delta_{ts}\sigma_{ij}^{T}$ for all i, j, t and s; and T = 0, 1,

where $\delta_{ts} = 1$ for s = t, and 0 otherwise, and

(14) $E(u_{it}^{0}u_{js}^{1}) = 0$ for all i, j, t and s.

We consider y and the p's as non-stochastic variables. From (11) follows

(15) $\sum_{i}\sigma_{ij}^{T} = 0$ (j = 1,..., N; T = 0, 1),

i.e., the covariance matrix (σ_{ij}^{T}) is singular. The assumption of no correlation between disturbances relating to different household reports from the same period (cf. (13)) seems reasonable in view of the technique of random sampling used. Zero correlation between disturbances from different periods (cf. (14)) is clearly realistic provided that no household is included in both samples. If, on the other hand, the samples have at least one household in common, and if the disturbances contain "household specific" components, this assumption is violated. However, the probability that some households are selected twice is negligible, as each sample includes less than 0.4 per cent of the population of Norwegian households.[1)]

One implication of the above assumptions is worth noting: From (9) and (13) we obtain

(16) $E(v_i v_j) = \sigma_{ij}^{T}/(p_i^{T}p_j^{T})$ (i, j = 1,..., N; T = 0, 1).

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1) Stochastic specification of disturbances when using repeated sampling with partly overlapping samples is discussed in some detail in Biørn [2].

Combining (15) and (16) we observe that the covariance matrix of the v's cannot, in general, be independent of the value of the price vector. Only in the particular (and uninteresting) case where all prices change in the same proportion from period 0 to period 1, this possibility exists.[2)]

On the other hand, restricting the covariance matrix of the disturbances u_{it}^{T} to be the same for all observation points, i.e. $\sigma_{ij}^{1} = \sigma_{ij}^{0}$ for all i and j, would not be in conflict with our specification. Alternatively, the covariance matrix (σ_{ij}^{T}) might be assumed to be independent of the price vector, but depending in a prescribed way on total expenditure. For instance, replacing (13) by $E(u_{it}^{T}u_{js}^{T}) = \delta_{ts}y_t^2\mu_{ij}$ for T = 0, 1, where $\Sigma_i\mu_{ij} = 0$ for all j, would imply homoscedasticity of the disturbances of the expenditure functions when transforming to budget shares.[3)] In this paper, we shall, however, stick to the general specification (13).

2.3. Parametric specification of the expenditure functions

The problem of specifying the expenditure functions parametrically, may be approached in two essentially different ways: (i) specifying a complete system of demand functions f_i which satisfy the adding-up condition and the conditions of homogeneity and symmetry of the Slutsky substitution matrix, and deriving the corresponding expenditure functions; and (ii) choosing the parametric form of the expenditure functions "directly", without requiring "exact" consistency with maximisation of a parametrically specified utility function.

The first approach, usually preferred by theorists, has the drawback that it is not easy to find functional forms that satisfy all the theoretical restrictions and are empirically flexible at the same time. For instance, expenditure functions that are linear in total expenditure are theoretically acceptable, as they may be derived from e.g. the Stone-Geary or the quadratic utility functions.[4)] On the other hand, strict linearity is not supported by cross section data; almost universally this hypothesis, if tested, is rejected in favour of more flexible specifications, e.g. polynomials of higher order.

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2) We suppose that the matrix (σ_{ij}^{0}) has rank N-1.
3) For a further elaboration of this point, see Biørn [1], pp. 1-8.
4) Gorman [12] and Somermeyer [16] have studied the class of utility functions which imply linear expenditure functions. Pollak [15] has delimited the class of additive utility functions with this property.

For these reasons, our approach will be the second one. More specifically, we approximate the "true" expenditure functions F_i^T by cubics in total expenditure,

$$(17) \quad F_i^T(y) = \alpha_i^T + \beta_i^T y + \gamma_i^T y^2 + \delta_i^T y^3 + w_i^T \qquad (i = 1,\ldots,N),$$

where w_i^T is a stochastic disturbance taking care of errors in approximation. This specification is flexible, as cubics may show one minimum, one maximum, and one point of inflection. On the other hand, expenditure functions of this form do not conform to constrained utility maximisation.[5)6)]

From (10) and (17) we get

$$(18) \qquad \sum_i (\alpha_i^T + \beta_i^T y + \gamma_i^T y^2 + \delta_i^T y^3 + w_i^T) = y \quad \text{(identically)}.$$

We assume that

$$(19) \qquad \sum_i w_{it}^T = 0, \quad \text{for all } t.$$

This restriction, however, cannot, in contrast to (11), be justified from theoretical considerations only: Even if the "true" expenditure functions $F_i^T(y)$ add to y identically, it is by no means obvious that their approximations should meet a similar constraint. When we deliberately desist from having all the restrictions of the demand theory satisfied simultaneously, it may be questioned whether we should insist on keeping the adding-up condition. This is the main reason why we distinguish conceptually between the u's and the w's.

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5) The latter statement is the author's conjecture. He has never seen this, nor the contrary, rigorously proved.
6) Nasse [14], and Carlevaro [6] have proposed recently generalizations of the Stone LES model which relax the restriction of linearity of the expenditure functions while retaining all the utility-theoretical constraints. The resulting parametrizations are rather complicated, however, and they would hardly be of practical use with our rather detailed commodity specification. It remains to be seen whether these generalized Stone functions are more successful in analysing cross section data than are the LES functions on the other hand and our cubic approximations on the other. Bojer [4], in a recent study based on more or less the same data as ours, assumes strict linearity throughout.

From (7), (11), and (17)-(19) we get

$$c_i = \alpha_i^T + \beta_i^T y + \gamma_i^T y^2 + \delta_i^T y^3 + \varepsilon_i^T, \tag{20}$$

where

$$\varepsilon_i^T = u_i^T + w_i^T, \tag{21}$$

$$\sum_i \beta_i^T = 1, \quad \sum_i \alpha_i^T = \sum_i \gamma_i^T = \sum_i \delta_i^T = \sum_i \varepsilon_i^T = 0 \quad (T = 0,\ 1). \tag{22}$$

We assume, tentatively, that

$$E(w_{it}^T) = 0, \tag{23}$$

$$E(w_{it}^T w_{js}^T) = \delta_{ts}\tau_{ij}^T, \tag{24}$$

$$E(w_{it}^1 w_{js}^0) = 0, \tag{25}$$

and moreover that

$$E(w_{it}^T u_{js}^T) = \delta_{ts}\mu_{ij}^T, \tag{26}$$

$$E(w_{it}^1 u_{js}^0) = 0 \qquad \text{(for all } i, j, t, s, \text{ and } T = 0,\ 1). \tag{27}$$

This in combination with (12)-(14) implies

$$E(\varepsilon_{it}^T) = 0, \tag{28}$$

$$E(\varepsilon_{it}^T \varepsilon_{js}^T) = \delta_{ts}\,(\sigma_{ij}^T + \mu_{ij}^T + \mu_{ji}^T + \tau_{ij}^T) = \delta_{ts}\lambda_{ij}^T, \tag{29}$$

$$E(\varepsilon_{it}^1 \varepsilon_{js}^0) = 0 \qquad \text{(for all } i, j, t, s, \text{ and } T = 0,\ 1). \tag{30}$$

Notice, in particular, that we may have $\lambda_{ij}^1 \neq \lambda_{ij}^0$ even if $\sigma_{ij}^1 = \sigma_{ij}^0$.

Our attention so far has been confined to the parametrization of the effect of the income variable. The effect of the number of household members, their ages and socio-economic background variables - i.e., the variables supposed to account for the main differences in household preferences - is represented by "parametrizing" the constant term and the coefficients of the linear and the quadratic term as follows:

$$(31)\begin{cases} \alpha_i^T = \alpha_{i0}^T + \sum\limits_k \alpha_{i1k}^T z_k + \alpha_{i2}^T n^2, \\ \beta_i^T = \beta_{i0}^T + \beta_{i1}^T n, & (i = 1,\dots, N), \\ \gamma_i^T = \gamma_{i0}^T + \gamma_{i1}^T n, \end{cases}$$

where n denotes the number of household members, and z_k are binary variables reflecting type of household (single person, married couple with 0, 1, 2,... children etc.; 10 variables), occupation of the head of household (wage earner, self-employed etc.; 3 variables), and geographic location (2 variables).[7)]

The resulting expenditure functions may be considered as cubics in total expenditure (y) and the number of household members (n) after having deleted the terms in n^3 and n^2y and modified the constant term. They are truncated Taylor series expansions of the underlying, unknown functions F_i^T in analogy to e.g. the'trans-log function'. The latter has been frequently used in describing consumer demand and producer cost structure empirically, despite the fact that this function can at most be a second-order logarithmic approximation to 'true' theoretical functions. Our expenditure functions should be interpreted in a similar way.

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7) Notice that the number of household members is one of the characteristics used in defining the household types. Hence, in order to avoid (exact) multicollinearity, n should not be specified as a separate argument when "parametrizing" the constant term α_i^T. Notice also that the geographic binary variables may, to some extent, reflect geographic price differences.

3. Data and estimation

The data base contains individual household reports from the Norwegian Surveys of Consumer Expenditure of 1967 (T = 0) and 1973 (T = 1). The two surveys are identically designed in almost all respects, the main difference concerns the length of the period of reporting. The households participating in the 1967 survey were asked to report their consumption expenditures during a period of one month; in the 1973 survey the period of reporting was two weeks. However, for certain commodities purchased rather infrequently, e.g. durables, the reports give the value of purchases during the year prior to the month, respectively the two weeks, of registration. All items are converted to a per annum basis. The two samples include 3 645 and 3 363 households respectively.[8)]

Expenditures on purchases of transport equipment (mainly motor cars) are excluded from consideration. As this is by far the most important group of durables[9)] with considerable individual variation in expenditures, inclusion of this component would tend to make y a poor indicator of the total value of consumption services. The remaining total is divided into N = 45 commodity groups; see table 1.

The price data are obtained from the basic data used in the construction of the Official Consumer Price Index.

The two sets of expenditure functions (20) (with (31) inserted) are estimated by application of (unconstrained) ordinary least squares (OLS) to each equation separately. If the coefficients are considered as free parameters - only subject to (22), which OLS estimates satisfy automatically - other methods would give no gain in efficiency. This is due to the fact that all equations contain the same vector of exogenous variables. In such cases, OLS, the Aitken generalized least squares method, and the Maximum Likelihood method yield identical results.[10)]

Otherwise, if the coefficients were restricted in some way, a gain in efficiency might be obtained. Generally, the homogeneity and symmetry constraints of the demand theory imply not only restrictions between coefficients relating to the same year, but also restrictions on the change of coefficients over time. This sort of restrictions, however, could be utilized efficiently only if we were in a position to specify parametrically the change in the disturbance variances and covariances between the two years. This is rather difficult, however, owing to the change in the length of the period of reporting.[11)]

---

8) More detailed information is found in [7], and [8].

9) Its average budget share was about 4 per cent in 1967 and 6 per cent in 1973.

10) Cf. Zellner [18], p. 351. The fact that our disturbances ε_i^T have a singular covariance matrix (cf. (22)) does not affect this conclusion.

11) See section 4.3, in particular footnote 15, below.

4. Empirical results

The two sets of estimated expenditure functions may be compared in a variety of ways. We shall confine our attention to: (i) a direct comparison of coefficient vectors (section 4.1); (ii) a comparison of the implied "average" values of budget shares and expenditure elasticities (section 4.2); (iii) an analysis of the residual standard errors of estimation (section 4.3); and, finally, (iv) an attempt to use the functions estimated from the 1967 sample in "forecasting" the 1973 functions, paying regard to the price changes between the two years (section 4.4). Hopefully, by concentrating on different aspects, we will not only gain useful insight, but may also call attention to general problems involved in comparing micro data collected at different points of time.

4.1. Comparison of coefficient estimates

The most straightforward way of comparing the coefficient estimates for 1973 with those for 1967 is perhaps to examine the structure of their ratios. A glance at table 1, containing the ratios of δ_i^T, γ_{i0}^T, γ_{i1}^T, β_{i0}^T, β_{i1}^T, and α_{i2}^T, and table 2, showing their frequency distributions, reveals considerable variation. Table 1 gives in fact a rather "chaotic" impression. In table 2, only the ratios δ_i^1/δ_i^0 are concentrated; 38 of the 45 values belong to the interval (-0.1, 0.1), i.e., the absolute value of the coefficient of y^3 in 1973 is less than 1/10 of the corresponding one in 1967 for more than 80 per cent of the commodity groups. The ratios of the γ's (i.e., the coefficients of y^2 and ny^2) and those of the β's (i.e., the coefficients of y and ny) belong to the interval (-1.0, 1.0) for the majority of commodity groups. The median values of the coefficient ratios are

$$
(32)\begin{cases}
M_{\alpha 2} = \underset{i}{\text{med}}\ (\alpha_{i2}^1/\alpha_{i2}^0) = 1.5887, \\
M_{\beta 1} = \underset{i}{\text{med}}\ (\beta_{i1}^1/\beta_{i1}^0) = 0.4477, \\
M_{\beta 0} = \underset{i}{\text{med}}\ (\beta_{i0}^1/\beta_{i0}^0) = 0.6826, \\
M_{\gamma 1} = \underset{i}{\text{med}}\ (\gamma_{i1}^1/\gamma_{i1}^0) = 0.0339, \\
M_{\gamma 0} = \underset{i}{\text{med}}\ (\gamma_{i0}^1/\gamma_{i0}^0) = 0.0593, \\
M_{\delta} = \underset{i}{\text{med}}\ (\delta_i^1/\delta_i^0) = -0.0012.
\end{cases}
$$

Table 1. Coefficients in 1973 divided by coefficients in 1967[a)]

i	Commodity group	δ_i^1/δ_i^o	$\gamma_{io}^1/\gamma_{io}^o$	$\beta_{io}^1/\beta_{io}^o$	$\gamma_{i1}^1/\gamma_{i1}^o$	$\beta_{i1}^1/\beta_{i1}^o$	$\alpha_{i2}^1/\alpha_{i2}^o$
1	Flour, grain	0.06653	-0.08195	-0.25316	-0.89026	71.41012	1.58870
2	Bread, cake	0.02528	13.36593	1.52310	0.38695	0.69284	-85.92051
3	Meat, eggs	0.04019	0.21416	0.68262	0.18494	0.62516	-3.79576
4	Fish	-0.01011	-0.11794	0.23560	0.01558	0.33037	1.74174
5	Canned food	0.07479	-0.33240	-1.41253	0.18089	0.47778	1.08664
6	Milk, cream	0.01071	0.00625	0.00876	0.20767	0.30485	22.06306
7	Cheese	0.02107	0.33329	1.18619	0.05074	0.34619	1.45187
8	Butter	0.05265	-0.43312	-0.18859	1.52954	2.06474	-7.56441
9	Margarine .	0.02775	0.02244	0.32787	0.76407	0.46734	1.57110
10	Fresh vegetables	0.00040	0.17464	0.53507	-0.25032	0.28471	5.15773
11	Fresh fruits	0.05532	0.02482	0.19460	0.31896	0.54642	1.08439
12	Preserved vegetables and fruits	-0.18252	-0.73431	1.65504	0.25916	0.66364	-0.78723
13	Potatoes ..	0.49425	0.09530	0.32926	-0.15675	-8.01033	-0.84599
14	Cocoa, chocolate .	0.07034	-0.75732	-1.43750	0.43140	0.66887	3.06706
15	Sugar, coffee, tea, ice-cream etc.	-0.10884	-0.44792	-0.36593	-1.13158	0.15710	21.30060
16	Mineral waters	-0.01430	0.04308	37.97834	1.20852	5.14653	1.98690
17	Beer	-0.01813	0.01095	0.30995	-0.40066	-1.99379	2.33969
18	Wines and spirits ...	0.02649	0.06717	3.21751	0.37551	0.42401	2.52719
19	Tobacco ...	-0.03732	0.48789	0.58851	0.02838	-0.19675	0.72353
20	Clothing ..	-0.00031	-1.30043	1.27813	0.59021	0.02225	-3.59371
21	Cloths, yarn	-0.13575	1.71355	1.13865	6.62775	0.79244	-8.02677
22	Footwear ..	-0.04704	1.16751	0.89815	-0.41614	3.28934	5.57787
23	Housing and maintenance	-0.04945	0.79281	0.98846	-0.34442	0.57418	93.19627
24	Electricity	0.00170	0.56353	0.73656	-15.35295	8.58740	8.39616
25	Fuel	-0.14567	0.54177	1.14383	1.41668	3.02858	3.84441
26	Furniture, household textiles etc.	0.01402	0.03463	0.83239	0.82228	-0.08163	-109.58376
27	Electric appliances, tableware etc.	-0.00453	0.05606	0.52006	-0.29279	-0.57484	4.25383

a) The equations estimated are: c_i = constant and dummy shift terms + $\beta_{io}^T y + \gamma_{io}^T y^2 + \beta_{i1}^T ny + \alpha_{i2}^T n^2 + \delta_i^T y^3 + \gamma_{i1}^T ny^2 + \varepsilon_i^T$.

Table 1 (cont.). Coefficients in 1973 divided by coefficients in 1967[a)]

i	Commodity group	δ_i^1/δ_i^o	$\gamma_{io}^1/\gamma_{io}^o$	$\beta_{io}^1/\beta_{io}^o$	$\gamma_{i1}^1/\gamma_{i1}^o$	$\beta_{i1}^1/\beta_{i1}^o$	$\alpha_{i2}^1/\alpha_{i2}^o$
28	Misc. household goods and services	-0.01434	0.31313	0.72770	5.14589	-0.06461	13.05756
29	Domestic services ..	0.00214	-0.10284	2.58653	0.04627	-0.06743	4.52432
30	Medical care	0.04436	-1.93808	2.50963	0.83551	5.19444	-1.92135
31	Petrol and oil	-0.07049	-1.57752	1.32492	-0.02376	-1.14344	0.85286
32	Maintenance of transport equipment	0.07572	0.05931	2.45363	0.73978	1.50173	0.38330
33	Public transport services ..	-0.05811	12.37241	-0.81660	-2.31054	-2.54150	-0.26207
34	Postal, telephone and telegraph services	-0.08957	0.66346	0.50092	-0.23278	-0.83333	-5.52092
35	Television and radio sets	0.00759	0.24432	0.97190	-0.16782	-0.42675	6.96639
36	Recreation equipment .	0.01400	-0.03267	-0.74319	-0.08216	0.51234	-0.36392
37	Public entertainment	0.07938	1.24234	1.33218	0.24159	0.87968	0.64193
38	Books, newspapers	-0.07437	0.23870	0.42129	-0.17383	-0.67697	4.46355
39	Magazines and periodicals	0.00874	0.09883	0.30023	-0.06203	0.44769	1.05552
40	School fees	-1.01285	-1.35004	-3.09511	-1.10815	-1.13201	8.70024
41	Cosmetic articles ..	-0.02219	-0.36008	1.32524	-0.10204	-2.53202	1.81571
42	Other toilet articles ..	-0.02855	4.78612	1.34140	-2.20462	-0.25484	1.99508
43	Travel goods, jewellery etc.	-0.00118	-0.06753	-1.95724	11.41234	0.73055	25.06723
44	Restaurants, hotels	-0.01112	0.42801	0.84285	-0.50096	1.40818	0.32917
45	Financial and other services ..	-0.18973	1.02209	-1.32698	0.03389	9.87319	2.33227

a) The equations estimated are: c_i = constant and dummy shift terms + $\beta_{io}^T y$ + $\gamma_{io}^T y^2 + \beta_{i1}^T ny + \alpha_{i2}^T n^2 + \delta_i^T y3 + \gamma_{i1}^T ny^2 + \varepsilon_i^T$.

Table 2. Frequency distributions of coefficient ratios[a)]

	Below -10	-10- -5	-5- -2	-2- -1	-1- -0.5	-0.5- 0	0- 0.5	0.5- 1	1- 1.5	1.5- 2	2- 5	5- 10	10- 50	Above 50
δ_i^1/δ_i^0	-	-	-	1	-	22[b)]	22[c)]	-	-	-	-	-	-	-
$\gamma_{i0}^1/\gamma_{i0}^0$	-	-	-	4	2	9	19	4	3	1	1	-	2	-
$\gamma_{i1}^1/\gamma_{i1}^0$	1	-	2	2	1	14	14	5	2	1	-	2	1	-
$\beta_{i0}^1/\beta_{i0}^0$	-	-	1	4	2	3	8	12	8	2	4	-	1	-
$\beta_{i1}^1/\beta_{i1}^0$	-	1	2	3	3	6	10	10	1	1	3	4	-	1
$\alpha_{i2}^1/\alpha_{i2}^0$	2	3	2	1	2	2	2	3	4	6	8	5	4	1

a) The equations estimated are: c_i = constant and dummy shift terms + $\beta_{io}^T y + \gamma_{io}^T y^2 + \beta_{i1}^T ny + \alpha_{i2}^T n^2 + \delta_i^T y^3 + \gamma_{i1}^T ny^2 + \varepsilon_i^T$.
b) Of these 17 belong to the interval (-0.1, 0.0).
c) Of these 21 belong to the interval (0.0, 0.1).

How could we explain the systematic pattern indicated in table 2 and accentuated in (32)? We notice that if our cubic expenditure functions (20) were to satisfy the homogeneity condition on $F_i^T(y)$ <u>exactly</u>, then their constant terms should be homogeneous in prices of degree one, and the coefficients of y, y^2 and y^3 homogeneous of degrees 0, -1, and -2, respectively. Let us consider a specification which satisfy these restrictions without violating the adding-up constraint, viz.[12)],

$$
(33)\begin{cases}
\alpha_i^T = \alpha_i P^T, \\
\beta_i^T = \beta_i, \\
\gamma_i^T = \gamma_i / P^T, \\
\delta_i^T = \delta_i / (P^T)^2,
\end{cases}
\qquad (i = 1, \ldots, N)
$$

12) In general, this parametrization is incompatible with constrained utility maximisation, according to the results of Fourgeaud and Nataf [10]. Moreover, the price responses are rather rigidly described: Price changes affect the budget shares at current prices only through their effect on the general price index P.

with

$$\sum_i \beta_i = 1, \ \sum_i \alpha_i = \sum_i \gamma_i = \sum_i \delta_i = 0,$$

where $P^T = P(p_1^T, \ldots, p_N^T)$ is a price index homogeneous of the first degree.

Taking (33) as a reference, we should expect the constant terms to increase more or less in proportion with the general price index P, the coefficients of the linear terms to be approximately unaffected, and the coefficients of the quadratic and cubic terms to decrease in inverse proportion with P and P^2 respectively. The "theoretical" values of the coefficient ratios corresponding to (33) are

$$(34)\begin{cases} \alpha_i^1/\alpha_i^0 = P^1 = 1.4441, \\ \beta_i^1/\beta_i^0 = 1.0000, \\ \gamma_i^1/\gamma_i^0 = 1/P^1 = 0.6925, \\ \delta_i^1/\delta_i^0 = 1/(P^1)^2 = 0.4795, \qquad (i=1,\ldots,N), \end{cases}$$

when representing P by the Official Consumer Price Index[13)] and imposing the normalization $P^0 = 1$. The median $M_{\alpha 2}$ is somewhat higher than its "theoretical counterpart", the five other ones are lower. The ranking of the medians, however, concurs with the ranking of the theoretical ratios. Thus, to some extent, the overall pattern of change of coefficients is in accordance with the homogeneity restriction of the demand theory. But as the values in (32) and (34) differ considerably, the results hardly support the simple parametrization (33).

Certainly, a formal test of whether the two sets of expenditure functions have been produced by the same set of demand functions would have been of interest. The well-known test of equality of two vectors of regression coefficients proposed by Chow [9] comes to mind. (This test would require - provided we take (33) as the null hypothesis - a deflation of all expenditures by P^0 and P^1 respectively before running the regressions.)

13) This is a Laspeyres index with weights approximately equal to the budget shares of the average household in 1967 as estimated from the Survey of Consumer Expenditure of that year.

However, the Chow test uses equality of all disturbance variances in both samples as an essential assumption. This is hardly satisfied in the present case because of the change in the length of the period of reporting. (See section 4.3.)

4.2. Comparison of estimated "average" budget shares and expenditure elasticities

It is not easy to see the practical consequences of the results reported above. Estimates of the implied values of budget shares and expenditure (Engel) elasticities give useful additional information. Table 3 contains estimates for a married couple with one child and with a total expenditure of N.kr 40 000 in 1973 and N.kr 27 699 = 40 000 P^0/P^1 in 1967. This roughly corresponds to the "average" household.[14] We let the value of total expenditure considered increase proportionally with the consumer price index, since the homogeneity restriction of the demand theory implies zero homogeneity in prices and total expenditure of all budget shares and expenditure elasticities.

The estimates agree more closely than might be anticipated from tables 1 and 2. The difference between the estimated expenditure elasticities is less than 0.40 for 37 of the 45 commodity groups; in this respect, the majority of food categories perform particularly well. However, for certain commodities, e.g. 36 Recreation equipment and 40 School fees, we find considerable discrepancies.

The average absolute value of the difference between the estimated budget shares is about 0.004. In some cases, e.g. groups 26 Furniture, household textiles etc. and 36 Recreation equipment, the differences are disappointingly large. Again, most of the food categories perform rather well. Of course, the change in relative prices from 1967 to 1973 may partly account for the changes in budget shares reported; our procedure pays regard only to the effect of the change in the general price level. In section 4.4, we attempt to carry this analysis a step further.

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14) The average household in the 1967 sample contained 3.43 persons with a total consumption expenditure of N.kr 20 766.57. In the 1973 sample, the corresponding averages were 3.08 persons and N.kr 35 696.35.

Table 3. Average estimates of budget shares and expenditure elasticities

i	Commodity group	Budget share		Expenditure elasticity	
		1967a)	1973b)	1967a)	1973b)
1	Flour, grain	0.0052	0.0044	0.08	0.25
2	Bread, cake	0.0183	0.0204	0.37	0.35
3	Meat, eggs	0.0824	0.0751	0.65	0.71
4	Fish	0.0176	0.0157	0.36	0.41
5	Canned food	0.0061	0.0053	0.41	0.56
6	Milk, cream	0.0284	0.0238	0.28	0.16
7	Cheese	0.0081	0.0079	0.43	0.40
8	Butter	0.0042	0.0026	0.72	0.68
9	Margarine	0.0073	0.0057	0.20	0.28
10	Fresh vegetables	0.0101	0.0102	0.63	0.55
11	Fresh fruits	0.0176	0.0149	0.70	0.45
12	Preserved vegetables and fruits	0.0161	0.0152	0.74	0.74
13	Potatoes	0.0099	0.0079	0.43	0.69
14	Cocoa, chocolate	0.0087	0.0074	1.04	0.59
15	Sugar, coffee, tea, ice-cream etc.	0.0294	0.0278	0.40	0.32
16	Mineral waters	0.0062	0.0095	1.34	0.77
17	Beer	0.0056	0.0074	1.43	0.96
18	Wines and spirits	0.0137	0.0087	2.60	2.25
19	Tobacco	0.0200	0.0208	0.63	0.60
20	Clothing	0.0916	0.0865	1.26	1.11
21	Cloths, yarn	0.0141	0.0134	0.96	0.75
22	Footwear	0.0225	0.0164	1.06	1.38
23	Housing and maintenance	0.0838	0.0971	0.96	1.09
24	Electricity	0.0236	0.0227	0.37	0.23
25	Fuel	0.0168	0.0111	0.30	0.50
26	Furniture, household textiles etc.	0.0453	0.0632	1.38	1.03
27	Electric appliances, tableware etc.	0.0410	0.0351	0.85	0.95
28	Misc. household goods and services	0.0188	0.0198	0.79	0.72
29	Domestic services	0.0075	0.0134	1.16	0.87
30	Medical care	0.0201	0.0232	1.50	1.70
31	Petrol and oil	0.0363	0.0445	1.09	0.83
32	Maintenance of transport equipment	0.0397	0.0426	1.66	2.28
33	Public transport services	0.0264	0.0207	1.88	2.62
34	Postal, telephone and telegraph services	0.0147	0.0158	1.46	1.57
35	Television and radio sets	0.0167	0.0155	0.64	0.73
36	Recreation equipment	0.0326	0.0493	2.38	1.16
37	Public entertainment	0.0191	0.0249	1.22	0.84
38	Books, newspapers	0.0137	0.0186	1.15	0.86
39	Magazines and periodicals	0.0066	0.0062	0.71	0.51
40	School fees	0.0046	0.0001	4.67	10.01
41	Cosmetic articles	0.0064	0.0072	1.24	1.00
42	Other toilet articles	0.0136	0.0158	0.83	0.69
43	Travel goods, jewellery etc.	0.0176	0.0174	1.94	1.48
44	Restaurants, hotels	0.0338	0.0218	2.28	1.89
45	Financial and other services	0.0183	0.0067	1.63	1.97

a) Calculated from the estimated expenditure functions for married couple with one child and total expenditure N.kr 27 699.
b) Calculated from the estimated expenditure functions for married couple with one child and total expenditure N.kr 40 000.

4.3. Analysis of residual standard errors

Estimates of the residual coefficients of variation (i.e., the residual standard errors divided by the corresponding sample average of expenditures) are given in the first two columns of table 4. (The estimate of $(\lambda_{ii}^T)^{\frac{1}{2}}$ is denoted by $\hat{\lambda}_i^T$.) Not unexpectedly, a considerable part of the individual differences in consumption expenditures is "unexplained"; the number of commodities with coefficients of variation below unity is only 12 in 1967 and 7 in 1973. More interesting is the fact that these coefficients are definitely higher in 1973 than in 1967 for the majority of commodities, showing a decrease in 3 cases only. To some extent, this increase is certainly due to the reduction of the period of registration from one month in 1967 to two weeks in 1973.[15] The ratios of the estimated residual standard errors, reported in column 3, are well above unity for all commodity groups; the corresponding variance ratios vary from 1.94 (for group 6 Milk and cream) to 16.17 (for group 32 Maintenance of transport equipment). Using a 1 per cent F-test, this implies rejection of the hypothesis $\lambda_{ii}^1 = \lambda_{ii}^0$ for all commodity groups.[16]

Can we find a systematic pattern in the increase of the residual standard errors from 1967 to 1973? In particular, is this increase correlated with the increase in (i) the volume component and/or (ii) the price component of the expenditure? By using a non-parametric method based on the Spearman rank correlation coefficient[17], such tests can be carried out rather easily. The ratios of the estimated standard errors, the expenditures at 1967 prices, and the price indices, as well as the corresponding ranks are reported in colums 3-8 of table 4. Letting X_i denote the rank of the standard error ratio of the commodity group with number i when ranking according to the expenditure ratios $(\bar{c}_i^1/(p_i^1\bar{c}_i^0))$, the rank correlation coefficient may be written as

---

15) This may be justified as follows: Households do not purchase consumer goods continuously, but at discrete points of time. It seems sensible, as a first approximation, to represent the activity of purchase by a "Poisson process", such that the number of purchases during a period of length θ has a Poisson distribution with parameter $\lambda\theta$, where the "intensity" λ is a function of income, prices, transaction costs etc. It then follows that the coefficient of variation of the expenditure, conditional on income, prices, transaction costs etc., is inversely proportional with the square root of θ, provided all purchases during the period have the same value. (For a general description of the Poisson process, see e.g. Sverdrup [17, Ch. VIII. 3].)

16) Of course, this does not necessarily justify acceptance of $\sigma_{ii}^1 > \sigma_{ii}^0$ (cf. (29)). It may be that the cubics (17) approximate the true expenditure functions F_i^T less well in 1973 than in 1967 $(\tau_{ii}^1 > \tau_{ii}^0)$, owing to the reduction of the period of reporting. The variances of u_i^1 and u_i^0 may still be equal.

17) See Kendall and Stuart [13, par. 31. 19-31. 23].

Table 4. Residual coefficients of variation. Ratios of residual standard errors, average expenditures, and prices[a), b)]

i	Commodity group	Residual coefficient of variation 1967 $\hat{\lambda}_i^0/\bar{c}_i^0$	1973 $\hat{\lambda}_i^1/\bar{c}_i^1$	Ratio of res.std. errors $\hat{\lambda}_i^1/\hat{\lambda}_i^0$	rank	Ratio of expenditures at 1967 prices $\bar{c}_i^1/(p_i^1\bar{c}_i^0)$	rank	Ratio of price indices p_i^1	rank
1	Flour, grain .	1.100	1.461	1.3976	44	0.9182	36	1.1452	45
2	Bread, cake .	0.607	0.751	1.8693	30	0.9391	34	1.6084	9
3	Meat, eggs ..	0.640	1.099	2.3620	18	0.9308	35	1.4788	18
4	Fish	0.851	1.015	1.6724	39	0.8454	41	1.6587	6
5	Canned food .	1.177	1.456	1.7009	38	0.8570	40	1.6035	10
6	Milk, cream .	0.420	0.466	1.3932	45	0.9526	31	1.3168	36
7	Cheese	0.714	0.852	1.7429	37	1.0515	25	1.3895	28
8	Butter	1.238	1.660	1.7631	36	1.1034	23	1.1918	42
9	Margarine ...	0.584	0.953	1.9563	28	0.8893	38	1.3488	33
10	Fresh vegetables	0.890	1.035	1.8681	31	1.0051	29	1.5985	11
11	Fresh fruits	0.766	0.862	1.4454	42	0.9432	33	1.3619	31
12	Preserved vegetables and fruits	1.115	1.303	1.8280	34	1.1148	22	1.4034	26
13	Potatoes	1.182	2.016	2.2297	22	0.7015	42	1.8648	2
14	Cocoa, chocolate	1.060	1.158	1.5231	41	0.9760	30	1.4285	22
15	Sugar, coffee, tea, icecream etc. ..	0.494	0.968	3.1840	6	1.0470	26	1.5517	14
16	Mineral waters	1.269	1.278	2.5402	11	1.9375	1	1.3020	41
17	Beer	2.182	2.085	2.4187	15	1.9082	3	1.3263	35
18	Wines and spirits	2.395	2.596	2.6744	10	1.7302	5	1.4264	23
19	Tobacco	1.063	1.167	1.8659	32	1.1849	19	1.4355	21
20	Clothing	0.775	1.164	2.3295	19	1.1322	21	1.3704	30
21	Cloths, yarn	1.667	2.629	1.9739	27	0.9122	37	1.3712	29
22	Footwear	1.389	2.178	2.2144	24	0.9468	32	1.4911	16
23	Housing and maintenance .	1.024	1.591	3.5825	3	1.6464	7	1.3996	27
24	Electricity .	0.496	0.458	1.4110	43	1.0775	24	1.4172	25
25	Fuel	1.057	1.601	1.6215	40	0.6841	43	1.5645	12
26	Furniture, household textiles etc.	1.403	1.578	2.2191	23	1.4566	11	1.3543	32
27	Electric appliances, tableware etc.	1.389	2.313	2.2572	21	1.0392	27	1.3051	39

a) $\hat{\lambda}_i^T$ and $\bar{c}_i^T$ denote the estimated residual standard error and the sample mean of the expenditure respectively of the i'th commodity in period T.
b) $p_i^0 = 1$ for all i.

Table 4 (cont.). Residual coefficients of variation. Ratios of residual standard errors, average expenditures, and prices[a), b)]

i	Commodity group	Residual coefficient of variation		Ratio of res.std. errors		Ratio of expenditures at 1967 prices		Ratio of price indices	
		1967	1973						
		$\hat{\lambda}_i^0/\bar{c}_i^0$	$\hat{\lambda}_i^1/\bar{c}_i^1$	$\hat{\lambda}_i^1/\hat{\lambda}_i^0$	rank	$\bar{c}_i^{\,1}/(p_i^{1-0}c_i)$	rank	p_i^1	rank
28	Misc. household goods and services	0.798	1.380	2.9379	7	1.2917	17	1.3158	37
29	Domestic services	4.938	5.866	2.7183	9	1.1630	20	1.9676	1
30	Medical care	2.158	3.475	3.3542	5	1.4261	12	1.4606	19
31	Petrol and oil	1.489	1.214	2.0597	25	1.9252	2	1.3120	38
32	Maintenance of transport equipment ...	1.885	2.917	4.0213	1	1.8249	4	1.4243	24
33	Public transport services	1.604	2.962	3.7326	2	1.3968	13	1.4476	20
34	Postal, telephone and telegraph services	2.491	3.633	3.3740	4	1.3495	14	1.7144	3
35	Television and radio sets	2.493	2.990	1.8415	33	1.3073	16	1.1744	43
36	Recreation equipment ...	2.240	2.488	2.4926	13	1.7219	6	1.3032	40
37	Public entertainment	1.645	1.714	2.5087	12	1.6078	9	1.4976	15
38	Books, newspapers	1.676	1.943	2.4034	16	1.2148	18	1.7069	4
39	Magazines and periodicals .	1.005	1.593	2.3250	20	0.8717	39	1.6830	5
40	School fees .	5.602	10.482	1.8064	35	0.5922	44	1.6301	8
41	Cosmetic articles	1.389	1.601	2.4643	14	1.5955	10	1.3361	34
42	Other toilet articles	1.005	1.224	2.3878	17	1.3160	15	1.4896	17
43	Travel goods, jewellery etc.	2.929	3.078	1.9756	26	1.6334	8	1.1508	44
44	Restaurants, hotels	1.771	1.992	1.8875	29	1.0252	28	1.6375	7
45	Financial and other services	4.100	17.519	2.8066	8	0.4206	45	1.5614	13

a) $\hat{\lambda}_i^T$ and $\bar{c}_i^T$ denote the estimated residual standard error and the sample mean of the expenditure respectively of the i'th commodity in period T.
b) $p_i^0 = 1$ for all i.

$$r_s = 1 - \frac{6}{N\ (N^2-1)} \sum_{i=1}^{N} (X_i - i)^2.$$

Then, the test statistic

$$t_s = \left\{ \frac{r_s^{\,2}(N-2)}{1-r_s^{\,2}} \right\}^{\frac{1}{2}}$$

has an approximate Student distribution with N-2 = 43 degrees of freedom when the two variables are independently distributed. From table 4 we get

$$r_s = 0.5491,\ t_s = 4.31.$$

In the same way, we find

$$r_s = 0.1302,\ t_s = 0.86,$$

when testing for correlation with the price ratio. As the critical Student value at a 1 per cent level is 2.42, the first hypothesis is rejected, the second is not.

Thus, there is evidence of heteroscedasticity in the following overall sense: The commodities with the largest increase in the residual standard errors from 1967 to 1973 have, by and large, had the largest increase in the average volume of consumption. On the other hand, the results do not indicate correlation with the price component of the expenditures. It should be admitted, of course, that the above analysis only gives a summary impression of the kind of heteroscedasticity involved. To get more precise conclusions, a detailed examination of the individual residuals would be needed.

4.4. Comparing estimates of expenditures based on the 1973 expenditure functions with estimates based on updated 1967 functions

Finally, we shall examine the predictive performance of the estimated 1967 expenditure functions. More precisely, we shall compare two sets of estimates of expenditures in 1973: a) estimates derived from the expenditure functions for this year, and b) estimates based on updated 1967 functions, paying regard to the observed changes in prices from 1967 to 1973. This is a more sophisticated, and in some way more satisfactory approach than that followed in section 4.2.

Let $c_i^0(y)$ and $c_i^1(y)$ denote the estimate of the expenditure on good i of a household with a total expenditure equal to y, as calculated from the expenditure functions for 1967 and 1973 respectively, and let $c_i^{*1}(y)$ denote the corresponding estimate based on the expenditure function for 1967 updated from 1967 to 1973. The method of updating used relies on the Frisch method of estimating a complete set of Cournot price elasticities. It is described in some detail in the Appendix. The corresponding estimates of budget shares are[18)]

$$a_i^0(y) = c_i^0(y)/y, \; a_i^1(y) = c_i^1(y)/y, \; a_i^{*1}(y) = c_i^{*1}(y)/y \qquad (i = 1,\ldots, N).$$

As a summary measure of the quality of the updating (the "prediction error") we use

$$U = U(y) = \sum_i |a_i^1(y) - a_i^{*1}(y)|,$$

U = 0 indicating "no prediction error". Values of U, as well as the corresponding average U/N, are given in table 5; part A showing the variation with total expenditure for a married couple with one child (i.e., the approximate "average" type of household in the sample), part B showing the variation with type of household for a total expenditure of N.kr 40 000 (which is approximately the average expenditure of the households in the 1973 sample). The average error U/N is about 0.4 per cent units for the average household; it is considerably larger for households strongly different from the average.

18) For simplicity, the superscripts on y used in the Appendix are omitted.

In relation to the level of the budget shares, the lack of precision is substantial. Since U/N is the average error, and the average budget share is 1/n, U simply indicates the relative error of the updated budget shares. Its value varies from 17 to 38 per cent for the households represented in table 5. This result, of course, should be interpreted with regard to our comparatively disaggregated commodity classification. A more aggregated specification would probably result in lower values of U (but not necessarily lower values of U/N).

Values of the differences $a_i^1 - a_i^{*1}$ as calculated for the average household are given in table 6. The absolute difference is largest for groups 26 Furniture, household textiles etc. (0.0182), 36 Recreation equipment (0.0151), and 23 Housing and maintenance (0.0135). On the other hand, the difference is less than 0.005 for 32 of the 45 commodity groups, including 14 of the 15 groups of food (the exception being group 3 Meat, eggs). Thus, the food commodities pass this test rather well, as was also the case with the tests in section 4.2.

Finally, it is interesting to compare the performance of the "predictor" a_i^{*1} with the performance of naive "forecasts". A naive way of forecasting consumption expenditures in 1973 from expenditure functions relating to 1967 would be to assume that all budget shares, given the values of the background variables, depend on total real expenditure only, i.e., using $a_i^0(y/P^1)$ (with P^1 indicating, as before, the average consumer price index in 1973, $P^0 = 1$) as a "forecast" of $a_i^1(y)$. Confining attention to the average household only, we find that $|a_i^1(y) - a_i^0(y/P^1)|$ is greater than $|a_i^1(y) - a_i^{*1}(y)|$ for 25 of the 45 commodity groups; the sums of the absolute errors are 0.1842 and 0.1832 respectively. Thus, on the average, the sophisticated method, which pays regard to the change in relative prices, performs only slightly better than the naive one, which neglects the price response.

Table 5. Summary measures of the precision of the updating (the "prediction error")

A. Variation with total expenditure; married couple with one child

Total expenditure, N.kr	10 000	20 000	30 000	40 000	50 000	60 000	80 000
U	0.3784	0.2774	0.2084	0.1832	0.1752	0.1770	0.2582
U/N	0.0084	0.0062	0.0046	0.0041	0.0039	0.0039	0.0057
$\max_i \lvert a_i^1 - \overset{*}{a}_i^1 \rvert$	0.0358	0.0274	0.0226	0.0182	0.0171	0.0219	0.0394

B. Variation with type of household; total expenditure N.kr 40 000

Type of household	Single	Married couple without children	Married couple, 1 child	Married couple, 2 children	Married couple, 3 children
U	0.2396	0.1678	0.1832	0.2308	0.2102
U/N	0.0053	0.0037	0.0041	0.0051	0.0047
$\max_i \lvert a_i^1 - \overset{*}{a}_i^1 \rvert$	0.0338	0.0150	0.0182	0.0490	0.0144

Table 6. "Observed" minus "predicted" budget shares. Married couple with one child and total expenditure N.kr 40 000

i a)	$a_i^1 - \overset{*}{a}_i^1$	i a)	$a_i^1 - \overset{*}{a}_i^1$	i a)	$a_i^1 - \overset{*}{a}_i^1$
1	+0.0002	16	+0.0033	31	+0.0090
2	+0.0006	17	+0.0018	32	+0.0025
3	-0.0089	18	-0.0053	33	-0.0059
4	-0.0039	19	+0.0007	34	+0.0012
5	-0.0013	20	-0.0048	35	+0.0009
6	-0.0025	21	-0.0005	36	+0.0151
7	0.0000	22	-0.0064	37	+0.0056
8	-0.0012	23	+0.0135	38	+0.0047
9	-0.0012	24	-0.0007	39	-0.0010
10	-0.0006	25	-0.0068	40	-0.0041
11	-0.0022	26	+0.0182	41	+0.0009
12	-0.0008	27	-0.0037	42	+0.0019
13	-0.0041	28	+0.0017	43	-0.0010
14	-0.0014	29	+0.0068	44	-0.0093
15	-0.0033	30	+0.0030	45	-0.0111

a) See table 4.

5. Conclusion

In this paper, we have dealt partly with methodological problems in comparing household budget data from two different years and partly with empirical results based on Norwegian data from 1967 and 1973. Initially, we pointed out that expenditure functions estimated from different data sets can hardly be adequately compared without reference to economic theory of consumer demand. However, recognizing the conflict between the claim that the functions agree perfectly with utility maximisation and the desire that they are sufficiently flexible to reflect adequately cross-sectional variations of consumption, our compromise choice is to use polynomials of the third degree in total consumption expenditure.

The coefficients of the 45 specified expenditure functions as estimated from the 1973 data differ considerably from those based on the 1967 data. An investigation of their ratios does not invite a definite conclusion, but to some extent the results agree with the homogeneity constraint of the demand theory. Moreover, the average estimates of expenditure (Engel) elasticities have the same order of magnitude for the majority of commodities. In this respect, the 15 food categories perform particularly well.

The reduction of the length of the period of reporting of expenditures in the 1973 survey as compared with the 1967 survey gives rise to a considerable increase not only in the residual standard errors of estimation, but also in the corresponding coefficients of variation. The relative increase in the residual standard error is positively correlated with the increase in the consumption expenditure at constant prices, but our test indicates no significant correlation with the increase in the price component of the expenditure.

Finally, a comparison of the 1973 expenditure functions with "updated" 1967 functions confirms that the two data sets have interesting properties in common. The average absolute difference between the estimated budget shares is, however, considerable - about 0.4 per cent for the average household - i.e., the relative "forecasting" error in a period of six years is about 20 per cent. Our method of updating, which pays regard to the change in relative prices, performs only marginally better than a naive method which neglects the price response.

Further research based on alternative models and methods and utilizing a greater body of data - particularly data from more than two years - is certainly needed.

Appendix

THE METHOD USED IN UPDATING THE EXPENDITURE ESTIMATES

The purpose of this appendix is to describe briefly the method used in section 4.4 in updating the expenditure estimates from the year 1967 to the year 1973.

Differentiating the demand functions f_i, including the first order terms only, gives

$$(1) \qquad \frac{\Delta x_i}{x_i} = E_i \frac{\Delta y}{y} + \sum_{j=1}^{N} e_{ij} \frac{\Delta p_j}{p_j} \qquad (i = 1, \ldots, N),$$

where x_i denotes the volume of consumption of good i, p_i its price index, y total expenditure, E_i the expenditure (Engel) elasticity of good i, and e_{ij} the price (Cournot) elasticity of good i with respect to good j. The elasticities refer to the point of f_i from which the differentials are taken. The updating is based on the assumption that the utility function generating the f_i's is additive (or rather, that it can be made additive by means of a suitable monotonic transformation). Additivity is, of course, a somewhat questionable assumption, in view of the rather detailed commodity classification used. We then have (cf. Frisch [11, pp 186-187])

$$(2) \qquad e_{ij} = (E_i/\omega)\,(\delta_{ij} - a_j E_j) - a_j E_i \qquad (i, j = 1, \ldots, N),$$

where δ_{ij} is the "Kronecker delta", a_j is the budget share of good j, and ω is an indicator of the overall degree of substitution. (As regards the additive members of the class of utility functions, ω has the alternative interpretation as the income elasticity of the marginal utility of income.) From (1) and (2) we obtain

$$(3) \qquad \frac{\Delta x_i}{x_i} = E_i \left(\frac{\Delta y}{y} - \sum_j a_j \frac{\Delta p_j}{p_j}\right) + \frac{E_i}{\omega}\left(\frac{\Delta p_i}{p_i} - \sum_j a_j E_j \frac{\Delta p_j}{p_j}\right) \quad (i = 1, \ldots, N).$$

Inserting $\Delta y/y = y^1/y^0 - 1$, $\Delta p_i/p_i = p_i^{\,1}/p_i^{\,0} - 1$, $\Delta x_i/x_i = (c_i^{*1}/c_i^{\,0})\,(p_i^{\,0}/p_i^{\,1}) - 1$, $a_i = a_i^{\,0}$, $E_i = E_i^{\,0}$, and $\omega = \omega^0$ (the superscripts 0 and 1 symbolizing, as before, the years 1967 and 1973 respectively), eq. (3) can be used to obtain estimates, c_i^{*1}, of the expenditure on good i in 1973, relating to total expenditure y^1, from estimates of $c_i^{\,0}$, $a_i^{\,0}$, $E_i^{\,0}$, and ω^0, relating to total expenditure y^0. In the calculations, the value of ω^0 is set equal to -2 for all households. This value, interpreted as an average estimate, agrees well with the majority of results concerning complete systems of demand functions (cf. Brown and Deaton [5, p. 1206]); the empirical evidence as regards the form of the function $\omega^0(y^0)$ is, however, scare and to some extent conflicting. The values of $c_i^0 = c_i^0(y^0)$, $a_i^0 = a_i^0(y^0)$, and $E_i^0 = E_i^0(y^0)$ are calculated from the expenditure functions for 1967 (using the normalisation $p_i^0 = 1$ for all i).[1) Finally, the combinations of values of y^0 and y^1 chosen are restricted to those satisfying $y^0 = y^1/P^1$ (P^1 denoting the average consumer price index in 1973, $P^0 = 1$). The intention is to prevent too large variations in E_i and a_i when extrapolating according to eq. (3), recalling that these parameters (as well as ω) are homogeneous of degree zero in total expenditure and prices.

In this way, we end up with the following expression for the updated expenditure on good i:

$$(4) \qquad c_i^{*1} = c_i^{*1}(y^1) = p_i^1 c_i^0 \left(\frac{y^1}{P^1}\right) \{1 + E_i^0\left(\frac{y^1}{P^1}\right) (P^1 - \sum_j a_j^0\left(\frac{y^1}{P^1}\right)p_j^1)$$

$$+ \frac{1}{\omega^0} E_i^0\left(\frac{y^1}{P^1}\right) (p_i^1 - \sum_j a_j^0\left(\frac{y^1}{P^1}\right) E_j^0\left(\frac{y^1}{P^1}\right)p_j^1)\} \qquad (i = 1,\ldots, N).$$

In order to ensure that the estimates add up exactly, i.e., $\sum c_i^{*1}(y^1) = y^1$ for all y^1, a final proportional adjustment is used.

1) The binary variables representing social status and geographic location (cf. chapter 3) are set equal to their sample means. Moreover, minor adjustments are made to ensure non-negativity of c_i^0, a_i^0, and E_i^0 for all income levels and household types of interest (recalling that additivity of the utility function implies absence of inferior goods).

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