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MODIS II
A MACRO-ECONOMIC MODEL FOR SHORT-TERM ANALYSIS AND PLANNING

By Per Sevaldson

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EN MAKROØKONOMISK MODELL FOR KORTTIDSANALYSE OG PLANLEGGING

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Contents

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Forord

Siden 1952 har Statistisk Sentralbyrå arbeidd med å utvikle makroøkonomiske modeller for analyse av problemer i Norges økonomi. Hovedelementene i disse modellene har vært kryssløpsrelasjoner mellom produksjon og bruk av innsatsfaktorer og videre relasjoner som forklarer det private konsum som funksjoner av inntekter og priser. I de senere utgaver av modellen er kryssløpssammenhengene også utnyttet til å beskrive hvordan priser og inntekter avhenger av kostnadsfaktorer som lønninger, utenlandspriser og produktivitet.

De forskjellige utgaver av modellen har fått fellesbetegnelsen MODIS, som står for modell av disaggregert type. Suksessive utgaver er nummerert I, II, osv.

Modellenes viktigste anvendelse hittil har vært som hjelpemiddel for Finansdepartementet i nasjonalbudsjettarbeidet.

I artikkelen «Teknisk revolusjon i økonomisk analyse og politikk?» av Petter Jakob Bjerve, Statsøkonomisk Tidsskrift nr. 1, 1966 (gjengitt som særtrykk i Artikler fra Statistisk Sentralbyrå nr. 18), er det gitt en alminnelig oversikt over utviklingen av modellarbeidet og bruken av modellene. Oversikten dekker utviklingen fram til utgangen av 1965 og gir henvisninger til publikasjoner med mer detaljerte redegjørelser.

I den artikkelen som legges fram her, er det gitt en beskrivelse av MODIS II, og av praksis ved bruken av den i nasjonalbudsjettarbeidet. Artikkelen ble utarbeidd til det fjerde møte av rådgivere for regjeringer i land knyttet til De forente nasjoners økonomiske kommisjon for Europa (Senior economic advisers to ECE-Governments) i Genève i juni 1966. Den er trykt i publikasjonen «Macro-Economic Models for Planning and Policy-making», redigert av ECE's sekretariat og utgitt av De forente nasjoner (Genève 1967). Statistisk Sentralbyrå er takknemlig for å ha fått anledning til å gjengi artikkelen her.

MODIS II er nå erstattet er en ny versjon, MODIS III. Men den vesentligste forskjell mellom disse to modellene er av teknisk art, slik at den prinsipielle beskrivelsen av MODIS II som er gitt i det følgende, i alt vesentlig også gjelder for MODIS III.

En formell beskrivelse av relasjonssystemet i MODIS III er gitt av Olav Bjerkholt i et arbeidsnotat fra Statistisk Sentralbyrå: «Likningssystemet i den økonomiske analysemodell MODIS III». (Arbeidsnotater IO 68/3).

Statistisk Sentralbyrå, Oslo, 27. mai 1968.

Odd Aukrust

Preface

The description of the macro-economic planning model MODIS II which is presented here, was submitted to the Fourth Meeting of Senior Economic Advisers to ECE Governments in Geneva in June 1966. It was included in the publication «Macro-Economic Models for Planning and Policy-making», edited by the Secretariat of the Economic Commission for Europe and published by the United Nations (Geneva 1967). The Central Bureau of Statistics is grateful for the permission to reprint this paper in the series ARTIKLER.

Central Bureau of Statistics, Oslo, 27 May 1968.

Odd Aukrust

A SHORT-TERM MODEL FOR PLANNING

1. Introduction

The Norwegian inter-industry consumption model MODIS II [1,2] has been developed by the Central Bureau of Statistics for use in economic analysis and planning. The version of this model which has been made operative is particularly designed for use in the preparation of the annual "national budget", the economic plan for the coming year which is presented every October by the Norwegian Government. This version of the model has been formulated in consultation with the Ministry of Finance and the University of Oslo.

The present model, MODIS II, was preceded by a somewhat simpler version (now given the name MODIS I) which was in regular use from 1960 in various types of economic analysis and in government economic planning for one- and four-year periods.[3]

Before the discussion of the formal framework of the model (Section 4), a more summary description (Section 2), as well as some words about how it is utilized in the administrative process of working out government plans (Section 3), may be in order.

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[1] Model of disaggregated type, second version.

[2] Arne Øien of the Central Bureau of Statistics was responsible for the work on MODIS II. The present description is based on Mr. Øien's presentation of the model in a Working Paper of the Central Bureau of Statistics (Series No. 10 66/3). The description of the administrative process is based on information supplied by the Ministry of Finance. See e.g. Royal Norwegian Ministry of Finance, *Concepts and Method in National Budgeting*, Oslo 1965.

[3] See Per Sevaldson, "An interindustry model of production and consumption for economic planning in Norway", in ed. Colin Clark and Geer Stuvel, *Income Redistribution and the Statistical Foundations of Economic Policy*, Income and Wealth, Series X, Bowes and Bowes, London 1964.

2. General Description

(a) *Basic philosophy of the model*

The model is based on the following assumptions:

1. Constant-price input-output coefficients are fixed.
2. Product prices are the sum of input prices, weighted by the fixed coefficients, plus profits per unit output subject to modification by indirect taxation.
3. Incomes are determined by production (through the input requirements), by unit labour costs (which depend in turn on wage and productivity rates), and by unit profits.
4. Private consumption is determined by disposable income and prices.

In order to formulate a model on these assumptions the following coefficients are needed:

5. Input-output coefficients for all production sectors.
6. A set of coefficients characterizing the relationships between incomes and prices on one hand, and private consumption of goods and services on the other.
7. A number of "transformation" coefficients, needed mainly for the transformation of exogenous estimates to the specifications of the model.

In order to make a model based on these assumptions determinate, the following variables must be given as exogenous estimates.

8. For each sector either the quantity of production or final demand other than private consumption must be given as an exogenous estimate.

9. For each sector the product price or the profit margin must be given as an exogenous estimate.
10. The prices of labour and other non-produced inputs (e.g. imports) must be given as exogenous estimates.
11. Tax rates must be given exogenously.

The determinate model gives estimates of:

12. Production levels and all deliveries from each sector of production, in both quantities and values.
13. All items of private consumption in quantities and values.
14. Requirements of labour, imports and other primary inputs in total and in each sector of production, and total value added in fixed prices in each sector and in total.
15. Prices and profit margins for all sectors of production. Price indexes based on fixed or current weights.
16. Income shares [4] and tax revenues.

In MODIS II the following choices have been made in regard to exogenous estimates:

17. For one group of production sectors it is assumed that the volume of production can most easily be estimated on the basis of exogenous information. Next, for some sectors in this group, supplementary imports are assumed to fill any gap remaining between demand and domestic production. For other sectors at least one item in final demand is determinated by the model. Finally, for some sectors it is assumed that intermediate demand for the products of the sector is determined as the difference between

[4] See below, section I (*b*), *Specifications in MODIS II*, (i).

exogenous estimates of production and exogenous estimates of final demand, the assumption of fixed input-output coefficients for the use of these products being abandoned. Sectors for which the volume of production is exogenously estimated are typically sectors where production levels are relatively independent of short run shifts in demand, e.g. agriculture, some sectors where production is mainly determined by capacity and some which are under direct government control.

18. For the remaining production sectors it is assumed that production is determined by demand, and that final demand other than private consumption is given by exogenous estimates.

19. For one group of production sectors, defined independently of the above grouping, it is assumed that their prices are given by world market conditions or by policy decisions and thus have to be estimated outside the model. This is the case for all export sectors, and for many sectors competing with imports in the domestic market, as well as for agriculture, fisheries and a number of sectors dominated by public enterprises. In these sectors profit margins will be residually determined.

20. For the remaining sectors it is assumed that profit per unit produced is determined by conditions in the domestic economy in much the same way as wage rates. These profit margins must be estimated exogenously. Then input costs, profit margins and indirect taxation will determine product prices.

(b) *Data requirements and specifications*

Data requirements

The Norwegian model has been formulated in such a way that:

1. Coefficients of the consumption relationships could be estimated on the basis of national accounts and consumer budget studies for the period 1952-1962.
2. National accounts and supplementary statistics for a "base year" provide the basis for all other data in the model (except, of course, for the exogenous estimates). Input-output coefficients, for instance, are computed on this basis. (When the model is used in national budgeting the base year is the year two years before the budget year.)
3. Model estimates are given in the form of estimated changes (i) from the base year to the "curent" year and (ii) from the "current" year to the "budget" year.[5] In order to find the absolute values for the budget year, therefore, accounts figures for the base year and estimates of exogenous variables both for the "current" year and for the "budget" year are needed.
4. The model has been programmed on an electronic computer (Univac 1107) in such a way that when national accounts for the base year and, in addition, up to ten alternative sets of exogenous variables for the current year and the budget year are fed into the machine, the results are produced in an integrated operation in the form of a booklet containing tables with complete texts, ordered and edited in the form which is most convenient for the user. Unfortunately,

[5] The terms "current" year and "budget" year here refer to the use of the model in national budgeting. In other uses of the model any set of "analysis" years may of course be studied, their relevant characteristics being given by the particular values chosen for the exogenous estimates and by the requirement that structural relationships and data from the "base" year be applicable.

this requires relatively much machine time (10 hours, or 7 hours if only one alternative is computed). Nevertheless, results can be produced from one day to the next.

5. Work on a simplified computation programme is in progress.

Specifications in MODIS II

(i) Internal specifications.

The model distinguishes between:

165 sectors of production
30 "import sectors" or groups of import commodities
9 categories of income shares (depreciation, direct and indirect taxes, subsidies, wages and profit by type of organisation). (These income shares can be further specified by sector or by group of sectors.)

(ii) Specifications for exogenous estimates.

(*a*) Price estimates:

59 estimates of product prices for Norwegian sectors of production
2 estimates of the rates of change of profit margins in the remaining 91 Norwegian sectors of production
29 estimates of import prices
13 estimates of wage rate changes in groups of Norwegian sectors of production
12 estimates of productivity changes in groups of Norwegian sectors of production

(*b*) Tax rate estimates:

20 estimates of indirect tax and subsidy rates
6 estimates of rates for direct taxes, social insurance and direct subsidies

(*c*) Quantity estimates:

49 estimates of production volumes in the same number of Norwegian sectors

380 estimates of final demand for the same number of types of goods of these:

7 estimates of exogenously determined items of private consumption.

18 estimates of items of government consumption

182 estimates of export items

70 estimates of inventory investments

103 estimates of other investment items.

(iii) Result specifications.

When the model is used for planning, the aim is to obtain a consistent set of estimates of key national accounting variables for the plan (budget) year, irrespective of whether these are endogenously or exogenously estimated. The model therefore produces as its end result a complete set of tables of such variables. These variables are to some extent more aggregated than they would be if the full internal specifications were maintained, and they are not systematically divided into exogenous and endogenous variables. The tables give figures in current and fixed prices, both absolute figures for the base year, the current year and the budget year, and figures for the changes between each pair of years, absolutely and per cent. Price indices are given for all items.

(iv) The main tables.

(*a*) Tables of gross national product and its components:

Domestic product by category of expenditure, 16 items

Exports by delivering group of sectors, 14 items

Gross investment by type of capital goods, 13 items

Government consumption, 11 items

Private consumption by type of goods, 17 items
Import by sector, 32 items
Gross national product by sector of production, 72 sectors and sector groups

(*b*) Tables of balance of payments and income accounts:
Balance of payments, 6 items
Income accounts: specification of incomes earned in each of 13 groups of sectors, altogether 120 items

(*c*) Tables of disposable incomes and savings.
Tables giving income and savings figures in total as well as separately for persons, corporations, central and local governments, and social security.

(c) *Price determination in the model*

The set of assumptions on which MODIS II is based makes prices independent of market conditions in the product markets: prices are either exogenously determined, or determined by unit costs together with exogenously given profit margins.

The prices which are not determined entirely outside the model are determined by the exogenous prices through the assumption of fixed input coefficients in production, and by the assumptions about tax and wage rates and profit margins. The model cannot take care of the possibility that entrepreneurs may vary profit margins in order to change prices in response to changes in demand. (Such a possibility must be handled outside the model, e.g. by iterative adjustments in the assumed profit margins.)

Formally, price determination is represented by a submodel which may be solved separately, independent of the quantity estimates, when the exogenous price estimates and estimates of tax, wage (and productivity) rates and profit margins are given.

In the model prices influence the volume and distribution of production and imports through their influence on the volume and distribution of private consumption. Adjustments to prices through substitutions in the production sectors are excluded by the assumption of fixed input-output coefficients.

Prices are also decisive for the level and distribution of private incomes and public revenues.

3. The Use of MODIS II in Government Planning

MODIS II is used as a tool in working out both four-year and one-year plans. But it is realized that the present version has serious shortcomings as a tool for planning over medium and longer terms, particularly because of its failure to account for capacity-production-investment relationships. As a consequence, its use in four-year planning is essentially of an auxiliary nature. Experimental computations with the model are used to investigate certain implications of policy and market alternatives as they emerge in the planning process. But over a four-year period other implications than those spelled out by the model may be of dominating importance. The use of the model has therefore not so far tended to be a focal operation around which planning routines could be organized. This is more nearly the case in the process of one-year planning or "national budgeting".

Standard routines for national budgeting were established before the first formal model, MODIS I, was available in 1960, and they were easily adapted to the use of the models, first MODIS I and now MODIS II.

In the present routines the use of MODIS II is quite crucial, but this has not meant that the model or the model operators have taken over any of the decision functions which were formerly exercised by political and administrative bodies.

The Norwegian National Budget is a government plan for policy in the coming year. This means that it is a declaration of the policy which the government intends to pursue in its recommendations to Parliament and in decision-making under its own authority. The plan is presented to and thoroughly discussed by Parliament, which may express its views on several issues. But the National Budget is not formally acted upon by Parliament until such times during the year as specific government recommendations are placed before it, and then Parliament is not bound to follow the plan.

In Norway planning is thought of as part of the responsibility of ordinary government agencies and the emergence of a separate planning organization with a life of its own, unrelated to the current administration, has been considered undesirable and has been avoided. Consequently, ministries, directorates and a number of other administrative institutions are all involved in the planning process, while the co-ordinating and central analytical work is the responsibility of the Ministry of Finance. Naturally, the various administrative agencies which take part in the planning process tend to single out and specialize the planning functions. In particular, the Ministry of Finance has a comparatively large and high-powered "Economic Department" which is responsible for its work on planning. The other agencies have at least a "contact man", who is responsible for contacts with the Ministry of Finance and with the contact men of other agencies over problems of planning.

A Working Group for the National Budget consisting of eight prominent economists from various branches of the administration, and appointed in their personal capacities by the Ministry of Finance, plays a central role in working out the national budget.

The period of the national budget is the calendar year, and is identical with the period for the fiscal budget, the major vehicle for parliament-approved government policy. However, it has so far not been possible entirely to integrate the process of working out the national budget and that of working out the government fiscal budget.

The process of working out the new national budget starts in the spring of the previous year with the issue of a circular from the Ministry of Finance to all agencies responsible for special aspects of the budget. In this circular general conditions for the plan are laid down; in this way a certain co-ordination in the use of available information is achieved.

In the administrative agencies work on their operating budgets (incomes and outlays covered by the government fiscal budget, etc.) and work on estimates for the national budget are co-ordinated. The national budget estimates are assembled by the various ministries into "sector budgets" referring to their respective fields of responsibility, and are submitted to the Economic Department in the Ministry of Finance for further processing.

The introduction of MODIS I and later of MODIS II led to a considerable reformulation of the estimates incorporated in the sector budgets, particularly in the specification of items. In general the models required estimates in much more detailed classifications than previous methods. However, the more aggregate classifications had often been sums of primary estimates worked out at the detailed level.

On the basis of an analysis of the sector budgets, both in detail and in terms of the implicit macrorelationships, the Economic Department works out preliminary estimates of the exogenous variables to be entered in MODIS II. Several alternatives are normally tried out. Computations are carried out towards the end of July.

The model estimates of endogenous variables are analysed by the Economic Department. They are checked for plausibility in the light of economic relationships that are not covered by the model, such as capacity and manpower restrictions in certain sectors, the possibilities of increases or decreases in inventories, reasonable limits on changes in entrepreneurial incomes, and so forth. The model estimates are also checked against corresponding estimates made directly by the budgeting (planning) agencies and incorporated in the sector budgets. Discrepancies are discussed with the agencies concerned and are removed either because these agencies accept the model results, because it is found that some of the exogenous estimates entered in the model ought to be revised, or because it is agreed that there are conditions that are not reflected in the structure of MODIS II. Discrepancies which cannot be resolved in this way will usually be few and of such a nature that they have to be referred to the Working Group for the National Budget or to the cabinet.

The adjusted estimates are put together in the preliminary budget; this is then reviewed by the Working Group for the National Budget, which makes a confidential report to the government. The report gives an assessment of the economic situation and recommendations for economic policy in the budget year.

In the second half of August the cabinet is able to discuss the report of the Working Group and to make its final decisions on the decisive policy measures and the general structure of the national budget. The cabinet has the final say both on the choice of assumptions about variables which are exogenous to the Norwegian economy and on the policy measures to be proposed. The final government decisions on the fiscal budget proposals for the coming year have to be made at about the same time, and decisions in the two fields can be co-ordinated.

For the document in which the national budget is presented to Parliament to be ready in time, i.e. in early October, all the estimates must be final by the end of August. This implies that a final run of the model must be made not later than the last week of August, before the cabinet has reached its conclusions on all controversial or doubtful points. In order to anticipate the cabinet's decisions and also to help it to decide, the final model computations cover all the most relevant alternatives.

When the results of the cabinet discussions as well as of the final model computations are available, the

Economic Department can finish its draft of the parliamentary report on the national budget. This draft is reviewed chapter by chapter by the Working Group and by the cabinet. The final document on the national budget is normally presented to Parliament at the same time as the fiscal budget, in early October.

MODIS II also plays a role in the follow-up of the budgeted policy: in the first quarter of the budget year a report to Parliament is prepared on the implementation of the national budget. This is about half a year after the policy decisions in the national budget were drafted, and a considerable amount of new information will be available, *inter alia* national accounts for the preceding year based on key indicators for the entire year. The need for adjustments of policy is therefore discussed and the effects of changes in external circumstances and policies are analysed. This analysis is carried out by means of the model.

4. The Formal Framework

(a) *A simplified model*

The system of relationships describing MODIS II is in principle quite simple, since the model is based on a rather simplified representation of a limited number of economic relationships. However, owing to the very detailed specification, and a relatively large number of cases which are given special treatment in the model, a symbolic representation nevertheless becomes rather cumbersome. The best way of introducing the model therefore seems to be to start from a simplified formulation, representing the basic principles, and then to introduce the complicating features one by one as expansions and modifications in the basic model. A simple input-output consumption model could be described by the following set of relationships:

$$
\begin{aligned}
&(1) \qquad & x &= A_x x + c_x + y_x, \\
&(2) \qquad & b &= A_b x + c_b + y_b, \\
&(3) \qquad & r &= \hat{p}_w W x, \\
&(4) \qquad & c &= c^0 + D\left(\frac{1}{\pi} r - r^0\right) + N\left(\frac{1}{\pi} p - i_c\right), \\
&(5) \qquad & p_x' &= p_x' A_x + p_b' A_b + p_w' W, \\
&(6) \qquad & \pi &= p' \frac{1}{\gamma} c^0, \\
&(7) \qquad & \gamma &= i_c' c^0, \\
&(8) \qquad & y &= \text{given}, \\
&(9) \qquad & p_b &= \text{given}, \\
&(10) \qquad & p_w &= \text{given}.
\end{aligned}
$$

Here

x = a column vector of production levels measured in constant-price values. The dimension n_x of this vector is equal to the number of production sectors (industries) in the model.

c_x = a column vector of sector deliveries to private consumption, measured in constant-price values and of dimension n_x.

y_x = a column vector of sector deliveries to final uses other than private consumption (exports, gross investment and government consumption) measured in constant-price values and of dimension n_x.

b = a column vector of imports measured in constant-price values. The dimension n_b of this vector is equal to the number of commodity types in the specification of imports in the model.

c_b = a column vector of import deliveries for private consumption, measured in constant-price values and of dimension n_b.

y_b = a column vector of import deliveries to final uses other than private consumption, measured in constant-price values and of dimension n_b.

r = a column vector of primary income shares (wages, depreciation, charges, indirect taxes and entrepreneurial incomes) measured in current-price values. The dimension n_r of this vector is equal to the number of primary income shares (specified by category and sector groups) in the model.

A_x = a n_x-by-n_x dimensional matrix of "input-output" coefficients, assumed to be constant.

A_b = a n_b-by-n_x dimensional matrix of "import coefficients", assumed to be constant.

W = a n_r-by-n_x dimensional matrix of "income share coefficients", estimated in the base year.

p_x = a column vector of price indices for all production sectors (dimension n_x).

p_b = a column vector of price indices for all import commodity types (dimension n_b).

p_w = a column vector of "price indices" for all incomes shares (dimension n_r). For each income share the price index is defined as current income in all sectors of production divided by the sum for all sectors of base year income times the index of production for the sector concerned.

D = a $(n_x + n_b)$-by-n^r dimensional matrix of "marginal propensities to consume".

N = a $(n_x + n_b)$ by $(n_x + n_b)$ dimensional matrix of price coefficients for consumption.

π = a price for index consumers' goods (a scalar).

i_c = a $(n_x + n_b)$ dimensional column vector with all elements = 1.

$$c = \begin{pmatrix} c_x \\ c_b \end{pmatrix}, \qquad p = \begin{pmatrix} p_x \\ p_b \end{pmatrix}, \qquad y = \begin{pmatrix} y_x \\ y_b \end{pmatrix} \quad \text{etc.}$$

A prime (′) denotes transposition. The superscript^0 denotes base year values. A "cap" (^) denotes that a vector is written as a diagonal matrix. This will also be denoted by a Λ and the symbol of the vector written within parenthesis, i.e.: $\hat{p}_w = \Lambda(p_w)$.

The first set of equations, (1), gives the levels of production in all production sectors (x) as the sum of deliveries to production sectors ($A_x\, x$), deliveries to private consumption (c_x), and deliveries to final uses other than private consumption (y_x). Deliveries to production sectors ($A_x\, x$) are assumed to be proportionate to production in the receiving sectors. The elements of A_x can be estimated on the basis of accounts for a base year.

The second set of equations, (2), gives the levels of demand for all import commodity-types (b) as the sum of deliveries to production sectors ($A_b\, x$), deliveries to private consumption (c_b), and deliveries to final uses other than private consumption (y_b). This implies that all imports are treated as "structural". The simple model has no provision for substitution between domestic and imported goods. The elements of A_b can be estimated on the basis of accounts for a base year.

The third set of equations, (3), gives the primary income shares in current values as determined by the production volumes in all production sectors. The income shares as fractions of total production in each production sector in a base year (W) can be established on the basis of accounts. Now the wage payment per unit produced in a given sector will change as a result of changes in productivity and wage rates. The indirect tax payment per unit produced in a given sector will change with the change in the tax rate calculated on a per unit basis. The depreciation charges per unit produced are assumed to be constant in volume, and will consequently change in proportion to a price index for these charges. Price indices expressing these types of change are assumed to be given. If we also assume changes in entrepreneurial income per unit produced in each sector to be known, then we have all the elements of p_w.

The fourth set of equations, (4), gives private consumption of products from each sector as the sum of consumption of the same product in the base year plus a set of terms depending on the change from the base year in "real income" $\left(\frac{1}{\pi} r - r^o\right)$ — defined as income deflated by a general price index (π) — plus a set of terms depending on the change from the base year in relative prices. c^0 can be found from accounts for a base year. The elements of D and N must be estimated by statistical studies of consumer behaviour.

The fifth set of equations, (5), gives the price relationships which obtain in the model. The price index for products from a given production sector equals the weighted sum of the price indices for all inputs and income shares in the sector, the weights being the fractions the elements constituted of total production in the base year. These fractions are given in the matrices A_x, A_b and W.

The sixth group of equations, (6) and (7), consists of only two. Taken together they give a formula for computing a general consumers' price index on the basis of base year weights.

By (8), (9) and (10) it is assumed that final demand other than private consumption (y), import prices (p_b) and "prices" for income shares (p_w) are given.

The system is solved by first solving (5) for p_x; then π may be computed by (6) and (7), and x, r and c are determined by simultaneously solving (1), (3) and (4). Finally b is computed from (2).

Having solved the equation system (1)-(10) we can also compute the vectors of current-price values: $\widehat{p_x}x$, $\widehat{p_b}b$, $\widehat{p}c$ and $\widehat{p}y$.

To make it easier to keep track of subsequent partitionings of variable vectors and coefficient matrices, we may write the equations (1)-(3) in the following form:

$$\begin{pmatrix} x \\ b \\ r \end{pmatrix} = \begin{pmatrix} A_x \\ A_b \\ \hat{p}_w W \end{pmatrix} x + \begin{pmatrix} c_x \\ c_b \\ 0 \end{pmatrix} + \begin{pmatrix} y_x \\ y_b \\ 0 \end{pmatrix}$$

and equation (5) in the following form:

$$p_x' = (p_x', p_b', p_w') \begin{pmatrix} A_x \\ A_b \\ W \end{pmatrix} = (p_x', p_b', i_r') \begin{pmatrix} A_x \\ A_b \\ \hat{p}_w \mathrm{W} \end{pmatrix}$$

Here

i_r = a n_r-dimensional column vector with all elements = 1.

The last form of (5) illustrates the close relationship between the production-income equations (1)-(3) and the price equations (5), through the identity of the coefficient matrices. This identity applies in all but the last of the following modifications of the model, and even in the last modifications the differences are not very striking.

(b) *Specification of final demand; "commodity converters"*

All variables in the model referring to goods delivered are measured in (current or base year) producers' market prices. This implies that trade and transportation margins on all goods delivered are entered as parallel deliveries from the trade and transportation sectors. The vector c, for instance, contains deliveries to private consumption from each sector at producers' prices, and separate items consisting of the trade and transportation margins on these deliveries.

Our first modification of the simple model will be to introduce "commodity converter matrices", which convert a vector of demands for private consumption and a vector of demands for final uses other than private consumption — both specified by classes of commodities and evaluated at (constant) purchasers' prices — into a vector of deliveries required from production sectors and import classes, evaluated at constant producers' prices.

$$
\begin{array}{lll}
(1') & x & = A_x x + C_x z + F_x u, \\
(2') & b & = A_b x + C_b z + F_b u, \\
(3') & r & = \hat{p}_w W x, \\
(4') & z & = z^0 + D\left(\frac{1}{\pi} r - r^0\right) + N\left(\frac{1}{\pi} p_z - i_z\right), \\
(5') & p_x' & = p_x' A_x + p_b' A_b + p_w' W, \\
(6a') & p_z' & = p_x' C_x + p_b' C_b, \\
(6b') & p_u' & = p_x' F_x + p_b' F_b, \\
(6c') & \pi & = p_z' \frac{1}{\gamma} z^0, \\
(7') & \gamma & = i_z' z^0, \\
(8') & u & = \text{given}, \\
(9') & p_b & = \text{given}, \\
(10') & p_w & = \text{given}.
\end{array}
$$

Here

z = a column vector of deliveries of consumers' goods measured in constant purchasers' prices (dimension n_z).

u = a column vector of deliveries of final demand goods other than private consumers' goods, measured in constant purchasers' prices (dimension n_u).

C_x = a n_x-by-n_z matrix of coefficients characterizing the composition of each consumers' good in terms of the fractions delivered by each production sector.

F_x = a n_x-by-n_u matrix of coefficients characterizing the composition of each final good other than consumers' goods in terms of the fractions delivered by each production sector.

C_b = a n_b-by-n_z matrix of coefficients characterizing the composition of each consumers' good in terms of the fractions delivered from each import commodity-type.

F_b = a n_b-by-n_u matrix of coefficients characterizing the composition of each final good other than consumers' goods in terms of the fractions delivered from each import commodity type.

i_z = a n_z-dimensional column vector with all elements = 1.

C_x, F_x, C_b and F_b are assumed to be constant, and estimated on the basis of base year accounts.

The equation of private consumption, (4), must be reformulated in terms of z instead of in terms of c.

The solution procedure is the same as for the simplified model. First, prices (p_x) can be computed by solving (5'); then p_z, p_u and π can be computed from (6a'), (6b'), (6c') and (7'). x, r and z are determined by simultaneously solving (1'), (3') and (4'), and b is computed from (2').

Equations (1') - (3') now may be written

$$\begin{pmatrix} x \\ b \\ r \end{pmatrix} = \begin{pmatrix} A_x & C_x & F_x \\ A_b & C_b & F_b \\ \hat{p}_w W & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ z \\ u \end{pmatrix}$$

and equations (5'), (6a'), (6b')

$$(p_x', p_z', p_u') = (p_x', p_b', p_w') \begin{pmatrix} A_x & C_x & F_x \\ A_b & C_b & F_b \\ W & 0 & 0 \end{pmatrix}$$

(c) *Indirect taxes*

The treatment of indirect taxes in the model will now be considered in more detail. Indirect taxes may be specified as a subvector of r. As sector deliveries are assumed to be given in producers' market prices, indirect taxes collected from the producers are included in the value of production of each sector, and the amount of each tax collected per unit value produced in the base year is characterized by an element in W. However, indirect taxes collected in trade are not included in the product deliveries from the ordinary production sectors, but are included in the deliveries from the trade sectors to the receivers of the goods on which the taxes are levied. For example, a general sales tax on consumers' goods is represented as deliveries from the trade sectors to private consumers' goods. The revenue from such a tax is represented in the matrix W by an item giving the revenue as a fraction of the total "deliveries" from the relevant trade sector in the base year. If we subdivide the trade sector functionally in such a way that the collection of each of the types of indirect taxes which are collected in trade is considered to be made in a separate subsector of trade which has no other function than the collection of this tax, then this fraction must be identically one.

Now indirect taxes may be collected on a quantity or on a value basis, and some indirect taxes, in particular subsidies — which may be considered as negative taxes — are levied in amounts which are independent both of the volume and the value of operations. If indirect taxes are collected on a quantity basis, and if all rates for a given tax are changed in the same proportion, then this change may be expressed by an index giving the (average) rate per quantity unit in a given year divided by the (average) rate per quantity unit in the base year. Such indices will then be elements of the vector p_w. They will not make necessary any reformulations in the simple model, either for the indirect taxes collected from the producers, or for those collected in trade. If the introduction of a new quantity tax which was not collected in the base year is contemplated, then it will be necessary to try to compute what this tax would have amounted to, if it had been collected on base year quantities at the contemplated rate. On this basis, items in an expanded r^0 vector and W matrix can be computed. The price index to be applied to the extra element of r is identically one.

For taxes collected on a value basis the situation is different: these items must be related to price or current value figures and not to quantity (constant-price value) figures. To account for these taxes the vector of income shares (r) is partitioned into three subvectors, r_1, r_2 and r_3, and p_w is correspondingly subdivided into p_{w1}, p_{w2}, and p_{w3}, and W into W_1, W_2 and W_3, in such a way that p_{w1} are the elements of p_w and W_1 are the rows of W corresponding to r_1, and so on.

r_1 contains all the incomes shares which are not the revenues of taxes collected on a value basis. So still

$$r_1 = \hat{p}_{w1} W_1 x \qquad (3a'')$$

In r_2 are the revenues from taxes collected from the production sectors on the basis of the value of production. For r_2,

$$r_2 = \hat{v} W_2 (\widehat{p_x x}) \qquad (3b'')$$

Here v is a vector of "tax rate indices" expressing the relative changes in rates from the base year. It replaces the vector p_w in (3′). v is assumed to be given by government decree.[6] Finally, the vector x in (3′) is substituted in this subset of equations by $(\widehat{p_x x})$, i.e. the volume figures are substituted by current-price values. (New taxes may be taken into account by expanding the r_2 vector to include items for the revenue of the new tax, and by entering the new tax coefficients in the corresponding places of an expanded $\hat{v} W_2$ matrix in equations ($3b''$).)

r_3 shows the revenue from indirect taxes collected on a value basis in trade. r_3 will be taken up again shortly. With the partitioning of W into W_1, W_2 and W_3, price equations (5′) can now be written

$$p_x' = p_x' A_x + p_b' A_b + p_{w1}' W_1 + v' W_2 \hat{p}_x + p_{w3}' W_3$$

This may also be written

$$p_x' = p_x' A_x + p_b' A_b + p_{w1}' W_1 + p_x' \Lambda (v' W_2) + p_{w3}' W_3,$$

or

[6] A complication occurs when not all the deliveries from a given sector are taxed. In this case the indices of tax rate changes (v) will not be the actual change in rates for that part of production which is taxed, but have to be derived from these rates. These calculations will not be described here.

$$p'_x \, [I - A_x - \Lambda \, (v' W_2)] = p'_b \, A_b + p'_{w1} \, W_1 + p'_{w3} \, W_3$$

Here I stands for a diagonal unit matrix of appropriate dimension (here n_x by n_x).

Taxes collected in trade on goods delivered to final uses are represented in the base year by elements of C_x and F_x, whether they are collected on a quantity or on a value basis. If they are collected on a value basis it makes little sense to compute the revenue on a "constant price" basis and to compute "price indices" for these concepts.

In order to be able to form the price equations for final goods, it is assumed instead that C_x and F_x may be partitioned horizontally into C_1 and C_2 and F_1 and F_2 respectively, by collecting in C_2 and F_2 those rows of the respective matrices which relate to indirect taxes on value. With a corresponding partitioning of p_x, the price equations ($6a'$) and ($6b'$) become instead

($6a''$) $$\begin{aligned} p'_z &= p'_1 \, C_1 + s' \, C_2 \, \hat{p}_z + p'_b \, C_b \\ &= p'_1 \, C_1 + p'_z \, \Lambda \, (s' \, C_2) + p'_b \, C_b, \end{aligned}$$

($6b''$) $$\begin{aligned} p'_u &= p'_1 \, F_1 + s' \, F_2 \, \hat{p}_u + p'_b \, F_b \\ &= p'_1 \, F_1 + p'_u \, \Lambda \, (s' \, F_2) + p'_b \, F_b. \end{aligned}$$

Here s is a vector of indices of changes in the value tax rates. s is assumed to be given, in the same way as v.

Corresponding to the partitioning of C_x and F_x is a partitioning of x into x_1 and x_2, where x_2 represents the "volumes of production" in those functionally defined trade sectors that are charged with the (sole) task of collecting these taxes. Correspondingly, p_x is partitioned into p_1 and p_2. x_2 and p_2 are of little interest separately, but the current value of the tax revenue is given by

$$p_2 \, x_2 = \hat{s} \, C_2 \, (\hat{p}_z \, z) + \hat{s} \, F_2 \, (\hat{p}_u \, u).$$

Since the whole activity in this group of sectors is the collection of the tax and since the taxes collected belong to the income share vector, these will also be the remaining elements of r, i.e. r_3.

($3c''$) $$r_3 = \hat{s} \, C_2 \, (\hat{p}_z \, z) + \hat{s} \, F_2 \, (\hat{p}_u \, u) \; (= \hat{p}_{w3} \, W_3 \, x).$$

Corresponding to the partitioning of x into x_1 and x_2, W_1, W_2 and W_3 may also be partitioned vertically into W_{11}, W_{12}, W_{21}, W_{22}, W_{31} and W_{32}, to get

$$W_1 x = W_{11}\, x_1 + W_{12}\, x_2,$$
$$W_2 x = W_{21}\, x_1 + W_{22}\, x_2,$$
$$W_3 x = W_{31}\, x_1 + W_{32}\, x_2.$$

Here all the elements of W_{31} must be identically zero, as the collection of taxes belonging to r_3 is by definition restricted to the sectors corresponding to x_2. W_{12} and W_{22} must both be zero matrices and W_{32} must be the identity matrix, I, since the corresponding sectors are functionally defined to have the sole task of collecting taxes, r_3.

It has already been mentioned that p_2 is without interest, and the price equations may therefore be written:

$$p_1' = p_1' A_{x1} + p_b' A_{b1} + p_{w1}' W_{11} + v' W_{21} \hat{p}_1,$$
$$(5'')\quad p_1' [I - A_{x1} - \Lambda (v' W_{21})] = p_b' A_{b1} + p_{w1}' W_{11}.$$

A_{x1} and A_{b1} are submatrices of A_x and A_b. Partitioning A_x and A_b in correspondence with the partitioning of x and p_x must give:

$$A_x = \begin{pmatrix} A_{x1} & 0 \\ 0 & 0 \end{pmatrix}, \quad A_b = (A_{b1}, 0).$$

(New taxes on the value of final deliveries and collected in trade can be handled in a way that is quite similar to the treatment of new taxes on the value of production.)

The treatment of value taxes collected in trade and levied on the use in production of certain raw materials has not been discussed. Since the revenue from such a tax per unit produced in the sector using the taxed raw material depends only on the quantity of the rax material used per unit of production and the price of the raw material, it might easily have been taken into account. Nevertheless, it was thought more convenient in MODIS II to treat all taxes collected in trade on the use of raw materials as if they were levied on a quantity basis. If this approximation is taken into account when the "price indexes" for these taxes are computed, the consequent errors in calculations will be negligible.

Subsidies and taxes levied independently of value and volume are also roughly recomputed on a per unit basis in the Norwegian model.

The modified system of equations may now be written out as follows:

$$(1'')\quad x_1 = A_{x1}\, x_1 + C_1 z + F_1 u,$$

$$(2'')\quad b = A_{b1}\, x_1 + C_b z + Fb\, u,$$

$$(3a'')\quad r_1 = \widehat{p}_{w1}\, W_{11}\, x_1,$$

$$(3b'')\quad r_2 = \widehat{v}\, W_{21}\, (\widehat{p}_1\, x_1),$$

$$(3c'')\quad r_3 = \widehat{s}\, C_2\, (\widehat{p}_z z) + \widehat{s}\, F_2\, (\widehat{p}_u\, u),$$

$$(4'')\quad z = z^0 + D\left(\frac{1}{\pi} r - r^o\right) + N\left(\frac{1}{\pi} p_z - i_z\right),$$

$$(5'')\quad p_1'\, [I - A_{x1} - \Lambda\,(v'\, W_{21})] = p_b'\, A_{b1} + p_{w1}'\, W_{11},$$

$$(6a'')\quad p_z'\, [I - \Lambda\,(s'\, C_2)] = p_1'\, C_1 + p_b'\, C_b,$$

$$(6b'')\quad p_u'\, [I - \Lambda\,(s'\, F_2)] = p_1'\, F_1 + p_b'\, F_b,$$

$$(6c'')\quad \pi = p_z' \frac{1}{\gamma} z^0,$$

$$(7'')\quad \gamma = i_z'\, z^0,$$

$$(8'')\quad u = \text{given},$$

$$(9'')\quad p_b = \text{given},$$

$$(10a'')\quad p_{w1} = \text{given},$$

$$(10b'')\quad v = \text{given},$$

$$(10c'')\quad s = \text{given}.$$

Equations (1″) — (3c″) can be written

$$\begin{pmatrix} x_1 \\ r_3 \\ b \\ r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} A_{x1} & C_1 & F_1 \\ 0 & \widehat{s}\, C_2\, \widehat{p}_z & \widehat{s}\, F_2\, \widehat{p}_u \\ A_{b1} & C_b & F_b \\ \widehat{p}_{w1}\, W_{11} & 0 & 0 \\ \widehat{v}\, W_{21}\, \widehat{p}_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ z \\ u \end{pmatrix}$$

and equations (5″) — (6b″)

$$(5b''') \qquad p'_{II} = p'_{II}\,A_{I\,II} + p'_{II}\,A_{II\,II} + p'_{II}\,\Lambda(v'\,W_{2\,II}) + p'_b\,A_{bII} + p'_{w1}\,\overline{W}_{1\,II} + e'.$$

The vector e now represents the entrepreneurial incomes per unit produced in each of those production sectors for which the product prices are given by p_{II}.

The incomes equation ($3a''$) must now be replaced by

$$(3a''') \qquad r_1 = \hat{p}_{w1}\,W_{1\,I}\,x_I + \hat{p}_{w1}\,W_{1\,II}\,x_{II} + E\,\hat{e}\,x_{II}.$$

Here E is a matrix which adds up the elements of $e\,x_{II}$ and puts them into the right category of incomes shares in r_1. x_I and x_{II} are the subvectors of x_1 resulting from a partitioning of x_1 corresponding to the partitioning of p_1 into p_I and p_{II}.

p_{II} will be assumed to be given exogenously:

$$(10d''') \qquad p_{II} = \text{given}.$$

The price indices of p_{II} may represent prices of goods that compete on the world market or prices which are determined directly by the government.

p_I will now be determined uniquely by ($5a'''$).

(e) *Wages and productivity*

Next, those elements of p_{w1} which represent the indexes of labour cost per unit produced will be discussed in more detail. It is obvious that there must be at least as many labour income elements in r_1 as there are sector groups for which separate indices of changes in labour cost per unit produced are to be estimated.

An index of unit labour cost will be determined by the average change in the wage rate and the average change in labour productivity in the sector group concerned.

Let

p_k = the subvector of p_{w1} containing the indexes of unit labour costs.

Let

k = the vector of wage indexes,

and let

$$(p_1', p_z', p_u') = (p_1', s', p_b', p_w', v') \begin{pmatrix} A_{x1} & C_1 & F_1 \\ 0 & C_2 \hat{p}_z & F_2 \hat{p}_u \\ A_{b1} & C_b & F_b \\ W_{11} & 0 & 0 \\ W_{21}\hat{p}_1 & 0 & 0 \end{pmatrix}$$

(d) *Endogenous and exogenous prices*

For the next modification of the model the starting point is equation (5″), written in the following form:

$$p_1' = p_1' A_{x1} + p_1' \Lambda (v' W_{21}) + p_b' A_{b1} + p_{w1}' W_{11}.$$

p_1 is now partitioned in to p_{I} and p_{II}, i.e. $p_1' = (p_{\text{I}}', p_{\text{II}}')$. Correspondingly, A_{x1} is partitioned into $A_{\text{I I}}$, $A_{\text{I II}}$, $A_{\text{II I}}$ and $A_{\text{II II}}$, A_{b1} into $A_{b\text{I}}$ and $A_{b\text{II}}$, W_{11} into $W_{1\,\text{I}}$, and $W_{1\,\text{II}}$, and W_{21} into $W_{2\,\text{I}}$ and $W_{2\,\text{II}}$, i.e.

$$A_{x1} = \begin{pmatrix} A_{\text{I I}} & A_{\text{I II}} \\ A_{\text{II I}} & A_{\text{II II}} \end{pmatrix},$$

$$A_{b1} = (A_{b\text{I}}\ A_{b\text{II}}),$$

$$\begin{pmatrix} W_{11} \\ W_{21} \end{pmatrix} = \begin{pmatrix} W_{1\,\text{I}} & W_{1\,\text{II}} \\ W_{2\,\text{I}} & W_{2\,\text{II}} \end{pmatrix}$$

Then

(5a‴)
$$p_{\text{I}}' = p_{\text{I}}' A_{\text{I I}} + p_{\text{II}}' A_{\text{II I}} + p_{\text{I}}' \Lambda \\ (v' W_{21}) + p_b' A_{b\text{I}} + p_{w1}' W_{1\,\text{I}},$$

$$p_{\text{II}}' = p_{\text{I}}' A_{\text{I II}}, + p_{\text{II}}' A_{\text{II II}} + p_{\text{II}}' \Lambda \\ (v' W_{2\,\text{II}}) + p_b' A_{b\text{II}} + p_{w1}' W_{1\,\text{II}}.$$

Finally, zeros are substituted for the coefficients in the row(s) corresponding to entrepreneurial incomes in $W_{1\,\text{II}}$ (but not in $W_{1\,\text{I}}$) and a vector *e* of the same dimension as p_{II} is added in the second equation. Writing $\overline{W}_{1\,\text{II}}$ for the matrix with the zero row(s),

g = the vector of productivity changes,
then

(10e''') $p_k = (\hat{g})^{-1}k.$

g and k will be assumed to be given.

(f) *Price index of depreciation charges*

In the Norwegian model entrepreneurial incomes are computed net of depreciation charges. This means that the sum of depreciation charges is a separate element in r_1 and that a price index is needed for this element.

The index used is computed on the basis of the prices for investment goods, i.e. some of the elements in p_u.

$$(6d''') \qquad p_d = d' p_u.$$

p_d will be an element of the vector of price indices for incomes shares, p_{w1}.

(g) *The system of price equations*

Writing p_{w0} for the part of the vector p_{w1} remaining when p_k and p_d are removed, writing $W_{k\mathrm{I}}$, $W_{k\mathrm{II}}$ and $W_{d\mathrm{I}}$ and $W_{d\mathrm{II}}$ for the rows of $W_{1\,\mathrm{I}}$ and $\overline{W}_{1\,\mathrm{II}}$ corresponding to p_k and p_d respectively (the "labour coefficients" and the "depreciation coefficients"), and writing $W_{0\mathrm{I}}$ and $W_{0\mathrm{II}}$ for the remaining rows of $W_{1\,\mathrm{I}}$ and $\overline{\mathrm{W}}_{1\,\mathrm{II}}$, the price equations of the system can now be written:

$$(5a^{iv}) \quad p'_{\mathrm{I}} = p'_{\mathrm{I}} A_{\mathrm{I\,I}} + p'_{\mathrm{II}} A_{\mathrm{II\,I}} + p'_{\mathrm{I}} \Lambda (v' W_{2\mathrm{I}}) + P'_b A_{b\mathrm{I}} + p'_{w0} W_{0\mathrm{I}} + p'_k W_{k\mathrm{I}} + p_d W_{d\mathrm{I}},$$

$$(5b^{iv}) \quad p'_{\mathrm{II}} = p'_{\mathrm{I}} A_{\mathrm{I\,II}} + p'_{\mathrm{II}} A_{\mathrm{II\,II}} + p'_{\mathrm{II}} \Lambda (v' W_{2\mathrm{II}}) + p'_b A_{b\mathrm{II}} + p'_{w0} W_{0\mathrm{II}} + p'_k W_{k\mathrm{II}} + p_d W_{d\mathrm{II}} + e',$$

$$(6a^{iv}) \quad p'_z \; [I - \Lambda (s' C_2)] = p'_1 C_1 + p'_b C_b,$$

$$(6b^{iv}) \quad p'_u \; [I - \Lambda (s' F_2)] = p'_1 F_1 + p'_b F_b,$$

$$(6c^{iv})\quad \pi = p_z' \frac{1}{\gamma} z^0,$$
$$(6d^{iv})\quad p_d = d' p_u,$$
$$(7^{iv})\quad \gamma = i_z' z^0,$$
$$(9^{iv})\quad p_b = \text{given},$$
$$(10a^{iv})\quad p_{wo} = \text{given},$$
$$(10b^{iv})\quad v = \text{given},$$
$$(10c^{iv})\quad s = \text{given},$$
$$(10d^{iv})\quad p_{\text{II}} = \text{given},$$
$$(10e^{iv})\quad p_k = (\hat{g})k^{-1},$$
$$(11a^{iv})\quad g = \text{given},$$
$$(11b^{iv})\quad k = \text{given},$$
$$(12^{iv})\quad p_1 = \begin{pmatrix} p_{\text{I}} \\ p_{\text{II}} \end{pmatrix}.$$

This is a determinate subsystem of the entire set of equations. It makes possible the determination of p_{I}, p_k, p_d, p_z, p_u and π from a set of given price, wage, tax and productivity variables.

This subsystem also gives a representation of the actual price relationships in the Norwegian model MODIS II.

In matrix, form equations $(5a^{iv})$ — $(6b^{iv})$ may be written:

$$(p_{\text{I}}', (p_{\text{II}} - e)', p_z', p_u') =$$

$$= (p_{\text{I}}', p_{\text{II}}', s', p_b', p_{w0}', p_k', p_d, v') \begin{pmatrix} A_{\text{I I}} & A_{\text{I II}} & C_{\text{I}} & F_{\text{I}} \\ A_{\text{II I}} & A_{\text{II II}} & C_{\text{II}} & F_{\text{II}} \\ 0 & 0 & C_2 \hat{p}_z & F_2 \hat{p}_u \\ A_{b\text{I}} & A_{b\text{II}} & C_b & F_b \\ W_{0\text{I}} & W_{0\text{II}} & 0 & 0 \\ W_{k\text{I}} & W_{k\text{II}} & 0 & 0 \\ W_{d\text{I}} & W_{d\text{II}} & 0 & 0 \\ W_{2\text{I}}\hat{p}_{\text{I}} & W_{2\text{II}}\hat{p}_{\text{II}} & 0 & 0 \end{pmatrix}$$

(h) *Exogenous estimates of production levels*

A new column vector $j = (j'_{I}, j'_{II})$ will now be added in the quantity and income relationships, giving:

$$(1a^{iv})\quad x_{I} = A_{I\,I}\, x_{I} + A_{I\,II}\, x_{II} + C_{I}\, z + F_{I}\, u + j_{I},$$

$$(1b^{iv})\quad x_{II} = A_{II\,I}\, x_{I} + A_{II\,II}\, x_{II} + C_{II}\, z + F_{II}\, u + j_{II},$$

$$(2^{iv})\quad b = A_{bI}\, x_{I} + A_{bII}\, x_{II} + C_{b}\, z + F_{b}\, u + K_{bI}\, j_{I} + K_{bII}\, j_{II},$$

$$(3a^{iv})\quad r_{o} = \hat{p}_{w0}\, W_{0I}\, x_{I} + \hat{p}_{w0}\, W_{0II}\, x_{II} + E\, \hat{e}\, x_{II} + K_{0I}\, \hat{p}_{I}\, j_{I} + K_{0II}\, \hat{p}_{II}\, j_{II},$$

$$(3b^{iv})\quad r_{k} = \hat{p}_{k}\, W_{kI}\, x_{I} + \hat{p}_{k}\, W_{kII}\, x_{II},$$

$$(3c^{iv})\quad r_{d} = p_{d}\, W_{dI}\, x_{I} + p_{d}\, W_{dII}\, x_{II},$$

$$(3d^{iv})\quad r_{2} = \hat{v}\, W_{2\,I}\, \hat{p}_{I}\, x_{I} + \hat{v}\, W_{2\,II}\, \hat{p}_{II}\, x_{II},$$

$$(3e^{iv})\quad r_{3} = \hat{s}\, C_{2}\, \hat{p}_{z}\, z + \hat{s}\, F_{2}\, \hat{p}_{u}\, u,$$

$$(8a^{iv})\quad y_{I} = F_{I}\, u + M_{I}\, j_{I},$$

$$(8b^{iv})\quad y_{II} = F_{II}\, u + M_{II}\, j_{II},$$

$$(8c^{iv})\quad u = \text{given.}$$

The elements of j fall into two categories: either they are identically zero, in which case equations ($1a^{iv}$) and ($1b^{iv}$) are used to determine the corresponding elements of $x = (x'_{I}, x'_{II})'$; or they are unknowns, to be determined by equations ($1a^{iv}$) and ($1b^{iv}$). The elements of x corresponding to nonzero elements of j must be exogenously given. They are the production levels that are considered to be determined by conditions of nature, or by the availability of factors of production.

The existence of nonzero elements of j implies that the remaining terms on the right hand sides of equations ($1a^{iv}$) and ($1b^{iv}$) do not account for the total of domestic production. The balancing items, the nonzero elements of j, are interpreted in three alternative ways:

(i) Some of them are considered to be adjustments in the exogenous estimates of inventory changes. In these cases final demand other than private consumption is not entirely exogenously determined, as is indicated by equations ($8a^{iv}$) and ($8b^{iv}$). (M_{I} and M_{II} are diagonal matrices with 1's and 0's on the diagonals.)

(ii) Some nonzero elements of j are considered to represent import substitutions, so that a positive element in j represents surplus supply from domestic production which is available for the replacement of a corresponding amount of imports, and a negative element in j represents the need for supplementary imports to cover demand which, if output could have been further expanded, would have been supplied from domestic production. This interpretation must have consequences for the import estimates. The matrices $K_{b\text{I}}$ and $K_{b\text{II}}$ in equation (2^{iv}) pick out those elements of j that are to be considered as affecting imports, and account for them in the import estimates of the model.

(iii) The third alternative is to consider the nonzero elements of j to be adjustments to the estimates of demand for inputs in the production sectors implied by the fixed coefficients in A. This interpretation implies that one or more of the production sectors use more or less of the product in question than it would have done if input-output coefficients were fixed. The corresponding saving or dissaving in production costs must be reflected in entrepreneurial incomes. Since there is no attempt to identify the sectors in which the changes of coefficients occur, neither can the corresponding adjustment in entrepreneurial income be specified by sector. (If the exogenous estimates of entrepreneurial income per unit produced in sectors with endogenous price determination are to be rigorously maintained, it must be assumed that the adjustments discussed here only affect sectors with exogenous price determination, where entrepreneurial income is residually determined.) The matrices $K_{0\text{I}} \hat{p}_{\text{I}}$ and $K_{0\text{II}} \hat{p}_{\text{II}}$ yield the cost savings, and these are added to those elements in r_0 which represent entrepreneurial incomes.

Written out in matrix form, the quantity and income equations become:

$$\begin{pmatrix} x_{I} \\ x_{II} \\ r_3 \\ b \\ r_o \\ r_k \\ r_d \\ r_2 \end{pmatrix} = \begin{pmatrix} A_{I\,I} & A_{I\,II} & C_{I} & F_{I} & I & 0 \\ A_{II\,I} & A_{II\,II} & C_{II} & F_{II} & 0 & I \\ 0 & 0 & s\hat{C}_2\hat{p}_z & s\hat{F}_2\hat{p}_u & 0 & 0 \\ A_{bI} & A_{bII} & C_b & F_b & K_{bI} & K_{bII} \\ \hat{p}_{wo} W_{oI} & (\hat{p}_{wo} W_{oII} + \hat{E} e) & 0 & 0 & K_{oI}\hat{p}_{I} & K_{oII}\hat{p}_{II} \\ \hat{p}_{wk} W_{hI} & \hat{p}_{wk} W_{kII} & 0 & 0 & 0 & 0 \\ p_{wd} W_{dI} & p_{wd} W_{dII} & 0 & 0 & 0 & 0 \\ \hat{v} W_{2I}\hat{p}_{I} & \hat{v} W_{2II}\hat{p}_{II} & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{I} \\ x_{II} \\ z \\ u \\ j_{I} \\ j_{II} \end{pmatrix}$$

(i) *Private consumption*

The final modification is in the consumption equation [(4), (4′), (4″)]. Here some items of private consumption are assumed to depend on other determinants than income and prices. These items are taken out of the vector z and are included among the exogenously estimated items of final demand, u. Except for the redefinition of the vectors z and u and some of the coefficients, which this implies, there are no effects for the formal presentation of the model. Further, disposable income is now substituted for gross income.

Finally, since it has not been possible to establish reliable estimates of marginal propensities to consume by income groups or by social groups in Norway, the matrix D has been replaced by one column (d) of estimates of marginal propensities to consume for the population as a whole. Only wage (r_k) and entrepreneurial incomes (r_0) are used in the consumption function; this gives:

$$(4a^{iv})\quad z = z^0 + d\left[\frac{1}{\pi}(i_k r_k^0 - T_k) - (i_k r_k^0 - T_k^0) + \frac{1}{\pi}(i_0 r_0 - T_0) - (i_0 r_0^0 - T_0^0)\right] + N\left(\frac{1}{\pi} p_z - i_z\right),$$

where

d = a column vector of marginal propensities to consume,

T_k = total direct taxes on labour income (wages),

T_0 = total direct taxes on entrepreneurial incomes.

In Norway the tax system is such that taxes on entrepreneurial incomes actually to be collected in the course of a given year are determined by the end of the preceding year, namely as the preliminary "advance tax" for the year plus or minus the adjustments due to final settlements for the preceding year. For one-year forecasts, taxes on entrepreneurial incomes are consequently considered to be exogenously given.

For wage and salary income, preliminary taxes depend on the current level of income. Thus

$$(4b^{iv})\quad T_k = \tau_m i_k\,(\widehat{k} - I)\,(\widehat{g})^{-1}\,W_k x^0 + \tau_g i_k\,\{\widehat{p}_k\,W_k\,(x - x^0) + [(\widehat{g})^{-1} - I]\,W_k x^0\} + T_{kc},$$

$$(4c^{iv})\quad T_0 = \text{given.}$$

Here

τ_m = an estimate of the marginal tax rate for wage and salary earners (in the "budget year"). It is applied to the increase in wage and salary income which would occur with the assumed changes in wage rates and productivity, if production in all sectors remained unchanged. This may be considered an approximation to the change in wage and salary income due to a change in average income per employed worker.

τ_g = an estimate of the average tax rate for wage and salary earners (in the "budget year"). It is applied to the remainder of the change in wage and salary incomes, which may be considered an approximation to the income change due to the change in employment.

T_{kc} = the tax revenues that would occur if present rates (for the "budget year") were applied to wage and salary incomes realized in the base year. T_{kc} can be estimated with a comparatively high degree of accuracy.

(k) *Incremental form of the equations*

Equation ($4b^{iv}$) is written in incremental form in x. It is easily seen that equation ($4a^{iv}$), which is in incremental form in z, can be written in incremental form in x by reformulation of ($3a^{iv}$) and ($3b^{iv}$) and insertion.

Finally, ($1a^{iv}$), ($1b^{iv}$) and (2^{iv}) can be written in incremental form in x_{I}, x_{II}, z, u and b; x_{I}^0, x_{II}^0, z^0, u^0, and b^0 disappear if the same equations are assumed to be valid for the base year with j identically zero.

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